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ACCURATE SOLUTIONS OF COLEBROOK- WHITE'S FRICTION FACTOR FORMULAE

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ABSTRACT
Estimations of friction factor (F_f) in pipeline systems and fluid transport are essential ingredients in engineering fields and processes. In this paper explicit friction factor formulae (F_{ff}) were proposed and evaluated with an aim of developing error free F_{ff}. General F_{ff} that relate F_f, Reynolds number (R_e) and relative roughness (R_r) were proposed. Colebrook – White's formula was used to compute different F_f for R_e between 4 x 10^3 and 1.704 x 10^8, and R_r between 1.0 x 10^{-7} and 0.052 using Microsoft Excel Solver to fix the F_{ff}. The fixed F_{ff} were used to compute F_f for R_e between 4 x 10^3 and 1.704 x 10^8 and R_r between 1.0 x 10^{-7} and 0.052. Accuracy of the fixed F_{ff} was evaluated using relative error; model of selection (MSC) and Akaike Information Criterion (AIC) and compared with the previous F_{ff} using Colebrook –White's F_f as the reference. The study revealed that F_f estimated using the fixed F_{ff} were the same as F_f estimated using Colebrook – White's F_{ff}. The fixed F_{ff} provided the lowest relative error of (0.02 %; 0.06 % and 0.04 %), the highest MSC (14.03; 12.42 and 13.07); and the lowest AIC (-73006; -64580 and -67982). The study concluded that modeling of F_{ff} using numerical methods and Microsoft Excel Solver are better tools for estimating F_f in pipeline flow problems.

Keywords: *Friction factor, MSC; AIC; Reynolds number; Engineering Field; pipe flow, statistical methods.*

1. INTRODUCTION

In pipeline systems, various parameters are involved in pipe network systems. Some of the parameters are the lengths, diameters and F_f of pipes, water levels in reservoirs and discharge characteristics of pumps, water demand at different nodes and performance characteristics of different valves and minor elements in the pipe systems [1, 2]. Part of these parameters (pipe length) remains constant at different ages of the pipe, and some parameters (pipe diameters, relative roughness, and friction coefficients) would change during the life of pipe system. The changing parameters can be considered to be imprecise information. Traditionally, the equation for computing the head loss for each pipe in the pipe network and pipeline system is the Hazen-Williams or Darcy-Weisbach's equation, which requires $\mathbf{F_f}$. Darcy – Weisbach's equation is expressed as follows (Equations 1 and 2):

$$h_l = \frac{fLV^2}{2gD} \tag{1}$$

$$h_l = \frac{16fLQ^2}{2g\pi^2D^5} \tag{2}$$

Where; h_l is the head loss; f is the F_f; L is the length of the pipeline or pipe system; D is the diameter of the pipe; V is the mean velocity of flow in the pipeline or pipe system; g is the acceleration due to gravity and Q is the flow rate (discharge) in the pipeline or pipe system.

Colebrook and White presented the initial F_f. The Colebrook –White [3; 4] expressions (Equation 3) are given by many researchers as follows [5; 6]:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k}{3.7D} + \frac{2.51}{R_e\sqrt{f}}\right) \tag{3}$$

Where; k is the relative roughness of the pipe and R_e is the Reynolds number.

Since the initial computation of F_f in pipeline system by Colebrook- White, the possibility of obtaining accurate friction factor in pipe line system and heat transfer in turbulent pipe flows have caught the attention of many researchers. Despite more than seven decades of research, a lower error and full understanding of the essentials of this phenomenon is still far from complete. This lack of accurate F_{ff} is perhaps not so surprising since the very nature of turbulent flows and the

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property of viscoelastic fluids in many numerical analysis fields are ongoing areas of research. Most of the advances in the studies of the turbulence of Newtonian fluids and progress in the computation has been made possible due to the development of semi-empirical models that describe various aspects of the transport of momentum (F_f, and heat). F_f computations are proposed as a viable resource to save energy (oil transport, ship-drag, sewers, fire-fighting, etc.). In applications, F_{ff} has produced fair results in the reduction of friction and the equivalent reduction in heat transport, treated water, chemical, fuel and water transportation. However, there are still many challenges regarding the accurate estimation of F_f in many other applications.

The estimation of F_f in pipes depicting turbulent fluid flow is critical in many engineering applications. In the transfer of fluid through chemical reactors and industrial processes involving single-phase, double-phase and more complicated pipe flow systems pipeline frictions is a critical issue. Nowadays, in medical sciences and biomedical engineering high local velocities are attained in blood vessels. Transportation of physiological fluid through catheter tube into the body of a sick human being (patients) is a daily activity, which requires accurate estimation of F_f in the catheter [7]. In order to estimate F_f, implicitly Colebrook - White formula is needed and one needs to use numerical algorithms, which are not as quick as the explicit approximations to the solution of Colebrook – White's formula. In complex and supercritical pipe-flow systems it becomes difficult to use Colebrook – White's formula. In such situations, quick and accurate estimation of F_f (reliable explicit approximations) are desired. The needs for more robust approximations to F_f estimate have led researchers to propose new explicit F_{ff} and models. A series of equations, which allows estimation of the friction factor in rough and smooth pipes, without carrying out iterative calculations has been proposed. Now, there are numerous explicit approximations to Colebrook – White's formula. These explicit formulae presented variations in the degree of accuracy [7, 8]. Some of the explicit formulae are Moody [9, 10], Wood [11] , Barr [12, 13] , Haaland [14] , Swamee and Jain [15] , Serghide [16] , Altshul-Tsal [17] , and Zigrang and Sylester [18] , Churchill [19, 20] , Jain [21] , Chen [22, 23] , Manadilli [24] , Romeo et al. [25] , Sonnad and Goudar [26] , Eck [27] , Round [28] , Vatankhah and Kouchakzadeh [29] , Buzzelli [30] , Avci and Kargoz [31] , Evangelids *et al* [32] Brkic [33, 34] , Danish *et al.* [35] , Fang *et al.* [36] , Mustafa *et al.* [37] , Vatankhah [38] , Cojbasic and Brikic [39] ,Shaikh *et al.*[7]]. More on F_{ff} and F_f computations can be found in Brikc [40, 41, 42, 43, 44, 45], Taler [46], Samadianfarad et al. [47], Dejan and Carko [48], Clamond [49] and [51] – [58].

The importance of F_f is well known in the selection of pipe size, determination of flows in a pipe, fluid transportation and in the design of potable water supply scheme. There are alot of researches and publications on the F_f estimation in pipe, but documentations on explicit F_{ff} for computing accurate F_f are rare in literature. Advancement in technology and development of high speed computer support the need to document explicit F_{ff} for computing accurate F_f and provide a performance evaluation of each of these F_{ff}. The key objective of this study therefore, is to provide explicit F_{ff} for computing F_f accurately and provide performance evaluation of each of these F_{ff} with particular attention to accuracy using statistical techniques with a larger aim of providing error free formula for F_f computation.

2. MATERIALS AND METHOD

General models that represent a generalization of Churchill [19; 20], Swamee and Jain [15]; Round [28] and Haaland [14]; Romeo *et al.* [25]; Zigrang and Sylester [18]. The models relating F_f, R_e and R_r were proposed. Romeo *et al.* [25] proposed a model of ten constants as follows:

$$\frac{1}{\sqrt{f}} = \left(\alpha_0 log\left(\left(\frac{k}{\alpha_1 D}\right) - \left(\frac{\alpha_2}{R_e}\right) log\left(\left(\left(\frac{k}{\alpha_3 D}\right) - \left(\frac{\alpha_4}{R_e}\right) - log\left(\left(\frac{k}{\alpha_5 D}\right)\right)^{\alpha_6} + \left(\frac{\alpha_7}{\alpha_8 + R_e}\right)^{\alpha_9}\right)\right)\right)\right)^{\alpha_{10}} \tag{4}$$

The proposed F_{ff} in this study are expressed as follows:

$$\text{Formula A:} = \left(\alpha_0 log\left(\left(\frac{k}{\alpha_1 D}\right) - \left(\frac{\alpha_2}{R_e}\right) log\left(\left(\left(\frac{k}{\alpha_3 D}\right) - \left(\frac{\alpha_4}{R_e}\right) - log\left(\left(\frac{k}{\alpha_5 D}\right)\right)^{\alpha_6} + \left(\frac{\alpha_7}{\alpha_8 + R_e}\right)^{\alpha_9}\right)\right)\right)\right)^{\alpha_{10}} \tag{5}$$

$$\text{Formula B: } \frac{1}{\sqrt{f}} = \left(\alpha_0 log\left(\left(\frac{k}{\alpha_1 D}\right) - \left(\frac{\alpha_2}{R_e}\right) log\left(\left(\left(\frac{k}{\alpha_3 D}\right) - \left(\frac{\alpha_4}{R_e}\right) - log\left(\left(\frac{k}{\alpha_5 D}\right)\right)^{\alpha_6} + \left(\frac{\alpha_7}{\alpha_8 + R_e}\right)^{\alpha_9}\right)\right)\right)\right)^{\alpha_{10}} \quad (6)$$

$$\text{Formula: } f = \left(\alpha_0 log\left(\left(\frac{k}{\alpha_1 D}\right) - \left(\frac{\alpha_2}{R_e}\right) log\left(\left(\left(\frac{k}{\alpha_3 D}\right) - \left(\frac{\alpha_4}{R_e}\right) - log\left(\left(\frac{k}{\alpha_5 D}\right)\right)^{\alpha_6} + \left(\frac{\alpha_7}{\alpha_8 + R_e}\right)^{\alpha_9}\right)\right)\right)\right)^{\alpha_{10}} \quad (7)$$

In (4) to (7), α_0 ; α_1; α_2; α_3 α_4 ; α_5 ; α_6 ; α_7; α_8 ; α_9; α_{10} and α_{11} are the constants for friction factor parameters, and f is the F_f. The F_{ff} were proposed and selected based on the complexity and expected accuracy [7; 8]. Colebrook and White's equation was used to estimate different F_f (5240) for R_e between 4 x 10^3 and 1.704 x 10^8 and the R_r of between 1.0 x 10^{-7} and 0.052 using Microsoft Excel Solver to fix the F_{ff}. Microsoft Excel Solver was used in this research for analysis based on easy accessibility and accuracy in numerical solutions. The fixed F_{ff} were used to estimate F_f for R_e between 4 x 10^3 and 1.704 x 10^8 and R_r between 1.0 x 10^{-7} and 0.052. Accuracy of the fixed F_{ff} was evaluated using relative error; model of selection (MSC) and Akaike Information Criterion (AIC) and compared with the previous F_{ff} using Colebrook –White's F_f as the reference. Procedure used in Microsoft Excel Solver can be summarized as follows:

a. Excel solver was added in the Microsoft excel;
b. Target; operation and changing cells were set; and
c. Solver was allowed to iterate at 200 iterations with 0.005 tolerance.

Figure 1 presents flow chart of the procedures for using Microsoft Excel Solver in the computation.

The model of selection criterion (MSC) interprets the proportion of expected F_f variation that are explained by the obtained F_f. A higher value of MSC indicates a higher accuracy, validity and the sound fitness of the method. MSC was computed using equation (8) as follows [6]:

$$MSC = In\left(\frac{\sum_{i=1}^{n}(Y_{obsi} - \bar{Y}_{obs})^2}{\sum_{i=1}^{n}(Y_{obsi} - \bar{Y}_{cali})^2}\right) - \frac{2p}{n} \quad (8)$$

Here, Y_{obsi} is the F_f estimated using Colebrook – White's formula; $\bar{Y}_{obs}$ is the average F_f estimated using Colebrook – White's formula; p is the total number of fixed parameters to be estimated in the equation; n is the total number of F_f estimated, and Y_{cali} is the F_f estimated using developed model equation.

Akaike Information Criterion (AIC) was developed by Akaike [59]. It allows a direct comparison of ***F_{ff}*** with a different number of parameters. The AIC represents a given set of parameter estimates by relating the coefficient of determination to the number of parameters. The Akaike Information Criterion (AIC) was determined using the following expression (Equation 9):

$$AIC = n\left(\text{In}\sum_{i=1}^{n}(Y_{obsi} - \bar{Y}_{cali})^2\right) + 2p \quad (9)$$

Relative error (RErr) was determined using equation (10) as follows [60; 61]:

$$RErr = \sum_{i=1}^{n} 100\frac{\left(\frac{Y_{obsi} - \bar{Y}_{cali}}{\bar{Y}_{cali}}\right)}{n} \quad (10)$$

3. RESULTS AND DISCUSSION

The results and discussion of the study are presented in three categories: the fixed F_{ff} (models); computed F_f using Colebrook – White's formula and fixed F_{ff} and statistical evaluations of the fixed F_{ff}. The fixed F_{ff} parameters obtained are as presented in Table 1.

The parameters are different from parameters in other F_{ff} found in literature such as Romeo *et al.* [25]; Zigrang and Sylvester [18], but the F_{ff} are similar to some of the previous F_{ff} such as Barr [12], Haaland [14]; Jain [15]; Eck [27]; Round [28]; Churchill [19]; Wood [11]; Swamee and Jain [15] ; Brkic [33;34]; Fang *et al.* [36] and Ghanbari *et al.*[52]. This result shows that Microsoft Excel Solver can be used to develop F_{ff} for estimating the F_f.

3.1 Estimated Friction Factors Using These Equations

Figures 2 (a to e) present relationship between R_e, R_r, and F_f. The figures showed that the F_f estimated were similar in shape, but some of the formulae provide sinusoidal nature instead of smooth nature produce in Colebrook – White's formula. Although, the figures were similar three figures were closer to figure from Colebrook – White's formula than other F_{ff}. The figures that are closer to Colebrook – White's F_{ff} are Shalkh *et al.*(Table 2); developed F_{ff} (Figure 2b) and Figure (2c). These results and figures show that these three F_{ff} are more accurate than the other selected F_{ff}. It was also revealed that there are ranges for the accuracy of the previous F_{ff}.

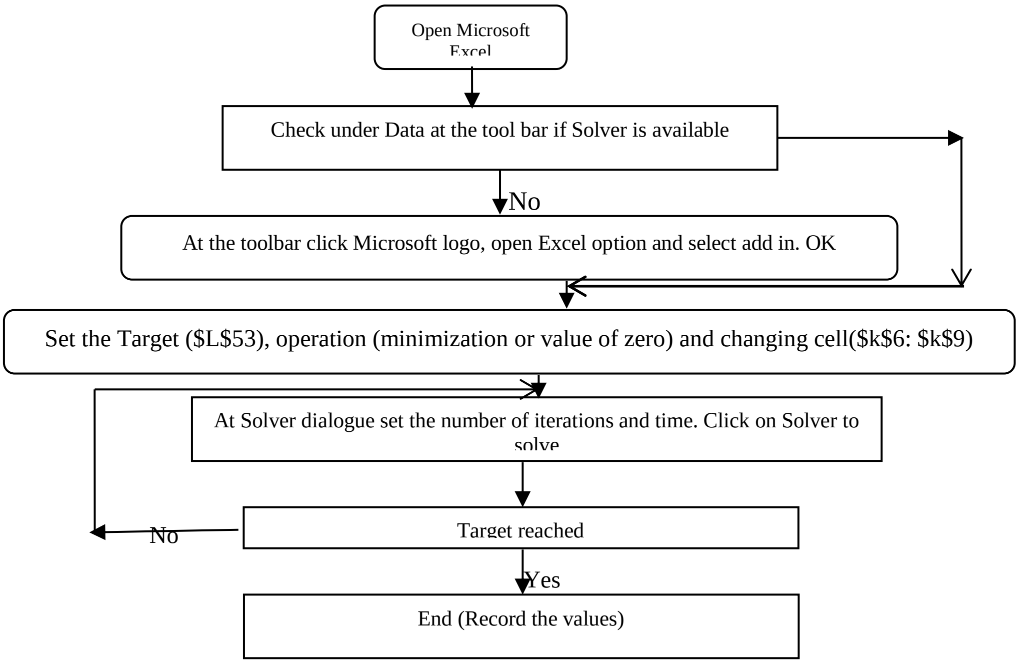

Figure 1: Procedure for using Microsoft Excel Solver in the computation of F_{ff} Parameters

Table 1: Values of the constants

Formula	Constants											
	α_0	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}
A	-1.9969	3.7063	0.1800	3.6636	5.3922	8.3556	1.7048	0.99119	1.0000	1.2744	-2.0017	
B	-2.0641	3.4833	0.1121	6.1374	1.2509	73.6856	0.0074	0.0121	1.9676	10.0947	0.9869	
C	-2.0460	3.5427	0.1120	6.1385	1.2512	73.6864	0.00742	0.0121	1.9676	10.0947	0.8317	-2.382
Romeo *et al* [25]	-2.0000	3.7065	5.0272	3.8270	4.5670	7.7918	0.9924	5.3326	208.815	0.9345	1.0000	

These values revealed that the new F_{ff} are as follows:

Formula A:

$$f=\left(-19969log\left(\left(\frac{k}{3.7063D}\right)-\left(\frac{0.1800}{R_e}\right)log\left(\frac{\frac{k}{3.6636D}-\frac{5.3922}{R_e}}{-log\left(\left(\frac{k}{8.3556D}\right)^{17048}+\left(\frac{099119}{1.000+R_e}\right)^{12744}\right)}\right)\right)\right)^{-2001} \tag{11}$$

Formula B:

$$\frac{1}{\sqrt{f}}=\left(-2064log\left(\left(\frac{k}{3.4833D}\right)-\left(\frac{0.1121}{R_e}\right)log\left(\frac{\left(\frac{k}{6.1374D}\right)-\left(\frac{1.2509}{R_e}\right)}{-log\left(\left(\frac{k}{73.6856D}\right)^{17048}+\left(\frac{0.0121}{1.9676+R_e}\right)^{10.0947}\right)}\right)\right)\right)^{0.9869} \tag{12}$$

Formula C:

$$f=\left(-2046log\left(\left(\frac{k}{3.5427D}\right)-\left(\frac{0.112}{R_e}\right)log\left(\frac{\left(\frac{k}{6.1385D}\right)-\left(\frac{1.2512}{R_e}\right)}{-log\left(\left(\frac{k}{73.6864D}\right)^{17048}+\left(\frac{0.0121}{1.9676+R_e}\right)^{10.0947}\right)}\right)\right)\right)^{-10.0947} \tag{13}$$

Romeo $et\ al$ [25]
$$\frac{1}{\sqrt{\lambda}} = -2\left[log_{10}\left(\left(\frac{k}{3.7065\ D}\right)-\left(\frac{5.0272}{R_e}\right)\right)log_{10}\left(\frac{k}{3.827\ D}\right)-\left(\frac{4.567}{R_e}\right)\right] \quad (14)$$

3.2 Statistical Evaluations

Table 2 provides values of AIC, MSC, and relative error. Figures 3 (a to d) presents variations of the relative errors in respect of R_e and R_r. Based on the statistical evaluations the F_{ff} can be grouped into five categories (based on relative error) as follows:

3.2.1 Perfect Friction Factor Formulae.

These are the formulae with relative error less than 0.5 %. The formulae are Shaikh et al (0.03%) the three fixed F_{ff} (0.02 %; 0.06 % and 0.04 %);

3.2.2 Highly accurate Friction Factor Formulae

These are the formulae with a relative error greater than 0,5% but less than 2.0 %. The formulae are Serghide (0.70%); Swamee and Jain (0.73%); Fang et al. (0.75%); Barr (0.73%); Zigrang and Sylvester (0.84%) ; Haaland (1.52 %); Eck (0.86%) ; Brikic (0.93%); Barr (0.77%); Swamee and Jain (0.73 %); Churchill (0.81 and 0.74 %); Jain (0.86 %); Chen (0.76 %); Buzzelli (0.70 %); Sounnad and Gouadar (0.71 and 0.72 %); Manadilli (0.73 %); Evangelids et al (0.80 %); Vatankhah and Kouchakzadeh (0.71 %); Romeo *et al* (0.71 %) ;

3.2.3 Moderately Accurate Friction Factor Formulae.

These are the formulae with a relative error greater than 2.0 %, but less than 5 %. The formula are Wood (3.48 %); Ghanbari *et al.* (2.17 %), and developed model c (2.15 %).

Table 2: Results of Statistical Evaluations

Friction factor formulae	Churchill	Jain	Chen	Round	Avci and Kargoz	Buzzelli	Swamee and Jain	Ghanbar et al	Shaikh et al	Model A
Relative Error (%)	0.81	0.86	0.76	5.53	47.12	0.70	0.73	2.17	0.03	0.02
AIC	-24191	-24223	-24416	-13827	6708	-24390	-24222	-21596	-55122	-73006
MSC	4.71	4.72	4.76	2.74	-1.18	4.75	4.72	4.22	10.65	14.03
Friction factor formulae	Sonnad and Gouadar[b]	Manadilli	Evangelids et al	Vatankhah and Kouchakzadeh	Romeo et al and Romeo et all Model III	Moody	Wood	Barr	Fang et al	Model B
Relative Error (%)	0.71	0.73	0.80	0.71	0.71	8.56	3.48	0.73	0.75	0.06
AIC	-24447	-24373	-24362	-24401	-24401	-8496	-19519	-24368	-24375	-64580
MSC	4.76	4.75	4.75	4.75	4.75	1.72	3.82	4.75	4.75	12.42
Friction factor formulae	Serghides	Barr[b]	Altshul-Tsal	Zigrang and Sylvester	Haaland	Brkic	Eck	Churchill	Sonnad and Gouadar	Model C
Relative Error (%)	0.70	0.77	17.34	0.84	1.52	0.93	0.86	0.74	0.72	0.04
AIC	-24399	-23632	-1757	-34327	-23895	-22690	-23732	-24204	-24204	-67982
MSC	4.75	4.61	0.43	6.65	4.66	4.43	4.63	4.72	4.72	13.07

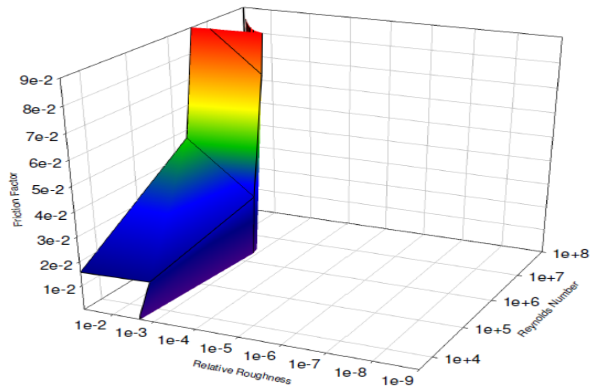

Figure 2a : Relationship between R_e, F_f and R_r using Colebrook- White formula

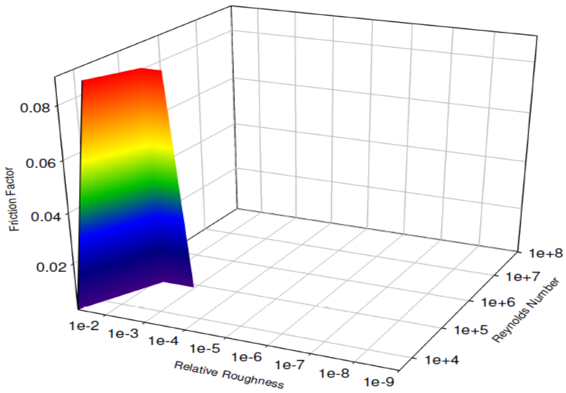

Figure 2e : Relationship between R_e, F_f and R_r using Romeo et al formula

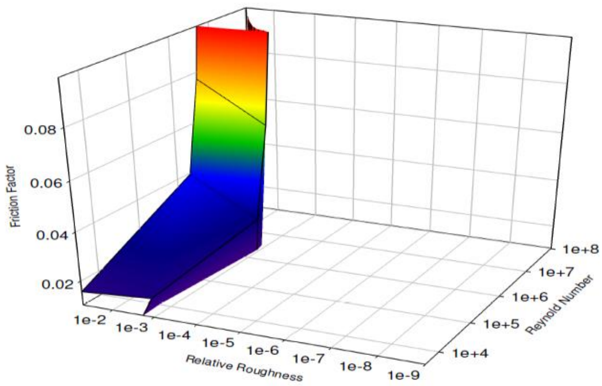

Figure 2b: Relationship between R_e, F_f and R_r using formula A

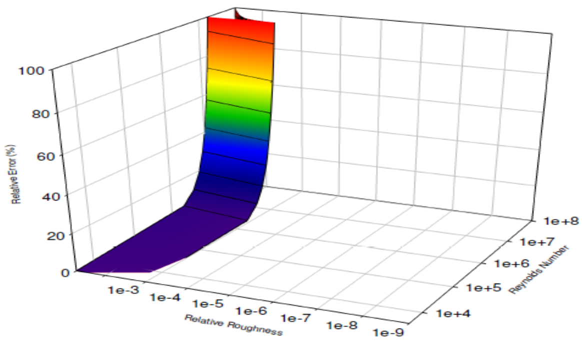

Figure 3a : Relationship between R_e, Relative Error and R_r using formula A

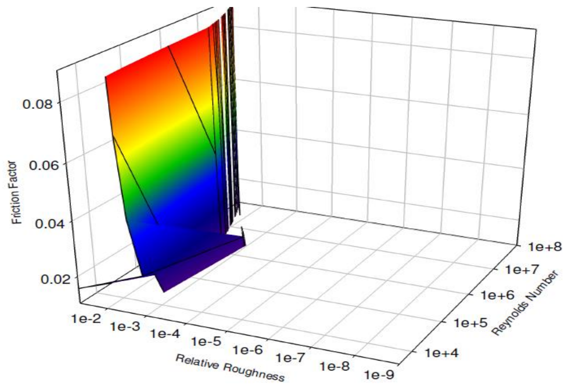

Figure 2c: Relationship between R_e, F_f and R_r using formula B

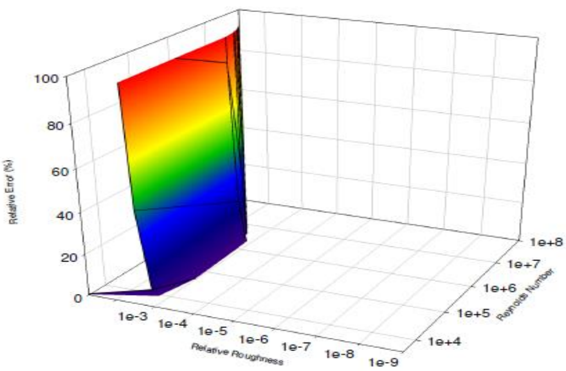

Figure 3b : Relationship between R_e, Relative Error and R_r using formula B

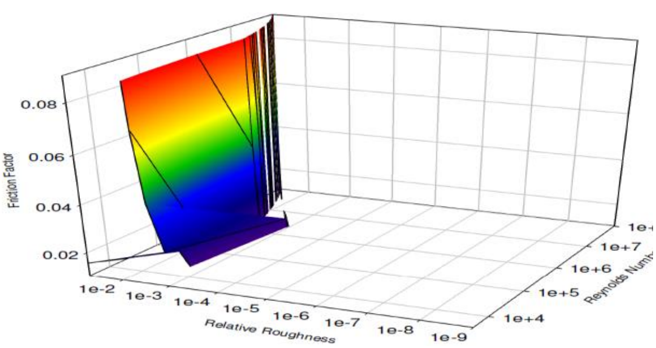

Figure 2d : Relationship between R_e, F_f and R_r using formula C

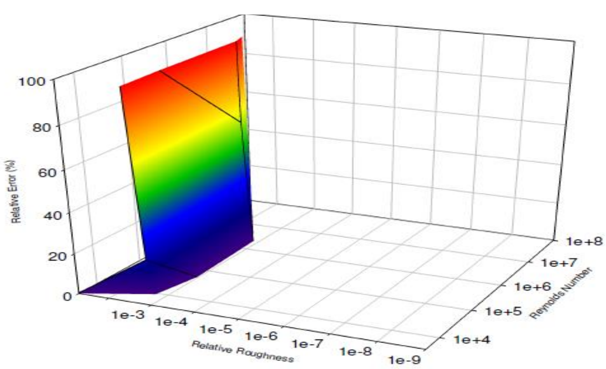

Figure 3c: Relationship between R_e, Relative Error and R_r using formula C

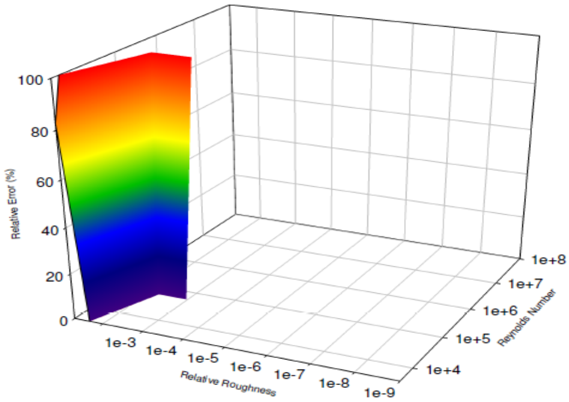

Figure 3d : Relationship between R_e, Relative Error and R_r using Romeo et al formula

3.2.4 Low accurate Friction Factor Formulae.

These are the formulae with a relative error greater than 5 %, but less than 10 %. The formulae are Moody (8.56 %); Round (5.53%).

3.2.5 Least accurate Friction Factor Formulae.

These are the formulae with a relative error greater than 10 %. The formulae are Avci and Kargoz (47.12 %) and Tsal (17.34 %).

3.3 Classification Based on the value of MSC,

The Friction factor formulae can be grouped into five categories as follows:

3.3.1 First choice Friction Factor Formulae.

These are the formulae with MSC greater than 10.00. The formulae are Shaikh et al (10.65), the three fixed F_{ff} (14.03; 12.42 and 13.07).

3.3.2 Highly Accurate Friction Factor Formulae.

These are the formulae with MSC less than 10.00 but greater than 4.00. The formulae are Serghide (4.75); Swamee and Jain (4.72); Fang et al. (4.75); Barr (4.61 and 4.75); Zigrang and Sylvester (6.65) ; Eck (4.63); Haaland (4.66) and Brikic (4.43); Churchill (4.71 and 4.72); Jain (4.72); Chen (4.76); Buzzelli (4.75); Ghanbari *et al.* (4.22); Sounnad and Gouadar (4.76 and 4.72); Manadilli (4.75); Evangelids *et al* (4.75); Vatankhah and Kouchakzadeh (4.75); Romeo et al (4.75; 4.71) ; Swamee and Jain (4.72).

3.3.3 Moderately Accurate Friction Factor Formulae.

These are the formulae with MSC greater than 2.20, but less than 4.00. The formulae are Round (2.74) and Wood (3.82).

3.3.4 Low Accurate Friction Factor Formulae.

These are the formulae with MSC greater than 0.00, but less than 2.20. The formulae are Moody (1.72) and Tsal (0.43).

3.3.5 Least Accurate Friction Factor Formulae.

These are the formulae with MSC less than 0.00. The formula is Avci and Kargoz (-1.13).

3.4 Classification

Based on the value of AIC, the Friction factor formulae can be grouped into four categories as follows:

3.4.1 Highly accurate Friction Factor Formulae.

These are the formulae with AIC less than -20000. The formulae are Shaikh *et al* (-55122), the three fixed F_{ff} (-73006; -64580 and -67982); Serghide (-24399); Swamee and Jain (-24222); Fang *et al.* (-24375); Barr (-23632; -24368); Zigrang and Sylvester (-34327) ; Eck (-23732); Haaland (-23895) and Brikic (-22690); Churchill (-24191 and -24204); Jain (-24223); Chen (-24416);Buzzelli (-24390); Ghanbari *et al.*(-21596); Sounnad and Gouadar (-24447 and -24204); Manadilli (-24373); Evangelids *et al* (-24362); Vatankhah and Kouchakzadeh (-24401); Romeo *et al* (-24401) ;

3.4.2 Moderately Accurate Friction Factor Formulae.

These are the formulae with AIC greater than -20000, but less than -18000. The formulae is Wood (-19519);

3.4.3 Low accurate Friction factor formulae.

These are the formulae with AIC greater than -18000, but less than -7000. The formulae are Round (-13827); and Moody (-8496);

3.4.4 Least accurate Friction Factor Formulae.

These are the formulae with AIC greater than -7000. The formulae are Avci and Kargoz (6708) and Tsal (-1757).

CONCLUSION

Based on the statistical evaluations, which have been done in this work, the most accurate and one of the easiest F_{ff} for use is known to be the current model formula. Being explicit, easy to use and very accurate are the most important characteristics, which cannot be found all together in any of the previous formulae. Based on the results of this study, one can state that this formula could be a better alternative to the existing ones. It can be concluded that:

i. New formulae (numerical formulae) are among the best F_{ff} tools for estimating F_f in pipe flow problems based on MSC, AIC and relative error;
ii. there is the need to perform economics evaluation on these F_{ff} and current F_{ff} to ascertain their reliability; and
iii. these current F_{ff} can be improved upon to 0.005 % relative error.

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