



FUZZY CONTROL TECHNIQUE APPLIED TO MODIFIED MATHEMATICAL MODEL FOR MALARIA CONTROL

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ABSTRACT

In this paper, fuzzy control technique is applied to the modified mathematical model for malaria control presented by the authors in an earlier study. Five Mamdani fuzzy controllers are constructed to control the input (some epidemiological parameters) to the malaria model simulated by 9 fully nonlinear ordinary differential equations (ODE) simulator of the Modified Ngwa and Shu's Model. Each of the controllers uses two different and unique variable parameters to calculate commanded control inputs to the simulator. The input domains are divided into 3 membership functions resulting in 7 fuzzy rules for each rule base and the output domains are partitioned with 5 membership functions. The Mamdani controllers use a standard max-min inference process and a fast centre of area method to calculate the crisp control signals. Numerical results obtained using Matlab 2008a software guarantee that malaria infection can be reduced to a tolerable limit.

Keywords: fuzzy control, malaria, drug effectiveness, mosquitoes, equilibrium state, dynamic equation

1. INTRODUCTION

Malaria is a vector borne infectious disease that has affected the human race since earliest times and an estimated 40% of the world's population lives in malaria endemic area [2]. Malaria is one of the most common infectious diseases and enormous public health problem. The yearly mortality from malaria is about 9 times that of HIV/AIDS. Malaria ranked among the major infectious disease causing deaths, after pneumococcal acute respiratory infection designated pneumonia and tuberculosis [3]. Malaria is a parasitic disease caused mainly by *plasmodium falciparum*. (*p. falciparum*) and three other malaria parasites that cause milder diseases in human (*p.vivax*, *p.ovale*, and *p. malariae*) are transmitted by more than a dozen species of Anopheles mosquitoes widely [4]. The disease kills about 1 to 3 million people yearly, 75% of whom are African children [5].

The incidence of malaria has been on the increase recently owing to parasite-drug-resistance and mosquito-insecticide-resistance [6]. Thus, this has lowered the productivity of the economy of the endemic zone [7]. Therefore, it is crucial to understand the important transmission parameters of

the disease and develop efficient and effective solution strategies for its prevention and control.

Fuzzy control research was started by Mamdani's pioneering work in 1974, which was motivated by Zadeh's two papers on fuzzy algorithms and linguistic analysis. Fuzzy logic is a logic system that provides human reasoning capabilities to capture uncertainties, which cannot be described by precise mathematical models. It has proven to be powerful in dynamic system control. Its main merit is that it gives the most efficient knowledge representation method that can be devised for rule-based systems that deals with continuous variables [8].

Fuzzy systems have attracted the interest of researchers in various scientific and engineering areas [9]. The number and variety of applications of fuzzy logic have been increasing, ranging from consumer products [10], attitude control in spacecraft [11], and industrial process control [13, 14] to medical instrumentation information system and decision analysis [9] and recently to diabetic control [12]. The main aim of using fuzzy logic in medicine is the fact that medical practitioners exhibit variation in decision making as a result of their approaches to dealing with uncertainties in knowledge and information. Also, the

diagnostic decisions depend on experience, expertise and perception of the practitioner. However, as the complexity of the system increases, it becomes necessary to use fuzzy logic as it is not easy to follow a particular path of diagnosis without any mistake. In other words, fuzzy logic renders precise what is imprecise in the world of medicine, as it is important for symptomatic diagnosis remedies.

In this paper, we apply fuzzy logic to the modified Ngwa and Shu's model presented in [1] which includes human recruitment rate, it does not restrict the total number of bites on humans only to the number of mosquitoes as the total number of bites depend on both the human and mosquito population size in our model. Likewise, we included not only the birth of susceptible mosquitoes' offsprings but also infectious offsprings of infected mosquitoes, the probability that a recovering individual become infected by one infected mosquito per contact unit time and effectiveness of drugs are included. But the model excludes direct human recovery from the infections to the susceptible class.

2. MATHEMATICAL MODEL

In this section, the modified mathematical model of Ngwa and Shu's model for the evolution of the malaria in a population formulated in [1] is presented in (1) – (9). The equations are based on both human and mosquito population, infection rates of human and mosquito per unit time, rescaling of critical population of each class by total species population and bifurcation analysis. Following the basic ideas and structure of mathematical modeling in epidemiology, the model for the malaria disease was developed under the terms in tables 1 and 2 (see [1] for detailed analysis).

$$\frac{du}{d\tau} = \frac{\Lambda}{N_h} + \beta R + \lambda - \left(\alpha_1 + \lambda + \frac{\Lambda}{N_h} \right) u + \gamma w u \quad (1)$$

$$\frac{dg}{d\tau} = \alpha_1 u - \left(v + \lambda + \frac{\Lambda}{N_h} \right) g + \gamma w g \quad (2)$$

$$\frac{dw}{d\tau} = v g - \left(\gamma + \alpha_2 A + \lambda + \frac{\Lambda}{N_h} \right) w + \gamma w^2 + \alpha_3 z N_v R \quad (3)$$

$$\frac{dR}{d\tau} = \alpha_2 A w - \left(\beta + \alpha_3 z N_v + \lambda + \frac{\Lambda}{N_h} \right) R + \gamma w R \quad (4)$$

$$\frac{dx}{d\tau} = a(1 + (1 - \theta))z - (\alpha_4 + a)x \quad (5)$$

$$\frac{dy}{d\tau} = \alpha_4 x - (a + e)y \quad (6)$$

$$\frac{dz}{d\tau} = ey + (\theta a - a)z \quad (7)$$

The equation for the total population of human and mosquitoes is now represented as:

$$\frac{dN_h}{d\tau} = \Lambda + \lambda N_h - (\mu_{1h} + \mu_{2h} N_h) N_h - \gamma w N_h \quad (8)$$

$$\frac{dN_v}{d\tau} = a N_v - (\mu_{1v} + \mu_{2v} N_v) N_v \quad (9)$$

Table 1: The State Variables for the Malaria Model

Symbol	Description
S_h	Population of susceptible humans
E_h	Population of exposed humans
I_h	Population of infectious humans
R_h	Population of recovered (immune and asymptomatic, but slightly infectious) humans
S_v	Population of susceptible mosquitoes
E_v	Population of exposed mosquitoes
I_v	Population of infectious mosquitoes
N_h	Total human population
N_v	Total mosquito population

Table 2: The Parameters for the Malaria Model

Symbol	Description
Λ_h	The recruitment rate
T	Time
β_h	Rate at which a human loses his immunity
λ_h	Per capita human birth rate
λ_v	Per capita mosquito birth rate
α_1	The probability that a susceptible individual become infected by one infected mosquito per unit contact time
γ_h	Infected humans who die from the disease
α_2	The recovery rate of the infected human
A	The measure of the effectiveness of anti-malaria drugs
α_3	The probability that a recovering individual become infected by one infected mosquito per contact unit time
α_4	The probability that a susceptible mosquito becomes infected
θ	Proportion of the offspring of the infected mosquito that are infected
$(1 - \theta)$	Proportion of the offspring of the infected mosquito that are susceptible
μ_{1h}	Density-independent part of the death (and emigration) rate for humans
μ_{2h}	Density-dependent part of the death (and emigration) rate for humans
μ_{1v}	Density-independent part of the death (and emigration) rate for mosquitoes
μ_{2v}	Density-dependent part of the death (and emigration) rate for mosquitoes
w	Ratio of population of infectious humans to total human population

Symbol	Description
β	Ratio of the rate at which a human loses his immunity to unit time rescale
V	Human incubation rate

An interesting note is that since the equations are with respect to dt, this means that the unit time measurement is with respect to the life span of a mosquito which is about 21 days [2].

3. MODIFIED MODEL OUTPUT CONTROL

The major task of malaria control system (model) is to keep the population in a healthy equilibrium while minimizing or reducing the infections in humans (by treatment, vaccination) and eliminating (reducing) vector populations by direct physical control and the use of insecticides within a space of time. In this work, the model is required to follow the predetermined parameter value.

Malaria dynamical model ((1)-(9)) is highly non-linear, multivariable with significant degree of time variation due to the life span of the anopheles mosquito and change in immunity and environmental conditions. The 9-ODE model consists of 5-human compartmental variables and 4-vector compartmental variables.

The core of a malaria control system is the parameters of ODE model through which infection equilibrium point movement and control is realized and designed (e.g. $\alpha_1, V, \alpha_2, \alpha_4,$ and β).

This work is focused on the 9-ODE input variable control of the modified malaria model with a Mamdani based direct control strategy as shown in Figure 1.

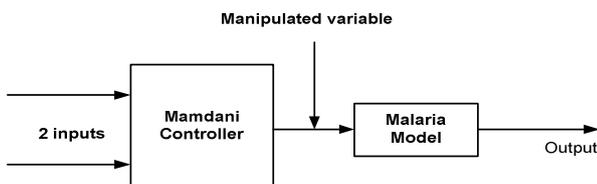


Figure 1: Mamdani controller for malaria transmission control

The two inputs to the Mamdani controller can be any of the following: Body exposure and water retention, medication and rest, infected host rise and biting rate, good diet and fitness, and dosage and tolerance. The inputs variable control requirements are described by the commanded dynamic variable, the infectious rate of human (α_1), the recovery rate of infected human (α_2), the mosquito rate of infection (α_4), the human incubation rate (V), and ratio of the rate at which human loses immunity to unit time rescaled (β),

which are predetermined dynamic parameters of the modified malaria model based on the inputs to the Mamdani controller. These variables result in favourable epidemiological changes required to change the actual malaria model population to a required (stable) equilibrium states i.e. increased recovered and immunized humans and decrease infectious humans and vectors.

The infectious human, and recovered human, are controlled by a Mamdani controller with medication and food (and/rest) as inputs and the commanded α_2 as output. The exposed human, g, and susceptible human, u, are controlled using degree of body exposure and water level as inputs and commanded α_1 as output. The susceptible mosquitoes, x, and exposed mosquitoes, y, are controlled using increase-in-infected host and biting rate as inputs and the calculated α_4 as output. Also the exposed humans, g, and the infected humans, w, are controlled using good diet and fitness as inputs and the commanded V as output. Finally, the susceptible human and the recovered human are controlled using dosage and tolerance as inputs and the calculated β as output.

The input and output range used in the Mamdani is as shown in Table 3. Figure 2 shows the controller structure for one of the output state variable.

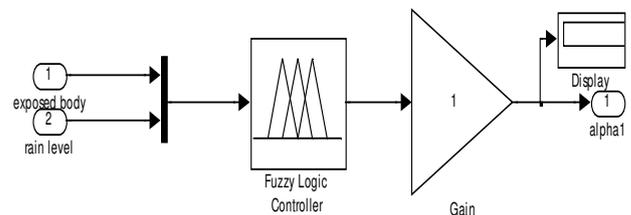


Figure 2: The structure of the Mamdani controller used in the Simulink simulation environment

Table 3: Input and output- domains used in the Mamdani output state variable controller

Parameter	Input range	Output range
α_1	$0 \leq \text{input} \leq 1$	$0 \leq \text{output} \leq 0.5$
V	$0 \leq \text{input} \leq 1$	$0 \leq \text{output} \leq 3$
α_4	$0 \leq \text{input} \leq 1$	$0 \leq \text{output} \leq 0.5$
α_2	$0 \leq \text{input} \leq 1$	$0 \leq \text{output} \leq 0.5$
β	$0 \leq \text{input} \leq 1$	$0 \leq \text{output} \leq 0.05$

4. RESULTS AND DISCUSSIONS

The modified malaria control model is tested in a simulation environment using the 9 ODE Simulators. The test system structure in Simulink environment is depicted in Figure 3. Figures 4 – 9 in which the results

of the fuzzy based method applied to the modified mathematical model is compared with the analytical results of [1] for exposed, infectious and susceptible humans (host) and mosquitoes (vectors).

Figure 4 shows the exposed populations for humans and the results are similar except for the analytical that peak and subsequently fall. Figure 5 presents the infectious population for humans which is similar too, except for the analytical result that peak and fall subsequently. The deviation in the Figures is based on the fact that malaria cannot be reduced to zero completely as indicated by the fuzzy technique. However, the fuzzy technique gives an indication that is comparable to the analytic method for diagnosis purpose. While Figure 6 shows the susceptible population for humans and the results indicates that there is decrease in population. Similarly, the results for exposed, infectious and susceptible mosquitoes are

presented in Figures 7 – 9. It is observed that they follow the same pattern with that for humans.

The peaks for the exposed population indicate movement of respective individual from the susceptible class into the exposed class. While the falls indicate that individual concerned eventually leaves the exposed class. For the infectious class, the fall is gradual in vector class than the host's. The peak shows movement of the respective individual into the infected class from the exposed class while the fall indicates that individual eventually leaves the infected class [1].

In the simulated cases using fuzzy control, it is observed that there is no peaking of any of the individuals and the fall was steep in the vector cases than the host cases.

From the results obtained, we can say that the fuzzy based method of malaria control is comparable to the analytical method.

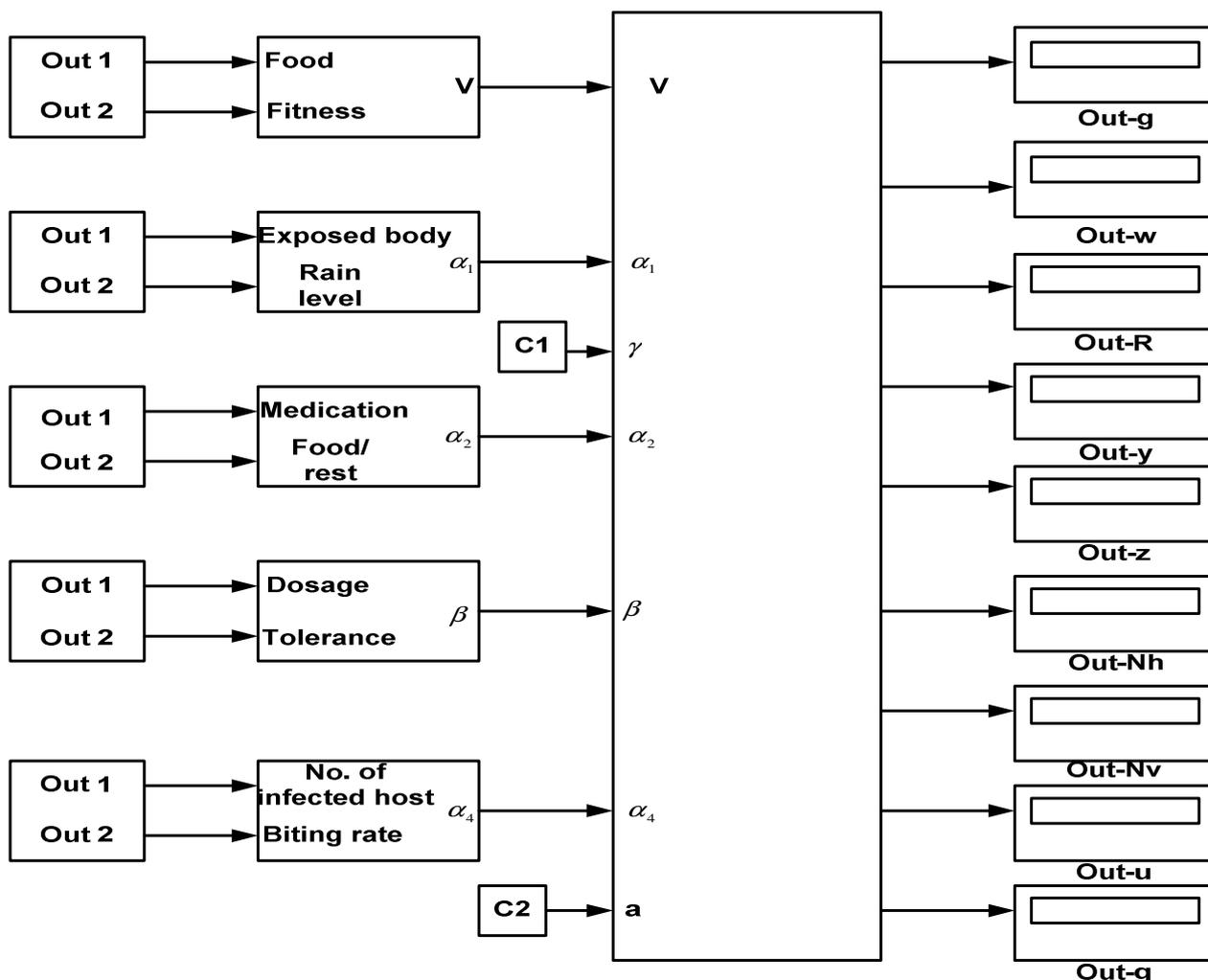


Figure 3: Simulink system structure

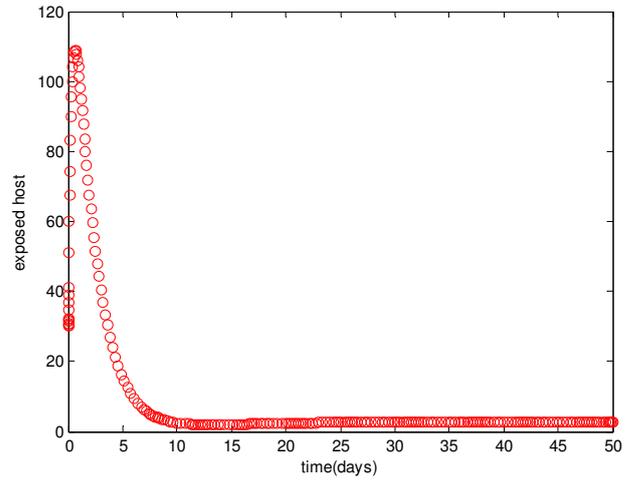
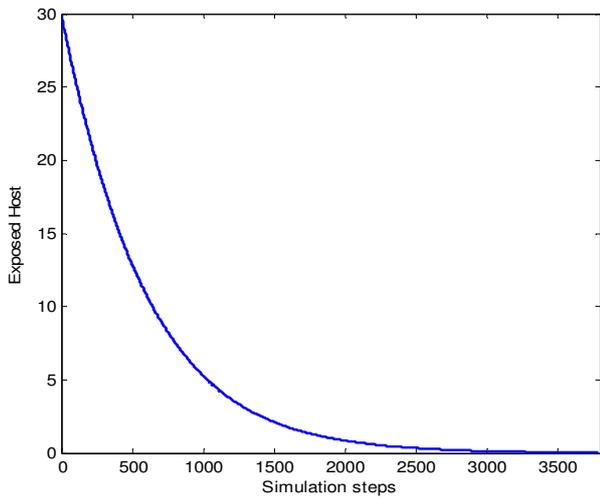


Figure 4: Exposed Host for fuzzy based method and analytical method respectively

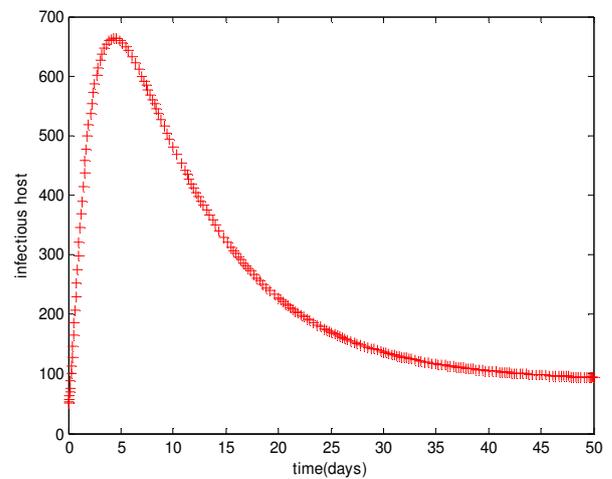
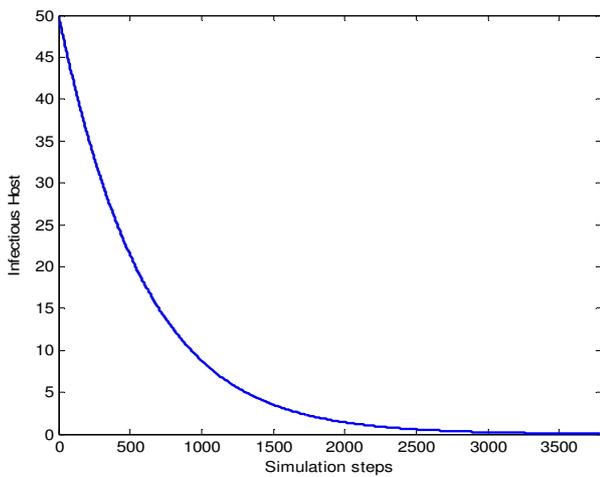


Figure 5: Infectious Host for fuzzy based method and analytical method respectively

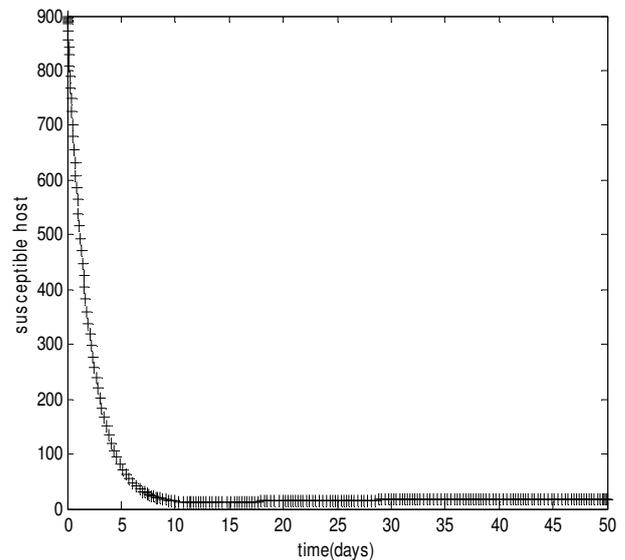
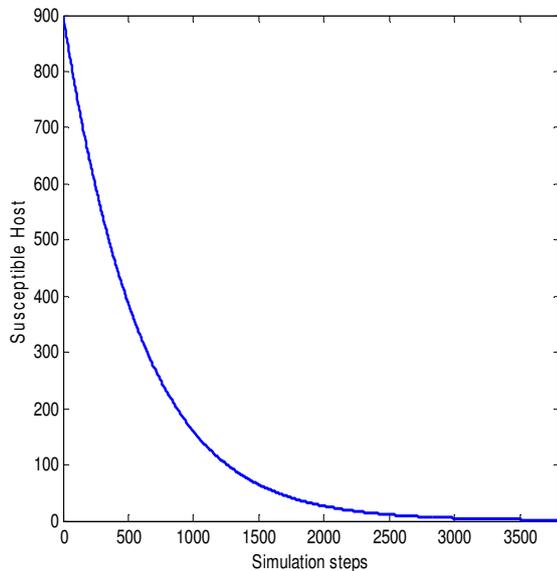


Figure 6: Susceptible Host for fuzzy based method and analytical method respectively

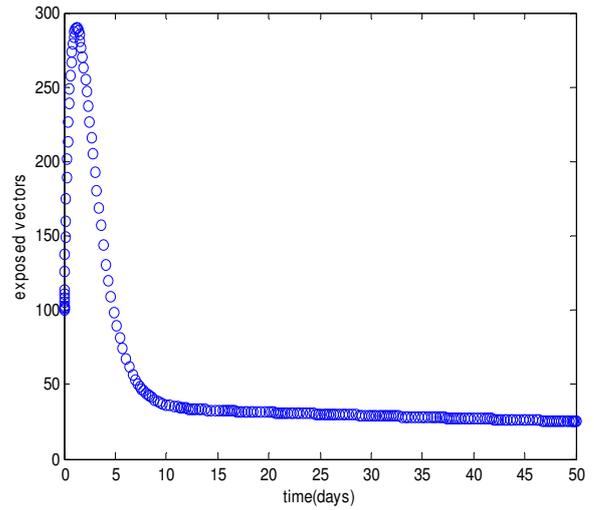
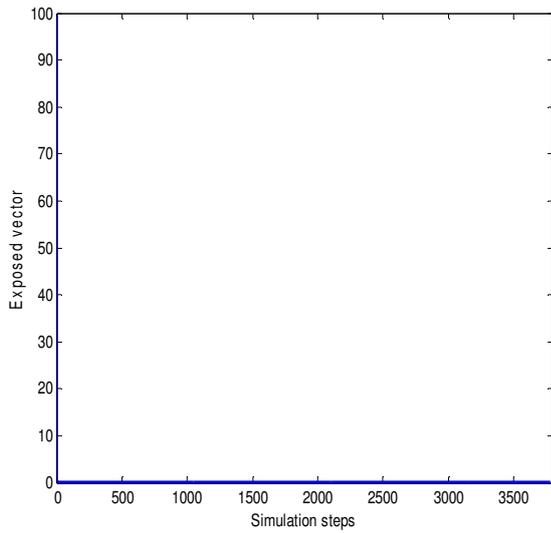


Figure 7: Exposed vector for fuzzy based method and analytical method respectively

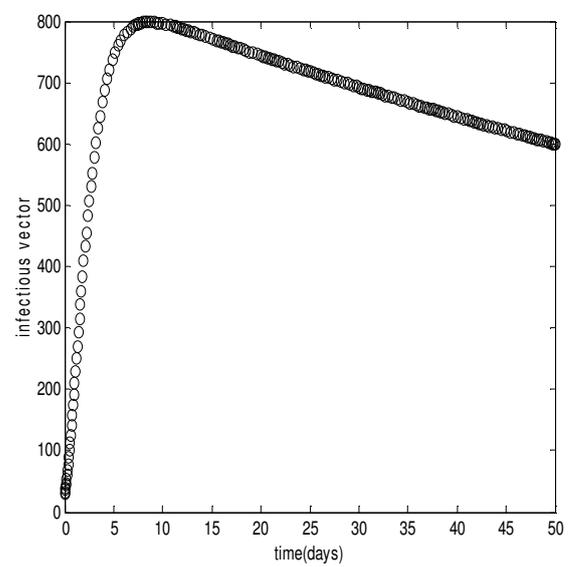
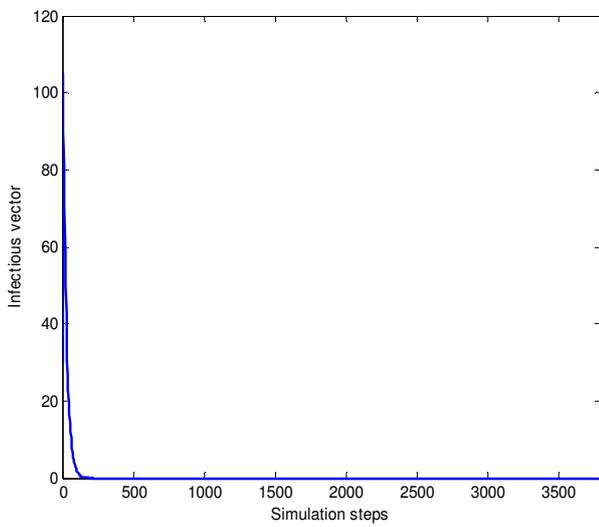


Figure 8: Infectious vector for fuzzy based method and analytical method respectively

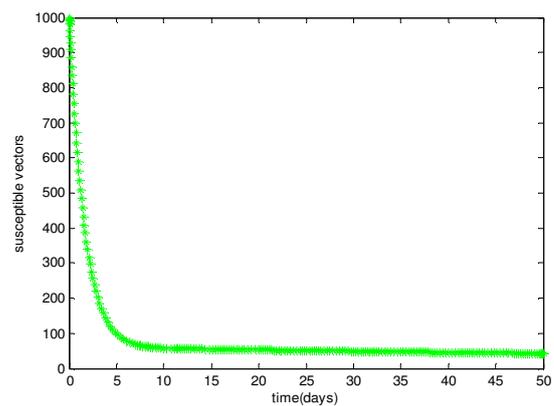
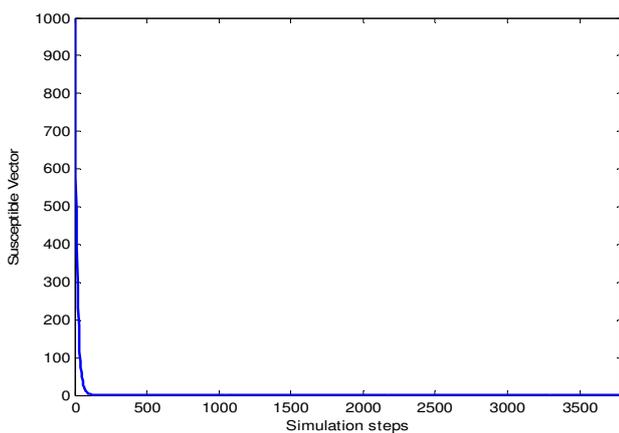


Figure 9: Susceptible vector for fuzzy based method and analytical method respectively

5. CONCLUSION

In this paper, fuzzy control was applied to the modified mathematical model developed for malaria control analyzed in [1]. The aim is to encourage

decision making by medical practitioners using vague description for diagnosis. The fuzzy model designed is to control some of the malaria parameters in a given environment in order to have an environment where

malaria is reduced to a tolerable limit. Tolerable limit implies that the fuzzy technique helps in reducing the infectious humans and mosquitoes population as presented in Figures 4 - 9. The results obtained shows that they are comparable with any other method.

6. REFERENCES

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