



POSITION ESTIMATION PERFORMANCE EVALUATION OF A LINEAR LATERATION ALGORITHM WITH AN SNR-BASED REFERENCE STATION SELECTION TECHNIQUE

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ABSTRACT

The position estimation accuracy of the multilateration system lateration algorithm depends on several factors such as the number of stations deployed, time difference of arrival (TDOA) estimation technique and the choice of reference station. In this paper, a technique to select the suitable reference station for the lateration algorithm based on received signal-to-noise (SNR) at each of the deployed stations is presented. The position estimation performance analysis of the lateration algorithm with the reference selection technique is carried out and improvement in the position estimation accuracy is determined by comparing it with the convention approach of using fixed reference station. Monte Carlo simulation results when compared with the conventional approach based on a square station configuration showed a reduction in the position estimation error of about 20%.

Keywords: reference selection, lateration algorithm, signal-to-noise-ratio, position estimation

1. INTRODUCTION

Multilateration is a type of wireless surveillance technology used by the air traffic monitoring centre to monitor and regulate air traffic [1]. It uses a two-stage process to estimate the position of the emitting source [1, 2]. The first stage involves time difference of arrival (TDOA) estimation of the received signal at spatially located station pairs [3–6]. In the second stage which is the scope of this paper, the estimated TDOA measurement from the first stage are used with the known deployed station coordinates to determine the location of the emitting source using a position estimation algorithm called lateration algorithm [2]. Due to the non-linear relationship between the TDOA measurements and the emitter position, several approaches have been developed to linearize this relationship. This resulted in the different type of lateration algorithms and can be grouped as linear and non-linear approach [2, 7, 8]. The nonlinear approach lateration algorithm utilizes linear approximation methods and iteration process to establish a linear relationship between the TDOA measurements and emitter position [2, 9, 10]. Due to the iteration process, convergence is an issue and thus, not suitable for a passive positioning application [11]. Algebraic manipulation is used in the linear approach of the lateration algorithm to obtain the linear relationship

[12–15]. It does not suffer convergence issues as no iteration process is used but very sensitive to error in the TDOA measurement resulting to high error in the position estimation process.

Several researchers have proposed techniques to reduce the error in the position estimation process of the lateration algorithm [9, 16–20]. Techniques like the Tikhonov regularization [9] and total least squares (TLS) [16] have been proposed which have shown to be very efficient. The use of multiple reference stations for the lateration algorithm has also been suggested to improve the position estimation accuracy of the lateration algorithm through reduction in position estimation error [17, 18]. Another approach which has been reported is choosing the suitable reference station to be used with the lateration algorithm [17, 19, 21]. A TDOA residual-based method to determine the suitable reference station has been suggested in [21]. It requires for an estimated position of the emitter to be known. Each of the stations is then used with the estimated emitter position to solve for the TDOA residual. The station with the least TDOA residual is chosen as the reference station for the lateration algorithm. In [17], a condition number based technique was proposed to determine the suitable station pair to be used as reference. A matrix whose entry consists of only TDOA measurements is derived and its condition number is

calculated using the TDOA measurement obtained with each of the stations as reference in the TDOA estimation process. The station pair that resulted in the least condition number of the derived matrix is chosen as the reference station for the lateration algorithm. An SNR-based approach was proposed in [19] but the improvement in the position estimation accuracy of the lateration algorithm with the reference selection technique has not determined. Thus, in this research work, the position estimation accuracy of the lateration algorithm is determined with the SNR-based reference selection technique. The Improvement in the position estimation of the lateration algorithm with the reference selection technique is determined through comparison with the conventional approach of using fix station as reference.

The remainder of the paper is organised as follows. Section 2 gives the detailed derivation of the linear approach to the lateration algorithm, which is followed by the methodology for the proposed SNR-based reference selection technique. The simulation, result and discussion is presented in Section 4 and finally the conclusion in Section 5.

2. MULTILATERATION SYSTEM POSITION ESTIMATION METHODOLOGY

In this section, summary of the position estimation methodology used by the multilateration system as reported in [17] to estimate the position of an emitter in 2-D based on the linear approach lateration algorithm is presented.

Let an emitter located at coordinates $\mathbf{x} = [x, y]^T$ transmit a signal that is detected at the i -th station with coordinates $\mathbf{S}_i = [x_i, y_i]^T$ and τ_i be the time taken by the signal to propagate from the emitter to the i -th station. The distance travelled by the signal is mathematically expressed as:

$$d_i = c \times \tau_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \tag{1}$$

whereas, $c=3 \times 10^8$ m/s; which is speed of light. The path difference (PD) measurement between the emitter, the i -th station and the m -th station with coordinates $\mathbf{S}_m = [x_m, y_m]^T$ is mathematically obtained as:

$$\begin{aligned} d_{i,m} &= d_i - d_m = c \times \tau_{i,m} \\ &= \sqrt{(x - x_i)^2 + (y - y_i)^2} \\ &\quad - \sqrt{(x - x_m)^2 + (y - y_m)^2} \end{aligned} \tag{2}$$

where $\tau_{i,m}$ is the measured TDOA of the signal between the i -th and m -th station.

Let the i -th station be chosen as reference while the m -th station, k -th station with coordinates $\mathbf{S}_k = [x_k, y_k]^T$, and n -th station with coordinates $\mathbf{S}_n = [x_n, y_n]^T$ be chosen as the non-reference stations. The PD

measurements for the k -th and n -th station with the i -th station as reference are [17]:

$$d_{i,k} = d_i - d_k \tag{3}$$

$$d_{i,n} = d_i - d_n \tag{4}$$

In practical application, signals are corrupted with noise which result in PD estimation (PDE) error. Modelling the PDE error as a zero mean Gaussian random variable with probability density function as $N(0, \sigma)$ [22], the estimated PD measurements based on Eq. (2), Eq. (3) and Eq. (4) are

$$\hat{d}_{i,m} = d_{i,m} + N(0, \sigma) \tag{5}$$

$$\hat{d}_{i,k} = d_{i,k} + N(0, \sigma) \tag{6}$$

$$\hat{d}_{i,n} = d_{i,n} + N(0, \sigma) \tag{7}$$

Algebraic manipulation of Eq. (5), Eq. (6) and Eq. (7) results in two plane equations which are presented as follows [15]:

$$A_{i,k,m} = xB_{i,k,m} + yC_{i,k,m} \tag{8}$$

$$A_{i,n,m} = xB_{i,n,m} + yC_{i,n,m} \tag{9}$$

where the coefficients of Eq. (8) and Eq. (9) are as follows:

$$\begin{aligned} A_{i,k,m} &= 0.5 \left(\hat{d}_{i,m} - \hat{d}_{i,k} + \frac{k_{i,m}}{\hat{d}_{i,m}} - \frac{k_{i,k}}{\hat{d}_{i,k}} \right), \\ A_{i,n,m} &= 0.5 \left(\hat{d}_{i,m} - \hat{d}_{i,n} + \frac{k_{i,m}}{\hat{d}_{i,m}} - \frac{k_{i,n}}{\hat{d}_{i,n}} \right), \\ B_{i,k,m} &= \frac{X_{k,i}}{\hat{d}_{i,k}} - \frac{X_{m,i}}{\hat{d}_{i,m}}, & B_{i,n,m} &= \frac{k_{n,i}}{\hat{d}_{i,n}} - \frac{k_{m,i}}{\hat{d}_{i,m}}, \\ C_{i,k,m} &= \frac{Y_{k,i}}{\hat{d}_{i,k}} - \frac{Y_{m,i}}{\hat{d}_{i,m}}, & C_{i,n,m} &= \frac{Y_{n,i}}{\hat{d}_{i,n}} - \frac{Y_{m,i}}{\hat{d}_{i,m}}, \\ k_{i,k} &= (x_i^2 + y_i^2) - (x_k^2 + y_k^2), \\ k_{i,m} &= (x_i^2 + y_i^2) - (x_m^2 + y_m^2), \\ X_{i,k} &= x_i - x_k, X_{i,m} = x_i - x_m, \\ Y_{i,k} &= y_i - y_k, Y_{i,m} = y_i - y_m \end{aligned}$$

Eq. (8) and Eq. (9) can be represented in matrix form as follows:

$$M_i \mathbf{x} = \mathbf{b}_i \tag{10}$$

where $M_i = \begin{bmatrix} B_{i,k,m} & C_{i,k,m} \\ B_{i,n,m} & C_{i,n,m} \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\mathbf{b}_i = \begin{bmatrix} A_{i,k,m} \\ A_{i,n,m} \end{bmatrix}$

The TLS equation in Eq. (10) is known as the multilateration 2-D position estimation mathematical model used for emitter locating. Given the PD measurements in Eq. (5) to Eq. (7) and station coordinates, the location of an emitter is obtained by finding the inverse matrix solution of Eq. (10). The matrix in Eq. (10) is a TLS problem which can be solved using Singular Value Decomposition (SVD) TLS approach as illustrated below;

Let $\mathbf{N} = [\mathbf{M}_i, \mathbf{b}_i]$, taking the SVD of matrix N as [16, 23]:

$$\mathbf{N} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \sum_{i=0}^{n+1} \mathbf{u}_i \sigma_i \mathbf{v}_i^T \tag{11}$$

The solution to Eq. (11) which is the estimated emitter position is obtained as:

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \frac{-1}{v_{n+1}, n+1} [v_{i,n+1} \quad v_{2,n+1} \cdots v_{n,n+1}]^T \quad (12)$$

Emitter positions in surveillance systems are presented in the cylindrical coordinate system that is (R_h, θ) where R_h is the horizontal range along the $x - y$ plane and θ is the emitter bearing from a reference point. Conversion from the cylindrical coordinate system to the Cartesian coordinate system can be done using Eq. (13) as follows

$$x = R_h \times \cos(\theta); \text{ and } y = R_h \times \sin(\theta) \quad (13)$$

In the remainder of the paper, the cylindrical coordinate system is used to represent emitter positions.

3. FORMULATION OF RECEIVED SNR-BASED REFERENCE SELECTION TECHNIQUE

The methodology for the proposed reference station to be used with the position estimation algorithm in Section 2 is presented in this section.

Let $\mathbf{x} = (R_h, \theta)$ be the position of an emitter in the cylindrical coordinate system. The received signal powers at each station that is transmitted by the emitter is calculated as:

$$P_r^i(R_h, \theta) = P_t + G_t + G_r - L_i \quad (14a)$$

$$P_r^k(R_h, \theta) = P_t + G_t + G_r - L_k \quad (14b)$$

$$P_r^m(R_h, \theta) = P_t + G_t + G_r - L_m \quad (14c)$$

$$P_r^n(R_h, \theta) = P_t + G_t + G_r - L_n \quad (14d)$$

where:

$$L_i = 32.44 + 20\log_{10}(d_i) + 20\log_{10}(f) \quad (15a)$$

$$L_k = 32.44 + 20\log_{10}(d_k) + 20\log_{10}(f) \quad (15b)$$

$$L_m = 32.44 + 20\log_{10}(d_m) + 20\log_{10}(f) \quad (15c)$$

$$L_n = 32.44 + 20\log_{10}(d_n) + 20\log_{10}(f) \quad (15d)$$

P_t is the emitter transponder transmit power in dBm, G_t is the emitter antenna gain in dBi, G_r the gain of the antenna at each station in dBi while L_i, L_k, L_m and L_n respectively are the propagation losses estimated at the i -th, k -th, m -th and n -th stations.

If the receiver sensitivity remains the same at each station, the received SNR at each station is obtained as:

$$SNR_i(R_h, \theta) = P_r^i(R_h, \theta) - P_n \quad (16a)$$

$$SNR_k(R_h, \theta) = P_r^k(R_h, \theta) - P_n \quad (16b)$$

$$SNR_m(R_h, \theta) = P_r^m(R_h, \theta) - P_n \quad (16c)$$

$$SNR_n(R_h, \theta) = P_r^n(R_h, \theta) - P_n \quad (16d)$$

The effective SNR of the system is obtained as [24]:

$$SNR_{eff} \approx \min\{SNR_i, SNR_k, SNR_m, SNR_n\} \quad (17)$$

From Eq. (17), the effective SNR is the received SNR at the station farthest away from the emitter position and this station has a significant effect on the overall performance of the system. Thus, the station with the least SNR is chosen as the reference station for the lateration algorithm. This is verified using Monte Carlo simulation in Section.

4. SIMULATION, RESULT AND DISCUSSION

In this Section of the paper, the proposed SNR-based reference selection technique presented in section 3 is validated. This is done through comparison of the position estimation error of the lateration algorithm with the reference selection technique and the conventional approach. The conventional approach involves using a fixed station as reference to estimate the position of all emitters within the system coverage. For the conventional approach, the station labelled 1 is chosen as the fixed reference station used for the position estimation as previously done in [25].

The position root mean square error (RMSE) is used as the performance measure to evaluate the position estimation accuracy of the lateration algorithm presented in Section 2 which is mathematically expressed as Eq. (18) at the bottom of this page. In (18), $(\hat{R}_{h,j}, \hat{\theta}_j)$ is the estimated emitter position at the j -th Monte Carlo simulation realization and (R_h, θ) is the known emitter position.

According to Chan et al. [26], simple configurations such as equilateral triangles and squares result in better position estimation accuracy. Thus, for this reason, the square station configuration as shown in Figure 1 is adopted for the performance analysis.

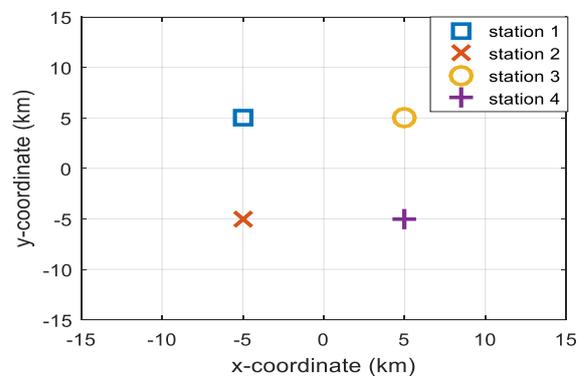


Figure 1: Square station configuration [10 km separation]

$$P_{mse} = \sqrt{\frac{1}{N} \sum_{j=1}^N \left[\left(\hat{R}_{h,j} \cos(\hat{\theta}_j) - R_h \cos(\theta) \right)^2 + \left(\hat{R}_{h,j} \sin(\hat{\theta}_j) - R_h \sin(\theta) \right)^2 \right]} \quad (18)$$

The transmitter/receiver parameters to be used for the SNR calculation are presented in Table 1 [27].

In earlier published articles [8,17,18,21], the choice of reference station is dependent on the emitter bearing relative to the configuration. Keeping the emitter range constant, Figure 2 shows the suitable reference station obtained using the proposed reference station selection technique presented in Section 3 against the emitter bearing from 0° to 359°. The station with the least received SNR value is considered as the reference station.

From Figure 2, at emitter bearing range of 0° to 90°, irrespective of the emitter range, the most suitable reference station to estimate the emitter at those positions with the lateration algorithm is station 2. For emitter bearing range of 91° to 180°, 181° to 270°, and 271° to 359°, the suitable reference stations are station 1, station 3 and station 4 respectively.

For the position estimation accuracy comparison, four emitter positions are considered with equal horizontal range but at different bearings as shown in Table 2.

By varying the PDE error standard deviation from 0m to 2m, the position RMSE based on Eq. (18) of the lateration algorithm with the reference selection technique and that of conventional approach were obtained and compared.

Figure 3 shows the position RMSE comparison between the two approaches at each of the emitter positions defined in Table 2 based on 100 realization Monte Carlo simulation. Comparison between the position RMSE of lateration algorithm with the reference selection technique and the conventional approach based on the square station configuration shows that there is an improvement in the position estimation accuracy by reduction in the position RMSE. Table 3 compares the position RMSE for the selected emitter positions at PDE error standard deviation of 1m. At emitter position A and PDE error standard deviation of 1m, there is a reduction in the position RMSE by about 3.38m (~18%) with the reference selection technique. Extending the analysis to emitter positions B, C and D, the position RMSE is reduced by about 0m (0%), 3.72m (~19%) and 4.99m (~25%) respectively. The reference station selected by the technique for the position estimation at emitter position B corresponds to the same station chosen for the conventional method hence the 0m position RMSE. On the average, about 20% reduction in the position RMSE is achieved with the reference selection technique at the selected emitter positions define in Table 2 based on the square station configuration in Figure 1.

Table 1: Simulation parameter

No.	Parameters	Value
1	Emitter transponder transmit power	30 dBm
2	Emitter carrier frequency	928 Hz
3	Receiver station sensitivity	-90 dBm
4	Receiver station antenna gain	12 dBi
5	Emitter antenna gain	6 dBi

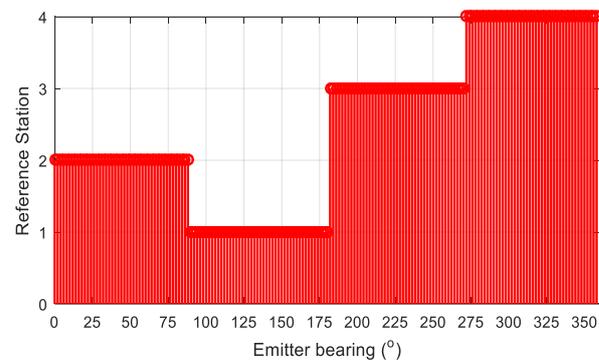


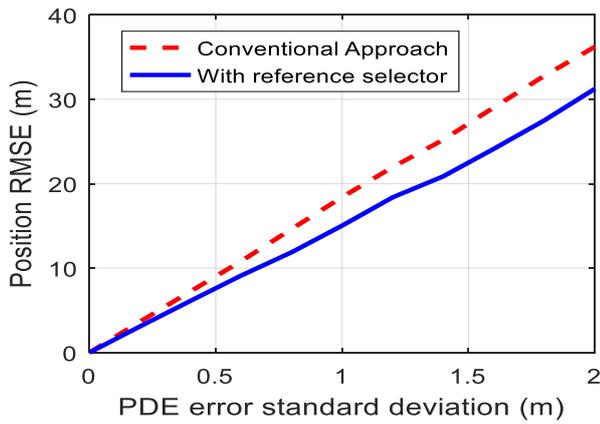
Figure 2: Emitter bearing against the suitable reference station

Table 2: Selected emitter positions for position estimation performance comparison.

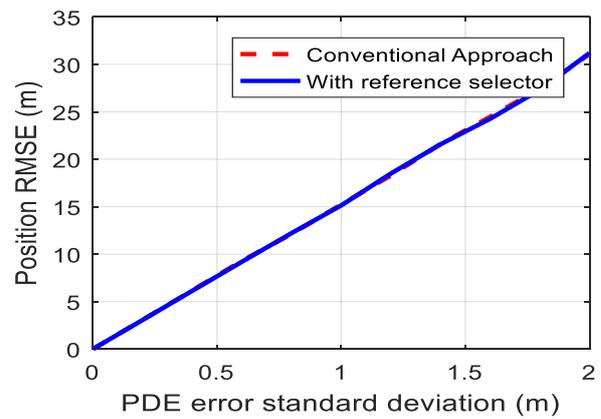
No.	Emitter position	Range (km)	Bearing (°)
1	A	20	60
2	B		120
3	C		240
4	D		330

Table 3: Position RMSE comparison. Green shade indicates the approach with the least position RMSE

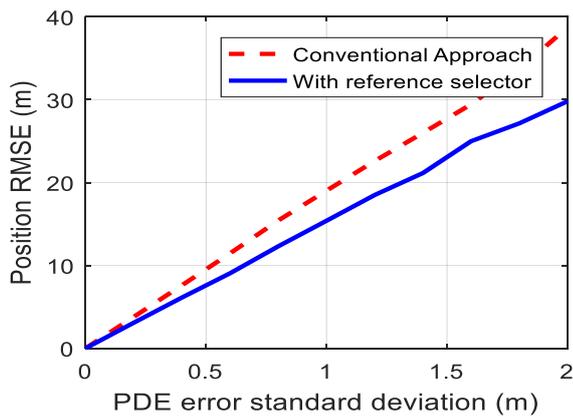
No.	Emitter position	Position RMSE (meters)		% reduction in error
		Conventional approach	With reference selection technique	
1	A	18.41	15.03	18%
2	B	15.14	15.14	0%
3	C	19.02	15.39	19%
4	D	20.19	15.20	25%



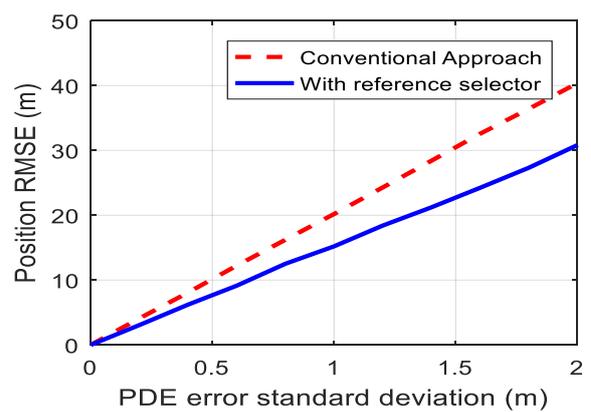
(a) Emitter position A



(b) Emitter position B



(c) Emitter position C



(d) Emitter position D

Figure 3: PE RMSE against PDE error standard deviation

5. CONCLUSION

In this study, a reference selection technique based on received SNR is proposed to improve the position estimation accuracy of a TDOA/PD based lateration algorithm. The position RMSE performance of the lateration algorithm with the reference selection technique is compared with the conventional approach of using a fixed reference station to estimate the positions of all emitters. Simulation results shows that the SNR-based reference selection technique depends on the emitter bearing. Position RMSE comparison shows that on the average, there is a reduction of about 20% in the position RMSE of the conventional approach when the reference selection technique is used. It is assumed that the TDOA/PD measurements have already been obtained but contain error which is modelled as a Gaussian random variable. The accuracy in estimating the TDOA/PD measurements depends on the algorithm used which will subsequently affect the position estimation accuracy of the lateration algorithm with or without the reference selection technique.

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