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DEVELOPMENT OF OPTIMIZED STRENGTH MODEL OF LATERITIC HOLLOW BLOCK WITH 4% MOUND SOIL INCLUSION

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ABSTRACT
The work is an investigation to develop and optimize a model of the compressive strength of lateritic hollow sandcrete block with mound soil inclusion. The study applies the Scheffe's optimization approach to obtain a mathematical model of the form $f(x_{i1},x_{i2},x_{i3},x_{i4})$, where x_i are proportions of the concrete components, viz: cement, mound soil, laterite, and water. Scheffe's experimental design techniques are followed to mould various hollow block samples measuring 450mm x 225mm x 150mm and tested for 28 days strength. The task involved experimentation and design, applying the second order polynomial characterization process of the simplex lattice method. The model adequacy is checked using the control factors. Finally a purpose-made software was applied to optimize the design computation process which takes the desired property of the mix, and generates the optimal mix ratios and conversely takes the desired mix ratio to generate the optimal compressive strength.

Keywords: *sandcrete, pseudo-component, simplex-lattice, optimization, ternary mixture.*

1. INTRODUCTION
The cost/stability of material has been a major issue in construction of structures where cost is a major index. This means that the locality and the usability of the available materials directly impact on the achievable development in any area as well as the attainable level of construction technology in the area.
As it is, concrete is the main material of construction, and the ease or cost of its production accounts for the level of success in the area of environmental upgrading involving the construction of new roads, buildings, dams, water structures and the renovation of such structures. To produce the concrete several primary components such as cement, sand, gravel and some admixtures are to be present in varying quantities and qualities. Unfortunately, the quality and availability of these components vary very randomly with location and hence the attendant problems of either excessive or scarce quantities of the different materials occurring in different areas. In situations where the scarcity of one component prevails exceedingly, the cost of the concrete production increase geometrically. Such problems obviate the need to seek alternative materials for partial or full replacement of the scarce component when it is possible to do so without losing the quality of the concrete.

1.1. Optimization Concept
Every venture that must be successful in human endeavour requires planning, the target of which is to maximize the desired outcome of the venture. As emphasized in their work on Orie and Osadebe [2], in order to maximize gains or outputs it is often necessary to keep inputs or investments at a minimum at the production level. The process involved in this planning activity of minimization and maximization is referred to as optimization. In the science of optimization, the desired property or quantity to be optimized is referred to as the objective function. The raw materials or quantities whose amount of combinations will produce this objective function are referred to as variables or constraints.
The variations of these variables produce different combinations and have different outputs. Often the space of variability of the variables is not universal as some conditions limit them. These conditions are called constraints. For example, money is a limited

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factor of production and hence, a constraint given to the entrepreneur at the time of investment. Hence, according to Majid [3], an optimization process is one that seeks for the maximum or minimum value and at the same time satisfying a number of other imposed requirements.

Everybody can make concrete but not everybody can make structural concrete which is made with specified materials for specified strength. Concrete is heterogeneous as it comprises sub-materials. Concrete is made up of fine aggregates, coarse aggregates, cement, water, and sometimes admixtures e.g. rice husk ash, mound soil, lime, etc. David and Galliford [4], report that modern research in concrete seeks to provide greater understanding of its constituent materials and possibilities of improving its qualities. For instance, Portland cement has been partially replaced with ground granulated blast furnace slag (GGBS), a by–product of the steel industry that has valuable cementitious properties [5].

1.2 Concrete Mix optimization

The task of concrete mix optimization implies selecting the most suitable concrete aggregates from the data base (Genadij and Juris, [6]. Several methods have been applied. Examples are by Mohan et al [7], Simon [8], Lech et al [9], Czarneki et al [10]. Nordstrom and Munoz [11] proposed an approach which adopts the equilibrium mineral assemblage concept of geochemical thermodynamics as a basis for establishing mix proportions. Bloom and Bentur [12] reports that optimization of mix designs require detailed knowledge of concrete properties. Orie. and Osadebe [2] adapted the simplex lattice design theory by Scheffe [1] to the design of concrete five-components mix including the mound soil admixture.

Low water-cement ratios lead to increased strength but will negatively lead to an accelerated and higher shrinkage. Also, apart from the larger deformations, the acceleration of dehydration and strength gain will cause cracking at early ages.

1.3. Modeling

Modeling involves setting up mathematical formulations of physical or other systems. Many factors of different effects occur in nature simultaneously dependently or independently. When they interplay they could inter-affect one another differently at equal, direct, combined or partially combined rates variationally, to generate varied natural constants in the form of coefficients and/or exponents. The challenging problem is to understand and asses these distinctive constants by which the interplaying factors underscore some unique natural phenomena towards which their natures tend, in a single, double or multi phase systems.

For such assessment a model could be constructed for a proper observation of responses from the interaction of the factors through controlled experimentation followed by schematic design where such simplex lattice approach of the type of Scheffe's [1] optimization theory could be employed. Also entirely different physical systems may correspond to the same mathematical model so that they can be solved by the same methods. This is an impressive demonstration of the unifying power of mathematics [13].

2. LITERATURE REVIEW

In the past ardent researchers have done works in the behavior of concrete under the influence of its components. With given proportions of aggregates the compressive strength of concrete depends primarily upon age, cement content, and the cement-water ratio [14]. Majid [3] reports that of all the desirable properties of hardened concrete such as the tensile, compressive, flexural, bond, shear strengths, etc., the compressive strength is the most convenient to measure and is used as the criterion for the overall quality of the hardened concrete.

Macrofaunal activities in soil are known to affect the nutrient and organic matter dynamics and structure of the soil. Such changes in soil properties have profound influences on the productivity of the ecosystem. Termites are the dominant macrofaunal group found in the tropical soils. Termites process considerable quantities of soil materials in their mound building activities. According to Fageria and Baligar [15] such activities might have potential effects on carbon sequestration, nutrient cycling, and soil texture.

The result of a study on some characteristics of laterite-cement mix containing termite mound soil (50% by weight of laterite) as replacement of laterite are presented by Udoeyo and Turman [16]. The study showed that laterite-mound soil mix stabilized with 6% cement could serve as a base course for roads for agricultural trafficking in rural areas where mound soils are abundant.

In the report by Udoeyo et al [17], the inclusion of mound soil in mortar matrix resulted in a compressive strength value of up to 40.08 N/mm^2, and the addition of 5% of mound soil to a concrete mix of 1:2:4:0.56

(cement: sand: coarse aggregate: water) resulted in an increase of up to 20.35% in compressive strength. The availability of mound soil is not regular over areas and, in some areas, quite low, just as the quantity in the mix is comparatively low.

Sandcrete blocks are masonry units used in all types of masonry constructions such as the interior and exterior load bearing and non load bearing walls, fire walls, party walls, curtain walls, panel walls, partition, backings for other masonry, facing materials, fire proofing over structured steel members, piers, plasters columns, retaining walls, chimneys, fireplaces, concrete floors, patio paving units, kerbs and fences. The block is defined by ASIM as hollow block when the cavity area exceeds 25% of the gross cross-sectional area, otherwise it belongs to the solid category. Obodo [18] stated that methods of compaction during moulding has a marked effect on the strength of sandcrete blocks.

2.1 Background Theory

This is a theory where a polynomial expression of any degrees, is used to characterize a simplex lattice mixture components. In the theory only a single phase mixture is covered. The theory lends path to a unifying equation model capable of taking varying mix component variables to fix mixture properties. The optimization that follows selects the optimal ratio from the component ratios list that is automatically generated. This theory is adapted to this work of formulation of response function for the compressive strength of sandcrete block.

2.2 Simplex Lattice

Jackson [19], states that Simplex is a structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atoms) of a mixture, and they are equidistant from each other. According to Akhnazarov and Kafarov [20], when studying the properties of a q-component mixture which are dependent on the component ratio, only the factor space is a regular (q-1)–simplex. Simplex lattice designs are saturated, that is, the proportions used for each factor have m + 1 equally spaced levels from 0 to 1 (x_i = 0, 1/m, 2/m, … 1), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used.

Mathematically, a simplex lattice is a space of constituent variables of X_1, X_2, X_3,……, and X_i which obey these laws:

$$X_i \neq negative, 0 \leq X_i \leq 1, \sum X_i = 1 \qquad (1)$$

That is, a lattice is an abstract space.

Scheffe [1] developed a model in which the response surfaces of the physical and chemical characteristics of a mixture can be approximated by a polynomial of the second degree. Using the approach, Akhnazarova and Kafarov [20] predicted the variations of reactivity and porosity of coke with the charges of four process groups of coal in a mixture. The approach could be adapted to predict the desired strength of concrete where the essential factors lies on the adequate proportioning of ingredients needed to make the concrete. With the compressive strength desired specified, possible combinations of needed ingredients to achieve the compressive strength can easily be predicted by the aid of computer, and if proportions are specified, the compressive strength can easily be predicted.

2.3 Simplex Lattice Method

According to Akhnazarova and Kafarov [20] in designing experiment to solve mixture problems involving component property diagrams the property studied is assumed to be a continuous function of certain arguments and with a sufficient accuracy it can be approximated with a polynomial. When investigating multi-components systems the use of experimental design methodologies substantially reduces the volume of experimental effort. Further, this obviates the need for a special representation of complex surface, as the wanted properties can be derived from equations while the possibility to graphically interpret the result is retained.

As a rule the response surfaces in multi-component systems are very intricate. To describe such surfaces adequately, high degree polynomials are required, and hence a great many experimental trials. A polynomial of degree n in q variable has C^n_{q+n} coefficients. If a mixture has a total of q components and x_i be the proportion of the i^{th} component in the mixture such that,

$$X_i \geq 0 \ \ (i = 1,2,\ldots q), \qquad (2)$$

then the sum of the component proportion is a whole unity, that is

$$X_1 + X_2 + X_3 + X_4 \quad or \quad \sum X_i - 1 = 0 \qquad (3)$$

where i = 1, 2, …., q… Thus the factor space is a regular (q-1) dimensional simplex. In (q-1) dimensional simplex if q = 2, we have 2 points of connectivity. This gives a straight line simplex lattice. If q=3, we have a triangular simplex lattice and for q = 4, it is a tetrahedron simplex lattice, etc. Taking a whole factor space in the design we have a (q,m) simplex lattice whose properties are defined as follows:

i. The factor space has uniformly distributed points,

ii. Simplex lattice designs are saturated.

That is, the proportions used for each factor have m + 1 equally spaced levels from 0 to 1 (x_i = 0, 1/m, 2/m, … 1), and all possible combinations are derived from such values of the component concentrations, tall possible mixtures, with these proportions are used. Hence, for the quadratic lattice (q,2), approximating the response surface with the second degree polynomials (m=2), the following levels of every factor must be used 0, ½ and 1; for the fourth order (m=4) polynomials, the levels are 0, 1/4, 2/4, 3/4 and 1, etc; Scheffe [1] showed that the number of points in a (q,m) lattice is given by:

$$C_{q+m-1} = q(q+1)\ldots(q+m-1)/m! \tag{4}$$

2.4 The (4,2) Lattice Model

The properties studied in the assumed polynomial are real-valued functions on the simplex and are termed responses. The mixture properties were described using polynomials assuming a polynomial function of degree m in the q-variable x_1, x_2,……, x_q, subject to equation (2), and will be called a (q, m) polynomial having a general form:

$$\hat{Y} = b_0 + \sum_{i\le i\le q} b_i X_i + \sum_{i\le i\le j\le q} b_{ij} X_i X_j + \ldots + \sum_{i\le i\le j\le k\le q} b_{ijk} X_i X_j X_k \ldots + \sum b_{i1i2..in} X_{i1} X_{i2} \ldots X_{in} \tag{5}$$

$$\hat{Y} = b_o + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{24}X_2X_4 + b_{23}X_{21}X_3 + b_{34}X_3X_4 + b_{11}X^2{}_1 + b_{22}X^2{}_2 + b_{33}X^2{}_3 + b_{44}X^2{}_4 \tag{6}$$

In (5) and (6), b is a constant coefficient. The relationship obtainable from Eqn (6) is subjected to the normalization condition of Eqn. (3) for a sum of independent variables. For a ternary mixture, applying some mathematical juggling the reduced second degree polynomial comes to

$$\hat{Y} = (b_o + b_1 + b_{11})X_1 + (b_o + b_2 + b_{22})X_2 + (b_o + b_3 + b_{33})X_3 + (b_o + b_4 + b_{44})X_4 + (b_{12} - b_{11} - b_{22})X_1X_2 + (b_{13} - b_{11} - b_{33})X_1X_3 + (b_{14} - b_{11} - b_{44})X_1X_4 + (b_{23} - b_{22} - b_{33})X_2X_3 + (b_{24} - b_{22} - b_{44})X_2X_4 + (b_{34} - b_{33} - b_{44})X_{31}X_4 \tag{7}$$

If we denote

$$\beta_i = b_o + b_i = b_{ii} \; and \; \beta_{ii} = b_i - b_{ii} - b_{ii}$$

then we arrive at the reduced second degree polynomial in 6 variables:

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{23}X_2X_{23} + \beta_{24}X_2X_4 + \beta_{34}X_3X_4 \,. \tag{8}$$

Thus, the number of coefficients has reduced from 15 in (7) to 10 in (8). That is, the reduced second degree polynomial in q variables is

$$\hat{Y} = \sum \beta_i X_i + \sum \beta_{ii} X_i X_i \tag{9}$$

2.5 Construction of Experimental/Design Matrix

From the coordinates of points in the simplex lattice, we can obtain the design matrix. We recall that the principal coordinates of the lattice, only a component is 1 (Table 1).

Hence if we substitute in Eqn. (8), the coordinates of the first point (X_1=1, X_2=0, X_3=0, and X_4=0, (Table 1), we get that $Y_1 = \beta_1$.

And doing so in succession for the other three points in the tetrahedron, we obtain

$$Y_2 = \beta_2,\; Y_3 = \beta_3,\; Y_4 = \beta_4 \tag{10}$$

Table 1: Design matrix for (4,2) Lattice

N	X_1	X_2	X_3	X_4	Y_{exp}
1	1	0	0	0	Y_1
2	0	1	0	0	Y_2
3	0	0	1	0	Y_3
4	0	0	0	1	Y_4
5	½	½	0	0	Y_{12}
6	½	0	½	0	Y_{13}
7	½	0	0	½	Y_{14}
8	0	½	½	0	Y_{23}
9	0	½	0	½	Y_{24}
10	0	0	½	½	Y_{34}

The substitution of the coordinates of the fifth point yields

$$Y_{12} = \tfrac{1}{2}X_1 + \tfrac{1}{2}X_2 + \tfrac{1}{2}X_1 . \tfrac{1}{2}X_2$$
$$= \tfrac{1}{2}\beta_1 + \tfrac{1}{2}\beta_2 + \tfrac{1}{4}\beta_{12}$$

Thus

$$\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2 \qquad (11)$$

And similarly,

$$\beta_{13} = 4Y_{13} - 2Y_1 - 2Y_3$$
$$\beta_{23} = 4Y_{23} - 2Y_2 - 2Y_3 \text{ ,etc.}$$

On generalizing,

$$\beta_i = Y_i \quad \text{and} \quad \beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \qquad (12)$$

which are the coefficients of the reduced second degree polynomial for a q-component mixture, since the four points defining the coefficients β_{ij} lie on the edge. The subscripts of the mixture property symbols indicate the relative content of each component X_i alone and the property of the mixture is denoted by Y_i.

2.6 Actual and Pseudo Components

The requirements of the simplex that $\sum_{X=1} X_1 = 1$ makes it impossible to use the normal mix ratios such as 1:3, 1:5, etc, at a given water/cement ratio. Hence a transformation of the actual components (Ingredient proportions) to meet the above criterion is unavoidable. Such transformed ratios say $X_1^{(i)}$, $X_2^{(i)}$, and $X_3^{(i)}$ and $X_4^{(i)}$ for the i[th] experimental points are called pseudo components. Since X_1, X_2 and X_3 are subject to $\sum Xi = 1$, the transformation of cement : mound soil : laterite : water at say 0.30 water/cement ratio cannot easily be computed because X_1, X_2,X_3 and X_4 are in pseudo expressions $X_1^{(i)}$, $X_2^{(i)}$, and $X_3^{(i)}$ .For the i[th] experimental point, the transformation computations are to be done.

The arbitrary vertices chosen on the triangle are A_1(1:0.64:15.72:0.49), A_2(1:0.62:15.12:0.52), A_3(1:0.58:14.21:0.35), and A_4(1:0.69:16.80:0.55), based on experience and earlier research reports.

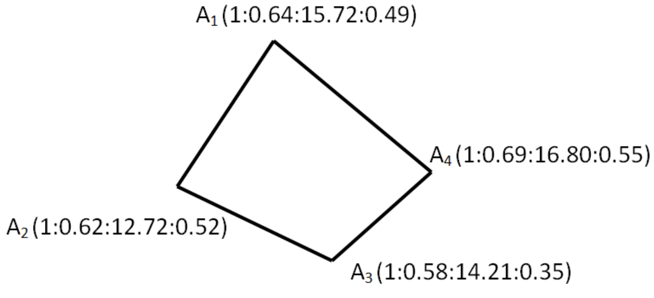

Figure 1: Tetrahedral simplex

2.7 Transformation matrix

If Z denotes the actual matrix of the i[th] experimental points, observing from Table 2 (points 1 to 4)

$$BZ = X = 1 \qquad (13)$$

where B is the transformed matrix.

Therefore,

$$B = IZ^{-1} \; or \; B = Z^{-1} \qquad (14)$$

For instance, for the chosen ratios A_1, A_2, A_3 and A_4 (Figure 1),

$$Z = \begin{bmatrix} 1 & 0.64 & 15.72 & 0.49 \\ 1 & 0.62 & 15.21 & 0.52 \\ 1 & 0.58 & 15.21 & 0.35 \\ 1 & 0.69 & 16.80 & 0.55 \end{bmatrix}$$

$$Z^{-1} = \begin{bmatrix} -3.01 & 8.20 & 2.42 & -6.61 \\ -224.80 & 52.74 & 59.53 & 112.53 \\ 9.31 & -2.96 & 2.35 & -4.00 \\ 3.05 & 9.31 & -7.31 & -5.05 \end{bmatrix}$$

Hence, $BZ^{-1} = Z.Z^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Thus, for actual component Z, the pseudo component X is given by:

$$X \begin{bmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \\ X_4^{(i)} \end{bmatrix} = B \begin{bmatrix} -3.01 & 8.20 & 2.42 & -6.61 \\ -224.80 & 52.74 & 59.53 & 112.53 \\ 9.31 & -2.96 & 2.35 & -4.00 \\ 3.05 & 9.31 & -7.31 & -5.05 \end{bmatrix} Z \begin{bmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \\ Z_4^{(i)} \end{bmatrix}$$

The inverse transformation from pseudo component to actual component is expressed as:

$$AX = Z \qquad (15)$$

where A is the inverse matrix.

From Eqn (13),

$$A = Z(BZ)^{-1} = Z.Z^{-1}B^{-1} = IB^{-1} = B^{-1} \qquad (16)$$

This implies that for any pseudo component X, the actual component is given by:

$$Z \begin{bmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \\ X_4^{(i)} \end{bmatrix} = B \begin{bmatrix} 1 & 0.64 & 15.72 & 0.49 \\ 1 & 0.62 & 15.21 & 0.52 \\ 1 & 0.58 & 15.21 & 0.35 \\ 1 & 0.69 & 16.80 & 0.55 \end{bmatrix} X \begin{bmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \\ Z_4^{(i)} \end{bmatrix} \qquad (17)$$

Eqn (17) is used to determine the actual components from points 5 to 10 , and the control values from experimental points 11 to 13 (Table 2).

2.8 Use of values in experiment

During the laboratory experiment, the actual components were used to measure out the appropriate proportions of the ingredients: cement, mound soil, laterite and water were for casting the samples. The values obtained are presented in Table 2.

2.9 Adequacy of tests

This is carried out by testing the fit of a second degree polynomial [20]. After the coefficients of the regression equation has been derived, analysis is considered necessary. The equation should be tested for goodness of fit, and the equation and surface values bound into the confidence intervals. In experimentation following simplex-lattice designs there are no degrees of freedom to test the equation for adequacy, so, the experiments are run at additional so-called control points.

The number of control points and their coordinates are conditioned by the problem formulation and experiment nature. Besides, the control points are sought so as to improve the model in case of inadequacy. The accuracy of response prediction is dissimilar at different points of the simplex. The variance of the predicted response, S_Y^2, is obtained from the error accumulation law. To illustrate this by the second degree polynomial for a quarternary mixture, the following points are assumed:

- X_i can be observed without errors.
- The replication variance, S_Y^2, is similar at all design points, and
- Response values are the average of n_i and n_{ij} replicate observations at appropriate points of the simplex

Then the variance $S_{\hat{Y}i}$ and $S_{\hat{Y}ij}$ will be:

$$\left(S_{\hat{Y}}^{\ 2}\right)_i = S_Y^{\ 2}/n_i \qquad . \qquad . \qquad (18)$$

$$\left(S_{\hat{Y}}^{\ 2}\right)_{ij} = S_Y^{\ 2}/n_{ij} \qquad (19)$$

By some mathematical transformations and introducing the designation

$$a_i = X_i(2X_i - 1) \quad and \quad a_{ij} = 4X_iX_j \qquad (20)$$

$$S_{\hat{Y}} = S_Y^{\ 2}\Big(\sum_{1\le i\le q} a_i/n_i + \big(\sum_{1\le i\le j\le q} a_{ij}/n_{ij}\big) \qquad (21)$$

If the number of replicate observations at all the points of the design are equal, i.e. $n_i = n_{ij} = n$, then all the relations for $S_{\hat{Y}}^2$ will take the form

$$S_{\hat{Y}} = S_Y^{\ 2}\xi/n \qquad (22)$$

where, for the second degree polynomial,

$$\xi = \sum_{1\le i\le q} a_i^{\ 2} + \sum_{1\le i\le j\le q} a_{ij}^{\ 2} \qquad (23)$$

As in equation (23), ξ is only dependent on the mixture composition. Given the replication Variance and the number of parallel observations n, the error for the predicted values of the response is readily calculated at any point of the composition-property diagram using an appropriate value of ξ taken from the curve.

Adequacy is tested at each control point, for which purpose the statistic is built:

$$t = \Delta_Y/(S_{\hat{Y}}^{\ 2} + S_Y^{\ 2}) = \Delta_Y n^{1/2}/(S_Y(1+\xi)^{1/2}) \qquad (24)$$

where

$$\Delta_Y = Y_{\exp} - Y_{theory} \qquad (25)$$

and n is the number of parallel observations at every point.

The t-statistic has the student distribution [23], and it is compared with the tabulated value of $t_{\alpha/L}(V)$ at a level of significance α, where L = the number of control points, and V = the number for the degrees of freedom for the replication variance.

Table 2: Values for Experiment

N	X_1	X_2	X_3	X_4	RESPONSE	Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	1.00	0.64	15.72	0.49
2	0	1	0	0	Y_2	1.00	0.62	15.12	0.52
3	0	0	1	0	Y_3	1.00	0.58	14.21	0.35
4	0	0	0	1	Y_4	1.00	0.69	16.80	0.55
5	½	1/2	0	0	Y_{12}	1.00	0.63	15.42	0.51
6	½	0	1/2	0	Y_{13}	1.00	0.32	14.97	0.42
7	1//2	0	0	1/2	Y_{14}	1.00	0.67	16.26	0.52
8	0	1/2	1/2	0	Y_{23}	1.00	0.31	14.67	0.44
9	0	1/2	0	1/2	Y_{24}	1.00	0.66	15.96	0.54
10	0	0	1/2	1/2	Y_{34}	1.00	0.35	15.51	0.45
Control points									
11	0.25	0.25	0.25	0.25	Y_{1234}	1.00	0.49	15.46	0.48
12	0.5	0.25	0.25	0	Y_{1123}	1.00	0.48	15.19	0.46
13	0.25	0.5	0	0.25	Y_{1224}	1.00	0.64	15.69	0.52

The null hypothesis is that the equation is adequate and accepted if $t_{cal} < t_{Table}$ for all the control points. The confidence interval for the response value is

$$\hat{Y} - \Delta \le Y \le \hat{Y} + \Delta \tag{26}$$

$$\Delta = t_{\alpha/L,k} S_{\hat{Y}} \tag{27}$$

where k is the number of polynomial coefficients determined.

Using (22) in (27) we have:

$$\Delta = t_{\alpha/L,k} S_Y (\xi/n)^{1/2} \tag{28}$$

3. METHODOLOGY

To be a good structural material, the material should be homogeneous and isotropic. The Portland cement, sandcrete or concrete are none of these, nevertheless they are popular construction materials [24].The necessary materials required in the manufacture of the sandcrete in the study are cement, mound soil, laterite, and water.

The research was done by assembling mix materials and preparing samples of lateritic concrete or latcrete (cement-laterite-water) mix using mix proportions generated from Scheffe's simplex lattice design theory after the laterite and mound soil were adequately characterized. The compressive stress responses were obtained from crushing tests of the samples. The responses were applied to the reduced and normalized polynomial equation assumed to characterize the lines of components' concentrations in the mix to obtain the compressive stress model capable of predicting the components' ratio for any desired lateritic concrete compressive stress using the laterite and mounds soil within the same range of physical characteristics. Reversibly, the optimal compressive stress of the lateritic concrete could be predicted by the model given some desired mix ratios, using an application programs designed for the property optimization process. It was found that it is possible to quickly

(i) fix the ratio of latcrete components without going through the rigors involved in most existing mix design methods

(ii) select latcrete component ratios for mixes to solve specific construction engineering tasks

(iii) apply the ubiquitously locally available laterite in construction works to achieve construction cost reduction and ultimately increase our foreign exchange earnings and conserve same.

3.1 Materials

The laterite materials were collected at the Emene area of Enugu State and conformed to BS 882 and belongs to zone 1 of the ASHTO classification. The mound soil was collected from the bushes around Emene area of Enugu State.

The water for use is pure drinking water which is free from any contamination i.e. nil Chloride content, pH =6.9, and Dissolved Solids < 2000ppm. Ordinary Portland cement is the hydraulic binder used in this project and sourced from the Dangote Cement Factory, and assumed to comply with the Standard Institute of Nigeria (NIS) 1974, and kept in an air-tight bag.

3.2 Preparation of samples

The sourced materials for the experiment were transferred to the laboratory. The pseudo components of the mixes were designed following the background theory from where the actual variables were developed. The component materials were mixed at ambient temperature according to the specified proportions of the actual components generated in Table 2. In all, two hollow blocks of 450mm x225 x150mm for each of ten experimental points and three control points were cast for the compressive strength test, cured for 28 days after setting and hardening.

3.3 Strength Test

After 28 day of curing, the blocks were crushed to determine the sandcrete block strength, using the compressive testing machine to the requirements of BS 1881: Part 115 of 1986.

4. RESULT AND ANALYSIS

4.1 Determination of Replication Error and Variance of Response

To raise the experimental design equation model by the lattice theory approach, two replicate experimental observations were conducted for each of the ten design points. Table 3 contain the results of two repetitions each of the 10 design points plus three control points of the (4, 2) simplex lattice, and show the mean and variance values per test of the observed response, using the following mean and variance equations below:

$$\bar{Y} = \sum (Y_r)/r \tag{29}$$

where $\hat{Y}$ is the mean of the response values and r = 1, 2, ..., i

$$S_Y{}^2 = \sum \left(Y_i - \overline{Y}\right)^2 /(n-1) \quad (30)$$

where n = 13.

Replication Variance

$$S_Y{}^2 = \sum S_i{}^2 /(n-1) = 0.03 \quad (31)$$

Replication Error

$$S_Y = (S_Y{}^2)^{1/2} = (0.03)^{1/2} = 0.1732 \quad (32)$$

4.2 Determination of Regression Equation for the Compressive Strength

From equations (10), (12) and Table 3, all the coefficients of the reduced second degree polynomial were determined and therefore from (8), we have:

$$\begin{aligned}\widehat{Y_1} = 1.95X_1 + 2.02X_2 + 2.15X_3 + 2.01X_4 + 0.63X_1X_2 \\ + 0.34X_1X_3 + 0.43X_1X_4 + 1.83X_2X_3 \\ + 0.20X_2X_4 + 0.31X_3X_4 \quad (33)\end{aligned}$$

Equation (33) is the mathematical model of the compressive strength of the hollow sandcrete block based on the 28-day strength.

4.3 Test of Adequacy of the Compressive strength Model

Equation (33), the equation model, will be tested for adequacy against the controlled experimental results.

We recall our statistical hypothesis as follows:

1. ***Null Hypothesis (H_0):*** There is no significant difference between the experimental values and the theoretical expected results of the compressive strength.
2. ***Alternative Hypothesis (H_1):*** There is a significant difference between the experimental values and the theoretical expected results of the compressive strength.

4.4 t-Test for the Compressive strength Model

If we substitute for X_i in equation (33) from Table 3, the theoretical predictions of the response ($\hat{Y}$) can be obtained. These values can be compared with the experimental results (Table 3). For the t-test (Table 4), a, ξ, t and Δ_y are evaluated using equations (21), (24), (25) and (26) respectively.`

Significance level α = 0.05,

i.e. from $t_{\alpha/L}(V_c) = t_{0.053}(13)$ where L=number of control point.

From student's t-Table [23] the tabulated value of $t_{0.05/3}$ (13) is found to be 3.01 which is greater than any of the calculated t-values in Table 4. Hence we can accept the Null Hypothesis.

With $k = 10, \quad t_{\alpha/k,y} = t_{0.05/k}(13) = 3.01$;

$$\Delta = 0.66 \ for\ C_{1234},\ 0.68\ for\ C_{1124},\ and\ 0.70\ for\ C_{1224}$$

which satisfies the confidence interval equation (26) when viewed against the response values in Table 4.

Table 3: Result of the Replication Variance of the Compressive Strength Response for 450x225x150 mm Block

Experiment No (n)	Repetition	Response Y (N/mm²)	Response Symbol	$\sum Y_r$	$\overline{Y}$	$\sum(Y_r - \overline{Y})^2$	S_i^2
1	1A 1B	1.94 2.10	Y_1	4.04	2.02	0.01	0.00
2	2A 2B	2.01 1.88	Y_2	3.89	1.95	0.01	0.00
3	3A 3B	2.34 1.96	Y_3	4.30	2.15	0.07	0.01
4	4A 4B	1.99 2.02	Y_4	4.01	2.01	0.00	0.00
5	5A 5B	2.30 1.98	Y_{12}	4.28	2.14	0.05	0.00
6	6A 6B	2.01 2.33	Y_{13}	4.34	2.17	0.05	0.00
7	7A 7B	1.89 1.92	Y_{14}	3.81	1.91	0.00	0.00
8	8A 8B	2.44 2.57	Y_{23}	5.01	2.51	0.00	0.00
9	9A 9B	1.79 2.26	Y_{24}	4.05	2.03	0.11	0.01
10	10A 10B	2.00 2.00	Y_{34}	4.00	2.00	0.00	0.00

Experiment No (n)	Repetition	Response Y (N/mm²)	Response Symbol	$\sum Y_r$	$\overline{Y}$	$\sum(Y_r - \overline{Y})^2$	S_i^2
			Control Points				
11	11A 11B	2.31 1.97	C_1	4.28	2.14	0.06	0.00
12	11A 11B	2.07 2.11	C_2	4.18	2.09	0.00	0.00
13	13A 13B	2.21 2.01	C_3	4.22	2.11	0.02	0.00
							$\sum$0.03

Table 4: t-Test for the Test Control Points

N	CN	I		a_i	a_{ij}	a_i^2	a_{ij}^2	ξ	$\overline{Y}$	$\hat{Y}_a$	Δ_y	t
1	C_1	1	2	-0.125	0.250	0.016	0.063	0.469	2.14	2.17	-0.03	-0.20
		1	3	-0.125	0.250	0.016	0.063					
		1	4	-0.125	0.250	0.016	0.063					
		2	3	-0.125	0.250	0.016	0.063					
		2	4	-0.125	0.250	0.016	0.063					
		3	4	-0.125	0.250	0.016	0.063					
						0.094	0.375					
2	C_2	1	2	0.000	0.500	0.000	0.250	0.563	2.09	2.27	-0.18	1.12
		1	3	0.000	0.500	0.000	0.250					
		1	4	0.000	0.000	0.000	0.000					
		2	3	0.000	0.250	0.000	0.063					
		2	4	0.000	0.000	0.000	0.000					
		3	4	0.000	0.000	0.000	0.000					
						0.000	0.563					
3	C_3	1	2	-0.125	0.500	0.016	0.250	0.656	2.11	2.06	-0.05.	0.33
		1	3	-0.125	0.000	0.016	0.000					
		1	4	-0.125	0.250	0.016	0.063					
		2	3	-0.125	0.000	0.016	0.000					
		2	4	-0.125	0.500	0.016	0.250					
		3	4	-0.125	0.000	0.016	0.000					
						0.094	0.563					

5. DISCUSSION OF RESULT

The results of the pseudo-components and the actual-components (values for experiments) shown in Table 2 represent the obtained components that conformed with the analysis to the reduced polynomial coefficients under the $\sum_{I=1}^{4} X_i = 1$ normalizing condition of the simplex lattice design. The values of the component ratios were obtained by executing the matrix product in Excel program. The experimental and replication variance necessary for testing the model statistical adequacy was presented in Table 2. It was observed that the t-test values from calculation were lower than the t-values read from the statistical tables and this confirmed that the model is adequate. The user of "ONUAMAH.COM" only need specify the desired lateritic concrete strength and the component ratio associated with the desired compressive strength will be automatically generated, whereas, if it's out of range, the program reports sends an error message. Conversely, given some mix ratios, the optimal attainable compressive strength is automatically generated. It is the user's to choose from the generated list of the applicable mix ratios based on material cost as such factors as workability and honeycombing are not considered in the design approach.

The results also showed that laterized concrete mixes can be designed with mound soil as a percentage addition instead of the conventional undersigned addition which leaves the sandcrete production to the rule of the thumb. That is, the paper provides an optimized design perspective of the use of admixtures instead of undesigned percentage addition which is currently prevalent in the concrete industry of the world.

In this work, the laterized concrete mix with 4% mound soil inclusion yielded the mix ratios of 1.00:0.61:14.780:0.44, 1.00:0.60:14.72:0.44 and 1.00:0.69:15.12:0.35, at a tolerance of 0.00005 N/mm^2 for the desired compressive stress of 2.445 N/mm^2 and reports the maximum compressive strength predictable by the model as 2.51 N/mm^2. On the other hand, when the mix of ratio 1.00:0.69:15.12:0.35 was run in the program, the optimal compressive strength of 2.09 N/mm^2 was obtained.

The magnitude of the result is much affected by the source and quality of the laterite whose silt content is deleterious to concrete quality and strength. Depending on the use of the lateritic concrete, the laterite in an area can be utilized to some great and cost effective advantage.

Henry Scheffe's simplex design was applied successfully to prove that the compressive strength of lateritic concrete is a function of the proportion of the ingredients (cement, mound soil, laterite and water), but not the quantities of the materials.

One can also draw the conclusion that the maximum values achievable, within the limits of experimental errors, is quite below that obtainable using sand as aggregate. This is due to the predominantly high silt content of laterite.

The project work is a great advancement in the search for the applicability of laterite and mound soiJl in concrete mortar production in regions where sand is extremely scarce with the ubiquity of laterite and mound soil materials.

6. RECOMMENDATIONS

From the foregoing study, the following could be recommended:

(i) The model can be used for the optimization of the strength of concrete made from cement, mound soil, laterite and water.

(ii) More research work need to be done in order to match the computer recommended mixes with the workability of the resulting concrete.

(iii) The accuracy of the model can be improved by taking higher order polynomials of the simplex.

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