INVESTIGATING THE EFFECT OF VIBRATION ON THE STABILITY OF THE THREE-WHEELED SCOOTER TAXI

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ABSTRACT

The present three wheeled scooter taxi (TWST) that are widespread in Africa and Asian Countries are fuel economical and inexpensive. However, they are unstable due to their schematic layout. This instability places limitation on the usage of the vehicle. Researchers have investigated the rollover and lateral stabilities of this vehicle, including the effect of vibration on the comfort of the riders. However, not much work has been done on the impact of vibration on the stability of the vehicle. These instabilities could be induced by trenches, potholes, uneven and ungraded roads that are prevalent in developing countries. Therefore, this work modelled and analysed the effect of vibration on a TWST using a standard road bump as reference point. The results proved the vehicle to be unstable in the vicinity of excitation frequency of 15.95 rad/sec and spring constant of 68,600N/m due to resonance. This would affect safety of life and property. Therefore, it would be appropriate for some of the manufacturers of these vehicles to provide for enough safety margins in the design and selection of springs where the vehicles are rollover and laterally unstable. This will enhance the vehicle safety and receptivity.

Keywords: Three wheeled Vehicle, Vibration Modelling and Analysis, Design, Safety of Life and Property

1. INTRODUCTION

Three-wheeled vehicle (TWV) usage, in different countries, varies widely. In developed nations such as United States of America and Canada, they are used mainly for recreation purposes such as shopping, racing and exercises while in developing countries they are used mainly for commercial purposes [1-5]. Tricycles are either motorised or powered through pedals. They are also used by paraplegics [6].

According to history, the first known TWV or trike was developed by a German paraplegic called Stephan Farffler, in 1655 [7]. He was a horologist that wanted to be transportable. The tricycle was operated by hand cranks. In 1797, French inventors developed a trike powered by pedals. This was followed by series of inventions and development of tricycle in Britain in 19th century with different models produced by different manufacturers [7].

In recent decades, the global economy suffered series of downward trends, leading to unemployment worldwide. As a result, the commercial TWV, called three-wheeled scooter taxi (TWST) shown schematically in Figure 1, was introduced into the third world economy in the nineteen eighties as a means of reducing unemployment and poverty in urban environments [1, 8].

![Figure 1: Schematic Diagram of Three-wheeled Scooter (TWST)](http://dx.doi.org/10.4314/njt.v39i4.1)

However, the firmness of the vehicle on ground has been the main concern [1-4, 9-11]. Many researchers have investigated the rollover and lateral stabilities of the vehicle including the effect of vibration on the
comfort of the riders [1-6, 12]. However, not much work has been done on the impact of vibration on the stability of the vehicle. The instability could be induced by unbalanced wheel, potholes, unfilled trenches, bridge gaps, road bumps and humps, and uneven and ungraded roads [13] which these vehicles have to transverse daily, especially in developing economies.

The focus of this paper is to investigate the effect of vibration on the stability of TWST due to these factors, using standard road bumps as reference point, so as to avoid some of the possible negative impact on the rollover and lateral instabilities of the vehicle [1 – 4]. These factors induce amplitudes which could cause the vehicle to be unstable. However, road bumps are effective charmers of traffic in urban environments, thereby reducing pedestrian road accidents [14]. Nevertheless, improperly designed road bumps and vehicle suspensions, and uneven and ungraded roads coupled with excessive speeds could be inimical to commuters’ safety.

2. MATERIALS AND METHODS

In this section, mathematical models for the analysis of vibration of Three-Wheeled Vehicle (TWV) were developed with the analytical solution method taken into consideration.

2.1 Six degree-of-freedom System Forced Vibration Modelling

Vibration in dynamics has some effect on vehicle stability. In this subsection, six degree-of-freedom system vibration model was derived. It is assumed that vehicle body can only vibrate in x-direction, roll and pitch in y and z-directions respectively as shown in Figure 2. Also, the three wheels can only vibrate in x-direction. These make a total of six degree-of-freedom model. Mathematical model formulations involved the use of schematic models and free-body diagrams, Newton’s and Euler’s equations (force and torque equations). Shown in Figure 2 is a schematic model of TWV with six degree-of-freedom vibration. Shown in Figure 3 is a full suspension for TWV based on Figure 2, with $m_1$ as the mass of the body and $m_2$ as mass of each wheel as they are assumed to be the same. The wheels are assumed to have different excitation at any point in time due to road contours that are not uniform. Shown in Figure 4 is a free-body-diagram of the full suspension of a TWV based on Figure 3.

Equations (1) to (6) show the governing differential equations of a TWV for forced vibration that was derived, using Newton’s and Euler’s Equations and the Free-body-diagram shown in Figure 4:

\[
\begin{align*}
\dot{h} &= \ddot{x} \\
Mg &= \text{weight of vehicle} \\
C.G &= \text{center of gravity} \\
W_B &= \text{wheel base} \\
W_T &= \text{wheel track} = 2b \\
L_1 &= \text{distance from front wheel center to C.G} \\
L_2 &= \text{distance from rear wheel center to C.G} \\
k_1, k_2, k_3 &= \text{springs} \\
m_1, m_2 &= \text{masses} \\
x_1, x_2, x_3 &= \text{displacements} \\
\theta &= \text{angle}
\end{align*}
\]
\[ m\ddot{x}_1 + k_1(x_1-x_2 + L\dot{\alpha} + k_2(x_1-x_2 - b\theta - L\dot{\alpha}) + k_3(x_1-x_3 + b\theta - L\dot{\alpha}) - c_1(x_1-x_2 + L\dot{\alpha}) + c_2(x_1-x_3 + b\theta - L\dot{\alpha}) = 0 \quad (1) \]

\[ l_{xx}\ddot{\theta} - bk_2(x_1-x_3 + b\theta - L\dot{\alpha}) + bk_3(x_1-x_3 + b\theta - L\dot{\alpha}) - b\dot{c}_2(x_1-x_4 - b\theta - L\dot{\alpha}) + b\dot{c}_3(x_1-x_3 + b\theta - L\dot{\alpha}) = 0 \quad (2) \]

\[ m\ddot{x}_2 - k_1(x_1-x_2 + L\dot{\alpha}) - c_1(x_1-x_2 + L\dot{\alpha}) + k_3(x_2 - y_1) = 0 \quad (3) \]

\[ m\ddot{x}_3 - k_2(x_1-x_3 + b\theta - L\dot{\alpha}) - c_2(x_1-x_3 + b\theta - L\dot{\alpha}) + k_3(x_3 - y_2) = 0 \quad (4) \]

\[ m\ddot{x}_4 - k_2(x_1-x_4 - b\theta - L\dot{\alpha}) - c_2(x_1-x_4 - b\theta - L\dot{\alpha}) + k_3(x_4 - y_3) = 0 \quad (5) \]

\[ m\ddot{x}_5 - k_2(x_1-x_5 - b\theta - L\dot{\alpha}) - c_2(x_1-x_5 - b\theta - L\dot{\alpha}) + k_3(x_5 - y_4) = 0 \quad (6) \]

2.2. Analysis of the Effect of Vibration due to Bumpy-Roads on the Stability of TWST

The three parameters that are of interest under multi-bodies vibration dynamics are: Amplitude, Acceleration and Frequency. The effect of amplitude is to cause rollover, slip or slide of a vehicle. Moreover, it could generate gyroscopic couple and Coriolis acceleration, with their attendant negative affect on lateral stability [13, 15].

It is assumed that if the front wheel or the two rear wheels of a TWV are ridden over a road bump-like-obstacle on the roads that it cannot induce rollover instability. However, if one of rear wheels rides over the obstacle that it can induced rollover instability to the vehicle. If the roll and pitch movement of the vehicle body are neglected, the governing six-degree-of-freedom differential equations derived as shown in equations (1) to (6) can be reduced to two-degree-of-freedom for forced vibration, on any of the rear wheels as shown in Figure 5.

Equations (7) and (8) were derived based on the free body diagram in Figure 5 as follows:

\[ m_1\ddot{\bar{x}}_1 + k_1(\bar{x}_1 - x_2) + c_1(\bar{x}_1 - \dot{x}_2) = 0 \quad (7) \]

\[ m_2\ddot{x}_2 - k_1(x_1-x_2) - c_1(\bar{x}_1 - \dot{x}_2) + k_2(x_2) = F\sin wt \quad (8) \]

Where \( k_2 x_3 \) can be expressed as \( F \sin wt \)
Some parameters derived from equations (7) and (8) are extracted from study of [17] as:

\( e = \frac{m_1}{m_2} \)  
\( \omega_1 = \sqrt{\frac{k_1}{m_1}} \)  
\( \omega_2 = \sqrt{\frac{k_2}{m_2}} \)  
\( \alpha = \frac{\omega_1}{\omega_2} \)  
\( \gamma = \frac{\omega}{\omega_1} \)  
\( \xi = \frac{2m_2\omega_1}{w_1} \)  
\( w_1 = [y^2(\alpha^2 - 1) + (1 - (1 + \epsilon)\alpha^2)] \)  
\( w_2 = 2\gamma\alpha [(1 - (1 + \epsilon)\gamma^2)\alpha^2] \)  
\( \mu^2 = \frac{4\epsilon^2\gamma^2 + 1}{w_1^2 + w_2^2} \)  
\( \tau^2 = \frac{4\epsilon^2\gamma^2 + 1 + \gamma^2(y^2 - 2)}{w_1^2 + w_2^2} \)  
\( \eta^2 = \frac{\gamma^4}{w_1^2 + w_2^2} \)

The solutions are given as:

\[ x_1 = X_1 \cos(\omega t - \varphi_1) = \mu X_3 \cos(\omega t - \varphi_1) \]  

Therefore, from equation:

\[ \varphi_1 = \cos^{-1}\left(\frac{X_3}{X_1}\right) \]

### Table 1: Parametric values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rear weight x 0.5 less Mass of Rear Wheel</td>
<td>m₁</td>
<td>114.7kg</td>
<td>[4]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>131.7kg</td>
<td></td>
</tr>
<tr>
<td>Mass of Wheel</td>
<td>m₂</td>
<td>160.7kg</td>
<td>By Experiment</td>
</tr>
<tr>
<td></td>
<td></td>
<td>190.2kg</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>219.7kg</td>
<td></td>
</tr>
<tr>
<td>Mass of Wheel (may include: shaft and drum)</td>
<td>m₂</td>
<td>10.3kg</td>
<td>[9]</td>
</tr>
<tr>
<td>Spring constant for rear spring</td>
<td>k₁</td>
<td>50,400 N/m(kd) and 49,800 N/m(kc)</td>
<td>[19, 20]</td>
</tr>
<tr>
<td>Spring constant for front spring</td>
<td>k₁</td>
<td>44,100N/m(ka) and 68,600Nm(ke)</td>
<td>[1]</td>
</tr>
<tr>
<td>Spring constant for rear spring</td>
<td>k₂</td>
<td>48,000 N/m(kb)</td>
<td>[9]</td>
</tr>
<tr>
<td>Tire stiffness rear left or right</td>
<td>k₂</td>
<td>250,490 N/m</td>
<td>[1]</td>
</tr>
<tr>
<td>Hydraulic double acting shock absorber Rear (Left or Right)</td>
<td>C</td>
<td>2,207.5 Ns/m</td>
<td>[19, 20]</td>
</tr>
<tr>
<td>Road Bump Excitation Frequency Computed</td>
<td>ω</td>
<td>15.95 Rad/sec</td>
<td>[1]</td>
</tr>
</tbody>
</table>
INVESTIGATING THE EFFECT OF VIBRATION ON THE STABILITY OF THE THREE-WHEELED SCOOTER TAXI,

S. J. Ojolo, et al

Figure 6: Amplitude of force vibration over one complete cycle due to standard bump

Figure 7: Amplitude versus occupants with 15.95 rad/s and 10.3kg wheel weight

Figure 8: Amplitude versus occupants with 15.95 rad/s and 7kg wheel weight

Figure 9: Amplitude versus occupants with 18.18 rad/s and 10.3kg wheel weight

Figure 10: Amplitude versus occupants with 18.18 rad/s and 7kg wheel weight

Shown in Figures 7-10 are the amplitudes of the vehicle body due to vibration, over two different bump profiles, with two different rear wheel weights and varying spring constants based on Table 1 and equation (12). All the amplitudes were reduced by 0.08m for plot clarity purposes.

It is expected that the higher the spring constant and the more the load, the less would the amplitude of vibration. However, as can be observed from Figure 7-10 this is not the case. The reason is that the closer the natural frequency of vehicle is to that of road excitation, the higher the amplitude and the further the less. The amplitude of vibration increases towards the resonant frequency and thereafter begins to decrease.

From Figures 7 and 8 increase or decrease of the front wheel weight relative to the vehicle body weight has no much effect on amplitude of vibration of the vehicle. The same thing is applicable in Figures 9 and 10. It can be concluded that change in wheel weight has no significant effect on amplitude of vibration.

However, changes in frequencies of wheels over bumps do affect amplitude of vibration, as can be seen from Figure 7 and 8 when compared with Figures 9 and 10.

The implication of this is that vehicle designers should select appropriate spring in line with international standard bumps and local realities to minimize effect of resonance frequency, so as to avoid its negative impact on the vehicle stability. The average rollover angle of TWST that causes instability is 28° [3] which amounts to lifting one of rear wheels by about 0.30m.

4. CONCLUSION

This study came up with six degree-of-freedom vibration model for the determination of stability of TWV in order to enhance safety of life and property.

The model was used in the analysis of the rollover stability of one of the familiar TWST on African and Asian roads...
based on standard road bump-like profiles. The vehicle was found to be rollover unstable with three to four occupants by climbing over a bump-like profile of 15.95 rad/sec with spring constant of 68,600N/m on one of the rear wheels. However, with spring constant range 48,000N/m to 51,000N/m over these bump-like profiles the vehicle will be safe. The implication of this is that vehicle designers should select appropriate springs in line with international standard bumps and local realities to minimize effect of resonance frequencies, so as to avoid their negative impact on the vehicle stability.

In addition, gyroscopic couple would be developed due to oscillation of the rear axle of the vehicle by bumps, uneven and ungraded roads, while in motion, which could induce lateral instability to the vehicle [13]. Nonetheless, it can be reduced on the TWST by introducing tyres with higher cornering coefficient at the rear [4]. Else, the vehicle should be redesigned to take care of rollover and lateral stabilities, and the effect of resonance frequencies.

5. REFERENCES


