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Effect of Orientation Angles and Boundary Conditions on the Pure Bending Coefficient of Thin Laminated Composite Plate using Euler-Bernoulli Equilibrium Equation

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Abstract

This work presents the effect of orientation angles and boundary conditions on the pure bending coefficient of thin rectangular laminated composite plates using Euler-Bernoulli equilibrium equation. The total potential energy function was applied assuming that the thin rectangular laminated composite plate was subjected to only pure bending effect, through this means, the Euler-Bernoulli governing equation of equilibrium was obtained. The governing Equation obtained was minimized to obtain Equation for obtaining the pure bending coefficient of thin rectangular laminated composite plate. Different orientation angles of laminas and boundary conditions were considered to know their effect on pure bending coefficient. Some of the plate boundary condition considered were SSSS, CCCC SSCC and SCSC. The orientation angles considered were $0^0 0^0$, $0^0 90^0$, $0^0 0^0 0^0$, $0^0 90^0 0^0$, $0^0 0^0 0^0 0^0$, and $0^0 90^0 90^0 0^0$. The aspect ratio considered ranges from 1-2 for elastic modular ratio E1/E2 = 25; G12/E2 = 0.5; V12 = 0.25. For any laminated composite plate considered, the orthotropic plate has the same pure bending coefficients but the value increases with increase in the aspect ratio. When other orientations were considered, the change in orientation angles produces change in the pure bending coefficient which increases with increase in aspect ratio. The maximum pure bending coefficient was obtained when SSSS thin rectangular laminated composite Plate was considered for two laminas with orientation $0^0 90^0$ as 0.0174 while the minimum pure bending coefficient was obtained when CCCC thin laminate composite plate was considered for orthotropic plate as 0.00153.

Keywords: Pure Bending, Orientation angle, Boundary condition, Laminated thin plate

1.0 INTRODUCTION

Pure Bending failure is a prominent failure mode experienced when thin laminated composite plates are in use for engineering works which need to be examined critically. Other forms of failure modes include buckling failure and free vibration [1].

The goal of this paper is to present the effect of orientation angles and different boundary conditions on the pure bending coefficient of thin rectangular laminated composite plate. It will also present a particular equation for obtaining the pure bending analysis of thin rectangular laminated composite plate when free vibration and bucking are zeros.

There are different methods of analyzing pure bending of thin laminated composite plate but most of the methods are based on assumed displacement function [1−7]. Some also assume displacement function as well as finite element method of analysis which is a notable approximate analytic method [4]. It is widely agreed that assumed displacement function will always give assumed result except where the displacement function assumed is the exact displacement.

Based on the known fact that most pure bending analysis on thin laminated composite plate are done using assumed displacement function, the present research aim at using the Euler-Bernoulli equilibrium equation which has been accepted as the deflected shape of beam strip to analyze thin laminated composite plate.

The method applied is based on Classical plate theories which is widely acceptable for thin rectangular laminated plate analysis. Classical plate theories (CPT) are based on Kirchhoff's hypothesis which assumes that normal to the mid – surface of the plate before deformation

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remain straight and normal to the mid – surface after deformation. [3].

2.0 DISPLACEMENT FIELD AND KINEMATICS OF A LAMINA OF THIN LAMINATED PLATE

The governing assumptions in this study is based on the work of Ibeaugblem [3], the plane stress assumption used for thin plate analysis (normal stress along z-axis, x-z plane and y-z plane shear stresses are zeros) another assumption is normal strain along z axis is so small that neglecting it shall not affect the gross response of the plate. Itemizing the assumptions gives.

i. $\sigma_{zz} = 0$

ii. $\tau_{xz} = 0$

iii. $\tau_{yz} = 0$

iv. $\varepsilon_{zz} = 0$

Two in-plane displacements and one out-of-plane displacement (u, v and w respectively) constitute the displacement field. From the fourth assumption it is taken that the out-of-plane displacement (deflection) is constant along z-axis, which means it is not a function of z. However, the two in-plane displacements (u and v) are functions of all coordinates (x, y and z) from assumption ii and iii, it is taken that corresponding x-z and y-z planes' shear strains are zeros. Thus, the in-plane displacements are given as:

$$\mathrm{u} = -\mathrm{z}\frac{\mathrm{dw}}{\mathrm{dx}} + \mathrm{u}_0 \tag{1}$$

$$\mathrm{v} = -\mathrm{z}\frac{\mathrm{dw}}{\mathrm{dy}} + \mathrm{v}_0 \tag{2}$$

The in-plane displacements of the middle surface (u_0 and v_0) are not constants [8]. Using equations 1 and 2 and the no-constant values of u_0 and v_0, in-plane strains are defined as:

$$\varepsilon_{xx} = \frac{du}{dx} = \varepsilon_{xx}^0 + \varepsilon_{xx}^i = \frac{du_0}{dx} - z\frac{d^2w}{dx^2} \tag{3}$$

$$\varepsilon_{yy} = \frac{dv}{dy} = \varepsilon_{yy}^0 + \varepsilon_{yy}^i = \frac{dv_0}{dy} - z\frac{d^2w}{dy^2} \tag{4}$$

$$\gamma_{xy} = \varepsilon_{xy} + \varepsilon_{yx} = \left[-z\frac{d^2w}{dxdy} + \frac{du_0}{dy}\right] + \left[-z\frac{d^2w}{dxdy} + \frac{dv_0}{dx}\right]$$

That is:

$$\gamma_{xy} = \gamma_{xy}^0 + \gamma_{xy}^i = \left(\frac{du_0}{dy} + \frac{dv_0}{dx}\right) - 2z\frac{d^2w}{dxdy} \tag{5}$$

2.1 Constitutive relations for a lamina of thin laminated plate

The Hook's law equation for one lamina in laminated plate is given as:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{bmatrix} = E_0\begin{bmatrix}e_{11} & e_{12} & 0\\ e_{12} & e_{22} & 0\\ 0 & 0 & e_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{bmatrix} \tag{6}$$

$Where$:

$$e_{11} = \frac{E_{11}/E_0}{1-\mu_{xy}\mu_{yx}};\ e_{12} = \frac{\mu_{21}.E_{11}/E_0}{1-\mu_{xy}\mu_{yx}} = \frac{\mu_{12}.E_{22}/E_0}{1-\mu_{xy}\mu_{yx}};$$

$$e_{22} = \frac{E_{22}/E_0}{1-\mu_{xy}\mu_{yx}};\ e_{33} = \frac{G_{12}}{E_0}$$

E_0 is the reference Elastic modulus.it can be E_{11} or E_{22}. E_{ij} is the modulii of elasticity and μ_{ij} is and Poisson's ratios in the i and j directions of an anisotropic lamina.

Using the transformation matrix [T], Equation 6 is transformed from (1-2 local) coordinate system to (x-y global) coordinate system as [10].

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy2}\end{bmatrix} = E_0\left\{[T]^{-1}\begin{bmatrix}e_{11} & e_{12} & 0\\ e_{12} & e_{22} & 0\\ 0 & 0 & e_{33}\end{bmatrix}[T]^{-T}\right\}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} \tag{7}$$

Here the transformation matrix, [T] is defined as:

$$[T] = \begin{bmatrix}m^2 & n^2 & 2mn\\ n^2 & m^2 & -2mn\\ -mn & mn & (m^2-n^2)\end{bmatrix} \tag{8}$$

$Where$: $m = Cos\theta\ and\ n = Sin\theta$

Substituting Equation 8 into Equation 7 gives Equation 9

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} = E_0\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} \tag{9}$$

Where:

$$a_{11} = m^4 e_{11} + 2m^2 n^2 (e_{12} + 2e_{33}) + n^4 e_{22}$$

$$a_{12} = e_{12}(n^4 + m^4) + m^2 n^2 (e_{11} + e_{22} - 4e_{33})$$

$$a_{13} = m^3 n (e_{11} - e_{12} - 2e_{33}) + mn^3 (e_{12} - e_{22} + 2e_{33})$$

$$a_{22} = n^4 e_{11} + 2m^2 n^2 (e_{12} + 2e_{33}) + m^4 e_{22}$$

$$\begin{aligned} a_{23} = mn^3 (e_{11} - e_{12} - 2e_{33}) \\ + m^3 n (-e_{22} + e_{12} + 2e_{33}) \end{aligned}$$

$$a_{33} = m^2 n^2 (e_{11} - 2e_{12} + e_{22} - 2e_{33}) + e_{33}(m^4 + n^4)$$

$$a_{21} = a_{12}, \qquad a_{31} = a_{13} \ \ and \ \ a_{32} = a_{23}$$

Substituting Equations 3, 4 and 5 into Equation 9 gives Equation 10:

$$[\sigma] = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = E_0 [a_{ij}][\varepsilon] \qquad (10a)$$

Where:

$$[a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad (10b)$$

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{du_0}{dx} - z\frac{d^2w}{dx^2} \\ \frac{dv_0}{dy} - z\frac{d^2w}{dy^2} \\ \left(\frac{du_0}{dy} + \frac{dv_0}{dx}\right) - 2z\frac{d^2w}{dxdy} \end{bmatrix} \qquad (11)$$

2.2 Total potential energy functional for a laminated thin rectangular plate

The total potential energy functional for a laminated thin rectangular plate is given as:

$$\Pi = \frac{1}{2}\iiint [\sigma][\varepsilon] \ dx\,.dy\,.dz - \iint qw \ dx\,dy \qquad (12)$$

Substituting Equations 10a and 11 into Equation 12 gives Equation 13:

$$\Pi = \frac{E_0}{2}\iiint [\varepsilon]^T [a_{ij}][\varepsilon] \ dxdydz - \iint qw\,dxdy \qquad (13)$$

Carrying out the multiplication and closed domain integration of Equation 13 with respect to z gives Equation 14:

$$\begin{aligned} \Pi = \frac{E_0 t^3}{2}\iint \Bigg\{ & \left(\frac{A_{11}}{t^2}\left[\frac{du_0}{dx}\right]^2 + 2\frac{A_{12}}{t^2}\frac{du_0}{dx}\frac{dv_0}{dy} + \frac{A_{33}}{t^2}\left[\frac{du_0}{dy}\right]^2 + 2\frac{A_{33}}{t^2}\frac{du_0}{dy}\frac{dv_0}{dx} + \frac{A_{33}}{t^2}\left[\frac{dv_0}{dx}\right]^2 + \frac{A_{22}}{t^2}\left[\frac{dv_0}{dy}\right]^2\right) \\ & + 2\left(\frac{B_{11}}{t}\frac{du_0}{dx}.\frac{d^2w}{dx^2} + \frac{(B_{12} + 2B_{33})}{t}\frac{du_0}{dy}\frac{d^2w}{dxdy} + \frac{(B_{12} + 2B_{33})}{t}\frac{dv_0}{dx}\frac{d^2w}{dxdy} + \frac{B_{22}}{t}\frac{dv_0}{dy}\frac{d^2w}{dy^2}\right) \\ & + 2\left(\frac{A_{13}}{t^2}\frac{du_0}{dx}\frac{du_0}{dy} + \frac{A_{13}}{t^2}\frac{du_0}{dx}\frac{dv_0}{dx} - 3\frac{B_{13}}{t}\frac{du_0}{dx}\frac{d^2w}{dxdy} - \frac{B_{13}}{t}\frac{dv_0}{dx}\frac{d^2w}{dx^2} + 2D_{13}\frac{d^2w}{dx^2}\frac{d^2w}{dxdy}\right) \\ & + \left(D_{11}\left[\frac{d^2w}{dx^2}\right]^2 + 2(D_{12} + 2D_{33})\left[\frac{d^2w}{dxdy}\right]^2 + D_{22}\left[\frac{d^2w}{dy^2}\right]^2\right) \\ & + 2\left(\frac{A_{23}}{t^2}\frac{du_0}{dy}\frac{dv_0}{dy} + \frac{A_{23}}{t^2}\frac{dv_0}{dy}\frac{dv_0}{dx} - 3\frac{B_{23}}{t}\frac{dv_0}{dy}\frac{d^2w}{dxdy} - \frac{B_{23}}{t}\frac{du_0}{dy}\frac{d^2w}{dy^2} + 2D_{23}\frac{d^2w}{dy^2}\frac{d^2w}{dxdy}\right)\Bigg\} dx\,.dy - \iint qw\,dx\,dy \end{aligned} \qquad (14)$$

Where:

$$A_{ij} = \frac{\overline{A_{ij}}}{t} \ \ and \ \ \overline{A_{ij}} = t\sum_{m=1}^{m=n} a_{ij}(s_m - s_{m-1}) \qquad (15)$$

$$B_{ij} = \frac{\overline{B_{ij}}}{t^2} \ and \ \overline{B_{ij}} = \frac{t^2}{2}\sum_{m=1}^{m=n} a_{ij}(s_m^2 - s_{m-1}^2) \qquad (16)$$

$$D_{ij} = \frac{\overline{D_{ij}}}{t^3} \ and \ \overline{D_{ij}}. = \frac{t^3}{3}\sum_{m=1}^{m=n} a_{ij}(s_m^3 - s_{m-1}^3) \qquad (17)$$

"m" stands for the number of a lamina in the laminated plate, n is the total number of laminas "s" is the non dimensional coordinate along z-axis defined as s = z/t.

Let the summation of the following three constants be unity (one). That is:

$$n_1 + n_2 + n_3 = 1 \qquad (18)$$

Substituting Equation 18 into Equation 14 to multiply the load, q (that is: q = n_1q+ n_2q + n_3q) rearranging the resulting quation gives Equation 19:

$$\Pi = \Pi_1 + \Pi_2 + \Pi_3 \qquad (19)$$

Where:

$$\Pi_1 = \frac{E_0 t^3}{2} \iint \{ \left(\frac{A_{11}}{t^2}\left[\frac{du_0}{dx}\right]^2 + 2\frac{A_{12}}{t^2}\frac{du_0}{dx}\frac{dv_0}{dy} + \frac{A_{33}}{t^2}\left[\frac{du_0}{dy}\right]^2 + 2\frac{A_{33}}{t^2}\frac{du_0}{dy}\frac{dv_0}{dx} + \frac{A_{33}}{t^2}\left[\frac{dv_0}{dx}\right]^2 + \frac{A_{22}}{t^2}\left[\frac{dv_0}{dy}\right]^2 \right)$$
$$-2\left(\frac{B_{11}}{t}\frac{du_0}{dx}.\frac{d^2w}{dx^2} + \frac{(B_{12}+2B_{33})}{t}\frac{du_0}{dy}\frac{d^2w}{dxdy} + \frac{(B_{12}+2B_{33})}{t}\frac{dv_0}{dx}\frac{d^2w}{dxdy} + \frac{B_{22}}{t}\frac{dv_0}{dy}\frac{d^2w}{dy^2} \right)$$
$$+\left(D_{11}\left[\frac{d^2w}{dx^2}\right]^2 + 2(D_{12}+2D_{33})\left[\frac{d^2w}{dxdy}\right]^2 + D_{22}\left[\frac{d^2w}{dy^2}\right]^2 \right) - n_1 \iint qw \quad dx\,dy \qquad (20a)$$

$$\Pi_2 = \frac{2E_0 t^3}{2} \iint \left[\frac{A_{13}}{t^2}\frac{du_0}{dx}\frac{du_0}{dy} + \frac{A_{13}}{t^2}\frac{du_0}{dx}\frac{dv_0}{dx} - 3\frac{B_{13}}{t}\frac{du_0}{dx}\frac{d^2w}{dxdy} - \frac{B_{13}}{t}\frac{dv_0}{dx}\frac{d^2w}{dx^2} + 2D_{13}\frac{d^2w}{dx^2}\frac{d^2w}{dxdy} \right] dx\,.dy\ - n_2 \iint qw \quad dx\,dy \qquad (20b)$$

$$\Pi_3 = \frac{2E_0 t^3}{2} \iint \left[\frac{A_{23}}{t^2}\frac{du_0}{dy}\frac{dv_0}{dy} + \frac{A_{23}}{t^2}\frac{dv_0}{dy}\frac{dv_0}{dx} - 3\frac{B_{23}}{t}\frac{dv_0}{dy}\frac{d^2w}{dxdy} - \frac{B_{23}}{t}\frac{du_0}{dy}\frac{d^2w}{dy^2} + 2D_{23}\frac{d^2w}{dy^2}\frac{d^2w}{dxdy} \right] dx\,.dy - n_3 \iint qw \quad dx\,dy \qquad (20c)$$

For easy understanding of the meanings for m, n and z is illustrated with a four-lamina laminated plate shown on Figure 1

$z_0 = t\,s_0 = t/2$

$Lamina\ 1: m = 1;\ z_{m-1} = z_0;\ z_m = z_1$

$z_1 = t\,s_1 = t/4$

$Lamina\ 2: m = 2;\ z_{m-1} = z_1;\ z_m = z_2$

$z_2 = t\,s_2 = 0$

$Lamina\ 3: m = 3\,;\ z_{m-1} = z_2;\ z_m = z_3$

$z_3 = t\,s_3 - t/4$

$Lamina\ 4: m = 4;\ z_{m-1} = z_3;\ z_m = z_4$

$z_4 = t\,s_4 - t/2$

Figure 1: A laminated plate that is made of four *laminas*

2.3 General and direct Variation of Total potential energy functional for a laminated thin rectangular plate

Minimizing Equations 20a, 20b and 20c with respect to w, u_0 and v_0 and making some rearrangements shall give the respective Equations 21 to 23:

$$\frac{\partial \Pi_1}{\partial \mathrm{w}} = 0 = \iint \left[-\frac{1}{t}\left(B_{11}\frac{\partial^3 u_0}{\partial x^3} + (B_{12}+2B_{33})\frac{\partial^3 u_0}{\partial x \partial y^2} + (B_{12}+2B_{33})\frac{\partial^3 v_0}{\partial x^2 \partial y} + B_{22}\frac{\partial^3 v_0}{\partial y^3} \right) + \left(D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12}+2D_{33})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} \right) \right] dxdy + \frac{n_1}{E_0 t^3}\iint q\,dx\,dy \qquad (21a)$$

$$\frac{\partial \Pi_2}{\partial \mathrm{w}} = 0 = \frac{1}{t}\iint \frac{\partial^2}{\partial x^2}\left(-3B_{13}\frac{\partial u_0}{\partial y} - B_{13}\frac{\partial v_0}{\partial x} + 4D_{13}\frac{\partial^2 w}{\partial x \partial y} \right) dxdy + \frac{n_2}{E_0 t^3}\iint q\,dx\,dy \qquad (21b)$$

$$\frac{\partial \Pi_3}{\partial \mathrm{w}} = 0 = \frac{1}{t}\iint \frac{\partial^2}{\partial y^2}\left(-3B_{23}\frac{\partial v_0}{\partial x} - B_{23}\frac{\partial u_0}{\partial y} + 4D_{23}\frac{\partial^2 w}{\partial x \partial y} \right) dxdy + \frac{n_3}{E_0 t^3}\iint q\,dx\,dy \qquad (21c)$$

$$\frac{\partial \Pi_1}{\partial u_0} = 0 = \frac{1}{t^2}\iint\left(\frac{d^2}{dx^2}\left[A_{11}u_0 - B_{11}\frac{dw}{dx}\right] + \frac{d^2}{dxdy}\left[A_{12}v_0 - B_{12}\frac{dw}{dy}\right] + \frac{d^2}{dxdy}\left[A_{33}v_0 - B_{33}\frac{dw}{dy}\right] + \frac{d^2u_0}{dy^2}\left[A_{33}u_0 - B_{33}\frac{dw}{dx}\right]\right)dxdy \qquad (22a)$$

$$\frac{\partial \Pi_2}{\partial u_0} = \iint\left(2\frac{d^2}{dxdy}\left[\frac{A_{13}}{t^2}u_0 - \frac{B_{13}}{t}\frac{dw}{dx}\right] + \frac{d^2}{dx^2}\left[\frac{A_{13}}{t^2}v_0 - \frac{B_{13}}{t}\frac{dw}{dy}\right]\right)\ dxdy = 0 \qquad (22b)$$

$$\frac{\partial \Pi_3}{\partial u_0} = \iint \frac{d^2}{dy^2}\left[\frac{A_{23}}{t^2}v_0 - \frac{B_{23}}{t}\frac{du_w}{dy}\right]\ dxdy = 0 \qquad (22c)$$

$$\frac{\partial \Pi_2}{\partial v_0} = \iint \frac{d^2}{dx^2}\left[\frac{A_{13}}{t^2}u_0 - \frac{B_{13}}{t}\frac{dw}{dx}\right]\ dxdy = 0 \qquad (23b)$$

$$\frac{\partial \Pi_3}{\partial v_0} = \iint\left(\frac{d^2}{dy^2}\left[\frac{A_{23}}{t^2}u_0 - \frac{B_{23}}{t}\frac{dw}{dx}\right] + 2\frac{d^2}{dxdy}\left[\frac{A_{23}}{t^2}v_0 - \frac{B_{23}}{t}\frac{dw}{dy}\right]\right) dxdy = 0 \qquad (23c)$$

For Equations 22a, 22b, 22c, 23a, 23b and 23c to be true, the following shall hold (where c and d are yet to be determined constants):

$$u_0 = t\frac{B_{ij}}{A_{ij}}\frac{\partial w}{\partial x} = c.t\frac{\partial w}{\partial x} \qquad (24a)$$

$$v_0 = t\frac{B_{ij}}{A_{ij}}\frac{\partial w}{\partial y} = d.t\frac{\partial w}{\partial y} \qquad (24b)$$

Substituting Equations 24a and 24b into Equation 21a and making some rearrangements and observing that an integral can only be zero if its integrands gives Equation 25:

$$\iint\left([D_{11} - cB_{11}]\frac{\partial^4 w}{\partial x^4} + 2[D_{12} - cB_{12} - dB_{12} + 2D_{33} - 2cB_{33} - 2dB_{33}]\frac{\partial^4 w}{\partial x^2 \partial y^2} + [D_{22} - dB_{22}]\frac{\partial^4 w}{\partial y^4} + \frac{n_1 N_x}{E_0 t^3}.\frac{d^2 w}{dx^2}\right)dxdy = 0 \qquad (25)$$

Dividing Equation 25 by [D_{22} - dB_{22}] gives equation 26:

$$\iint\left[f_1\frac{\partial^4 w}{\partial x^4} + f_2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{n_1 N_x}{E_0 t^3}.\frac{d^2 w}{dx^2}\right]dxdy = 0 \qquad (26)$$

Where:

$$f_1 = \frac{[D_{11} - cB_{11}]}{[D_{22} - dB_{22}]}$$

$$f_2 = \frac{2[D_{12} - cB_{12} - dB_{12} + 2D_{33} - 2cB_{33} - 2dB_{33}]}{[D_{22} - dB_{22}]}$$

The exact solutions to Equation (26) (in terms of non-dimensional coordinates) for pure bending analysis, buckling analysis and free vibration analysis were obtained to be (see [Ibearugbulem (2020)] for details):

$$w = \beta_1\left(a_0 + a_1 x + \frac{a_2}{2!}x^2 + \frac{a_3}{3!}x^3 + \frac{a_4}{4!}x^4\right)\left(b_0 + b_1 y + \frac{b_2}{2!}y^2 + \frac{b_3}{3!}y^3 + \frac{b_4}{4!}y^4\right) \qquad (27a)$$

From Equations 27a, it was gathered that:

$$w = \beta_1 h \qquad (27b)$$

Substituting Equation (27d) into Equations (24a) and (24b) gives:

$$u_0 = c.t.\beta_1\frac{\partial h}{\partial x} = \beta_2\frac{\partial h}{\partial x} \qquad (28a)$$

$$v_0 = d.t.\beta_1\frac{\partial h}{\partial y} = \beta_3\frac{\partial h}{\partial y} \qquad (28b)$$

$$\beta_2 = c.t.\beta_1 \ \ and \ \ \beta_3 = d.t.\beta_1 \qquad (28c)$$

Substituting Equations (27b), (28a) and (28b) into Equations (20a), (20b) and (20c) and writing the outcomes in terms of non dimensional coordinates gives:

$$\Pi_1 = \frac{E_0 t^3 ab}{2a^4}\iint\left[\left(A_{11}{\beta_2}^2\left(\frac{\partial^2 h}{\partial R^2}\right)^2 + \frac{1}{\alpha^2}\left[2A_{12}\beta_2\beta_3 + 2A_{33}\beta_2\beta_3 + A_{33}{\beta_2}^2 + A_{33}{\beta_3}^2\right]\left(\frac{\partial^2 h}{\partial R\partial Q}\right)^2 + A_{22}\frac{{\beta_3}^2}{\alpha^4}\left(\frac{\partial^2 h}{\partial Q^2}\right)^2\right) - 2\left(B_{11}\beta_1\beta_2\left(\frac{\partial^2 h}{\partial R^2}\right)^2 + (B_{12}+2B_{33})\frac{\beta_1\beta_2}{\alpha^2}\left(\frac{\partial^2 h}{\partial R\partial Q}\right)^2 + (B_{12}+2B_{33})\frac{\beta_1\beta_3}{\alpha^2}\left(\frac{\partial^2 h}{\partial R\partial Q}\right)^2 + B_{22}\frac{\beta_1\beta_3}{\alpha^4}\left(\frac{\partial^2 h}{\partial Q^2}\right)^2\right) + \left(D_{11}{\beta_1}^2\left(\frac{\partial^2 h}{\partial R^2}\right)^2 + 2(D_{12}+2D_{33})\frac{{\beta_1}^2}{\alpha^2}\left(\frac{\partial^2 h}{\partial R\partial Q}\right)^2 + D_{22}\frac{{\beta_1}^2}{\alpha^4}\left(\frac{\partial^2 h}{\partial Q^2}\right)^2\right)\right] dRdQ - \frac{n_1 ab}{2a^2}\iint q\beta_1 h\, dR\, dQ \quad (29a)$$

$$\Pi_2 = \frac{2E_0 t^3 ab}{2a^4}\iint\left[A_{13}\frac{{\beta_2}^2}{\alpha}\frac{\partial^2 h}{\partial R^2}.\frac{\partial^2 h}{\partial R\partial Q} + A_{13}\frac{\beta_2\beta_3}{\alpha}\frac{\partial^2 h}{\partial R^2}.\frac{\partial^2 h}{\partial R\partial Q} - 3B_{13}\frac{\beta_1\beta_2}{\alpha}\frac{\partial^2 h}{\partial R^2}\frac{\partial^2 h}{\partial R\partial Q} - B_{13}\frac{\beta_1\beta_3}{\alpha}.\frac{\partial^2 h}{\partial R^2}\frac{\partial^2 h}{\partial R\partial Q} + 2D_{13}\frac{{\beta_1}^2}{\alpha}\frac{\partial^2 h}{\partial R^2}\frac{\partial^2 h}{\partial R\partial Q}\right] dR\,.dQ - \frac{n_2 ab}{2a^2}\iint q\beta_1 h\, dR\, dQ \quad (29b)$$

$$\Pi_3 = \frac{2E_0 t^3 ab}{2a^4}\iint\left[A_{23}\frac{\beta_2\beta_3}{\alpha^3}\frac{\partial^2 h}{\partial R\partial Q}.\frac{\partial^2 h}{\partial Q^2} + A_{23}\frac{{\beta_3}^2}{\alpha^3}\frac{\partial^2 h}{\partial R\partial Q}.\frac{\partial^2 h}{\partial Q^2} - 3B_{23}\frac{\beta_1\beta_3}{\alpha^3}\frac{\partial^2 h}{\partial R\partial Q}.\frac{\partial^2 h}{\partial Q^2} - B_{23}\frac{\beta_1\beta_2}{\alpha^3}\frac{\partial^2 h}{\partial R\partial Q}.\frac{\partial^2 h}{\partial Q^2} + 2D_{23}\frac{{\beta_1}^2}{\alpha^3}\frac{\partial^2 h}{\partial R\partial Q}.\frac{\partial^2 h}{\partial Q^2}\right] dR\,.dQ - \frac{n_3 ab}{2a^2}\iint q\beta_1 h\, dR\, dQ \quad (29c)$$

Minimizing Equations (29a), (29b) and (29c) with respect to β1 and rearrange gives respectively:

$$\frac{d\Pi_1}{d\beta_1} = 0 = -\left(B_{11}\beta_2 k_x + (B_{12}+2B_{33})\frac{\beta_2}{\alpha^2}k_{xy} + (B_{12}+2B_{33})\frac{\beta_3}{\alpha^2}k_{xy} + B_{22}\frac{\beta_3}{\alpha^4}k_y\right) + \beta_1\left(D_{11}k_x + \frac{2}{\alpha^2}(D_{12}+2D_{33})k_{xy} + \frac{D_{22}}{\alpha^4}k_y\right) - \frac{n_1 a^4}{E_0 t^3}qk_q \quad (30a)$$

$$\frac{d\Pi_2}{d\beta_1} = 0 = \left(4D_{13}\frac{\beta_1}{\alpha} - 3B_{13}\frac{\beta_2}{\alpha} - B_{13}\frac{\beta_3}{\alpha}\right)k_{xxy} - \frac{n_2 a^4}{E_0 t^3}qk_q \quad (30b)$$

$$\frac{d\Pi_3}{d\beta_1} = 0 = \left(4D_{23}\frac{\beta_1}{\alpha^3} - B_{23}\frac{\beta_2}{\alpha^3} - 3B_{23}\frac{\beta_3}{\alpha^3}\right)k_{xyy} - \frac{n_3 a^4}{E_0 t^3}qk_q \quad (30c)$$

Minimizing Equations (29a), (29b) and (29c) with respect to β2 gives:

$$\frac{d\Pi_1}{d\beta_2} = \frac{ab}{\alpha^4}\left[\left(A_{11}\beta_2 k_x + \frac{1}{\alpha^2}[A_{12}\beta_3 + A_{33}\beta_2 + A_{33}\beta_3]k_{xy}\right) - B_{11}\beta_1 k_x - (B_{12}+2B_{33})\frac{\beta_1}{\alpha^2}k_{xy}\right] = 0 \quad (31a)$$

$$\frac{d\Pi_2}{d\beta_2} = \frac{ab}{\alpha^4}\left[2A_{13}\frac{\beta_2}{\alpha}k_{xxy} + A_{13}\frac{\beta_3}{\alpha}k_{xxy} - 3B_{13}\frac{\beta_1}{\alpha}k_{xxy}\right] = 0 \quad (31b)$$

$$\frac{d\Pi_3}{d\beta_2} = 0 = \frac{ab}{\alpha^4}\left[A_{23}\frac{\beta_3}{\alpha^3}k_{xyy} - B_{23}\frac{\beta_1}{\alpha^3}k_{xyy}\right] \quad (31c)$$

Minimizing Equations 29a, 29b and 29c with respect to β3 gives:

$$\frac{d\Pi_1}{d\beta_3} = \frac{ab}{\alpha^4}\left[A_{22}\frac{\beta_3}{\alpha^4}k_y + \frac{1}{\alpha^2}[A_{12}\beta_2 + A_{33}\beta_2 + A_{33}\beta_3]k_{xy} - B_{22}\frac{\beta_1}{\alpha^4}k_y - (B_{12}+2B_{33})\frac{\beta_1}{\alpha^2}k_{xy}\right] = 0 \quad (32a)$$

$$\frac{d\Pi_2}{d\beta_3} = 0 = \frac{ab}{\alpha^4}\left[A_{13}\frac{\beta_2}{\alpha}k_{xxy} - B_{13}\frac{\beta_1}{\alpha}.k_{xxy}\right] \quad (32b)$$

$$\frac{d\Pi_3}{d\beta_3} = 0 = \frac{ab}{\alpha^4}\left[A_{23}\frac{\beta_2}{\alpha^3}k_{xyy} + 2A_{23}\frac{\beta_3}{\alpha^3}k_{xyy} - 3B_{23}\frac{\beta_1}{\alpha^3}k_{xyy}\right]$$

$$Where: k_x = \iint \left(\frac{\partial^2 h}{\partial R^2}\right)^2 \ dR\,.dQ:$$

$$k_{xy} = \iint \left(\frac{\partial^2 h}{\partial R \partial Q}\right)^2 \ dR\,.dQ \,:\, k_y = \iint \left(\frac{\partial^2 h}{\partial Q^2}\right)^2 \ dR\,.dQ$$

$$k_{xxy} = \iint \frac{\partial^2 h}{\partial R^2}\frac{\partial^2 h}{\partial R \partial Q} \ dR\,.dQ$$

$$k_{xyy} = \iint \frac{\partial^2 h}{\partial R \partial Q}.\frac{\partial^2 h}{\partial Q^2} \ dR\,.dQ \,:\, k_q = \iint h \ \ dR\,.dQ$$

$$k_N = \iint \left(\frac{dh}{dR}\right)^2 \ dR\,.dQ \,:\, k_\lambda = \iint h^2 \ \ dR\,.dQ$$

Adding the Equations 30a, 30b and 30c together and rearranging the outcome gives:

$$\frac{d\Pi}{d\beta_1} = \frac{d\Pi_1}{d\beta_1} + \frac{d\Pi_2}{d\beta_1} + \frac{d\Pi_3}{d\beta_1} = 0.$$

That is:

$$\frac{d\Pi}{d\beta_1} = \beta_1\left(D_{11}k_x + \frac{2}{\alpha^2}(D_{12}+2D_{33})k_{xy} + \frac{D_{22}}{\alpha^4}k_y + 4\frac{D_{13}}{\alpha}k_{xxy} + 4\frac{D_{23}}{\alpha^3}k_{xyy}\right) - \beta_2\left(B_{11}k_x + (B_{12}+2B_{33})\frac{k_{xy}}{\alpha^2} + 3B_{13}\frac{k_{xxy}}{\alpha} + B_{23}\frac{k_{xyy}}{\alpha^3}\right) - \beta_3\left((B_{12}+2B_{33})\frac{k_{xy}}{\alpha^2} + B_{22}\frac{k_y}{\alpha^4} + B_{13}\frac{k_{xxy}}{\alpha} + 3B_{23}\frac{k_{xyy}}{\alpha^3}\right) - \frac{\alpha^4}{E_0 t^3}(n_1 + n_2 + n_3)qk_x \qquad (33a)$$

Substituting Equation (18) into equation (33a) and rearranging the outcome gives equation (33):

$$\frac{\alpha^4}{E_0 t^3}qk_x = \beta_1\left(D_{11}k_x + \frac{2}{\alpha^2}(D_{12}+2D_{33})k_{xy} + \frac{D_{22}}{\alpha^4}k_y + 4\frac{D_{13}}{\alpha}k_{xxy} + 4\frac{D_{23}}{\alpha^3}k_{xyy}\right) - \beta_2\left(B_{11}k_x + (B_{12}+2B_{33})\frac{k_{xy}}{\alpha^2} + 3B_{13}\frac{k_{xxy}}{\alpha} + B_{23}\frac{k_{xyy}}{\alpha^3}\right) - \beta_3\left((B_{12}+2B_{33})\frac{k_{xy}}{\alpha^2} + B_{22}\frac{k_y}{\alpha^4} + B_{13}\frac{k_{xxy}}{\alpha} + 3B_{23}\frac{k_{xyy}}{\alpha^3}\right) \qquad (33)$$

Adding the Equations (31a), (31b) and (31c) together and rearranging the outcome gives:

$$\beta_2\left(A_{11}k_x + A_{33}\frac{k_{xy}}{\alpha^2} + 2A_{13}\frac{k_{xxy}}{\alpha}\right) + \beta_3\left(A_{12}\frac{k_{xy}}{\alpha^2} + A_{33}\frac{k_{xy}}{\alpha^2} + A_{13}\frac{k_{xxy}}{\alpha} + A_{23}\frac{k_{xyy}}{\alpha^3}\right) = \beta_1\left(B_{11}k_x + (B_{12}+2B_{33})\frac{k_{xy}}{\alpha^2} + 3B_{13}\frac{k_{xxy}}{\alpha} + B_{23}\frac{k_{xyy}}{\alpha^3}\right) \qquad (34a)$$

Adding the Equations (32a), (32b) and (32c) together and rearranging the outcome gives:

$$\beta_2\left(A_{12}\frac{k_{xy}}{\alpha^2} + A_{33}\frac{k_{xy}}{\alpha^2} + A_{13}\frac{k_{xxy}}{\alpha} + A_{23}\frac{k_{xyy}}{\alpha^3}\right) + \beta_3\left(A_{22}\frac{k_y}{\alpha^4} + A_{33}\frac{k_{xy}}{\alpha^2} + 2A_{23}\frac{k_{xyy}}{\alpha^3}\right) = \beta_1\left(B_{22}\frac{k_y}{\alpha^4} + (B_{12}+2B_{33})\frac{k_{xy}}{\alpha^2} + B_{13}\frac{k_{xxy}}{\alpha} + 3B_{23}\frac{k_{xyy}}{\alpha^3}\right) \qquad (34b)$$

Solving Equations (34a) and (34b) simultaneously gives:

$$\beta_2 = T_2\beta_1 = \beta_1\frac{(d_{12}.d_{23} - d_{13}.d_{22})}{\left({d_{12}}^2 - d_{11}d_{22}\right)} \qquad (35a)$$

$$\beta_3 = T_3\beta_1 = \beta_1\frac{(d_{12}.d_{13} - d_{11}d_{23})}{\left({d_{12}}^2 - d_{11}d_{22}\right)} \qquad (35b)$$

Where:

$$d_{11} = A_{11}k_x + A_{33}\frac{k_{xy}}{\alpha^2} + 2A_{13}\frac{k_{xxy}}{\alpha} \quad (36a)$$

$$d_{12} = A_{12}\frac{k_{xy}}{\alpha^2} + A_{33}\frac{k_{xy}}{\alpha^2} + A_{13}\frac{k_{xxy}}{\alpha} + A_{23}\frac{k_{xyy}}{\alpha^3} \quad (36b)$$

$$d_{22} = A_{22}\frac{k_y}{\alpha^4} + A_{33}\frac{k_{xy}}{\alpha^2} + 2A_{23}\frac{k_{xyy}}{\alpha^3} \quad (36c)$$

$$d_{13} = B_{11}k_x + (B_{12} + 2B_{33})\frac{k_{xy}}{\alpha^2} + 3B_{13}\frac{k_{xxy}}{\alpha} + B_{23}\frac{k_{xyy}}{\alpha^3} \quad (36d)$$

$$d_{23} = B_{22}\frac{k_y}{\alpha^4} + (B_{12} + 2B_{33})\frac{k_{xy}}{\alpha^2} + B_{13}\frac{k_{xxy}}{\alpha} + 3B_{23}\frac{k_{xyy}}{\alpha^3} \quad (36e)$$

Substituting Equations (35a) and (35b) into Equation 33b and rearranging gives:

$$\frac{qa^4}{E_0t^3.\beta_1}k_x = \left(D_{11}k_x + \frac{2}{\alpha^2}(D_{12} + 2D_{33})k_{xy} + \frac{D_{22}}{\alpha^4}k_y + 4\frac{D_{13}}{\alpha}k_{xxy} + 4\frac{D_{23}}{\alpha^3}k_{xyy}\right) - T_2\left(B_{11}k_x + \frac{(B_{12} + 2B_{33})}{\alpha^2}k_{xy} + \frac{4B_{13}}{\alpha}k_{xxy} + \frac{B_{23}}{\alpha^3}k_{xyy}\right) - T_3\left(\frac{(B_{12} + 2B_{33})}{\alpha^2}k_{xy} + \frac{B_{22}}{\alpha^4}k_y + \frac{B_{13}}{\alpha}k_{xxy} + \frac{4B_{23}}{\alpha^3}k_{xyy}\right) \quad (37)$$

Rearranging equations (37) gives:

$$\frac{E_0t^3}{qa^4}.\beta_1 = \frac{k_q}{k_{T1} + k_{T2} + k_{T3}} \quad (38)$$

Where:

$$k_{T1} = \left(D_{11}k_x + \frac{2}{\alpha^2}(D_{12} + 2D_{33})k_{xy} + \frac{D_{22}}{\alpha^4}k_y + 4\frac{D_{13}}{\alpha}k_{xxy} + 4\frac{D_{23}}{\alpha^3}k_{xyy}\right) \quad (40c)$$

$$k_{T2} = -T_2\left(B_{11}k_x + \frac{(B_{12} + 2B_{33})}{\alpha^2}k_{xy} + \frac{4B_{13}}{\alpha}k_{xxy} + \frac{B_{23}}{\alpha^3}k_{xyy}\right) \quad (40d)$$

$$k_{T3} = -T_3\left(\frac{(B_{12} + 2B_{33})}{\alpha^2}k_{xy} + \frac{B_{22}}{\alpha^4}k_y + \frac{B_{13}}{\alpha}k_{xxy} + \frac{4B_{23}}{\alpha^3}k_{xyy}\right) \quad (40e)$$

3.0 NUMERICAL EXAMPLES

A thin rectangular laminated composite plate with orientation angles 0^00^0, 0^090^0, $0^00^00^0$, $0^090^00^0$, $0^00^00^00^0$, and $0^090^090^00^0$, and with the following boundary conditions SSSS (simply supported in all the four edges), CCCC (Clamped in all the four edges), SSCC (simply supported in the two adjacent edges while the other two edges are clamped) and SCSC (simply supported in the two opposite edges while the other two are clamped) having aspect ratio ranging from 1-2. The plates have the following material properties E1/E2 = 25; G12/E2 = 0.5**;** V12 = 0.25. It is required to determine the deflection of the plate under uniformly distributed load. The reference elastic modulus, E_0 is taken to be E_2. Hence,

$$\frac{E_0t^3}{qa^4}.\beta_1 = \frac{E_2t^3}{qa^4}.\beta_1 = \frac{k_q}{k_{T1} + k_{T2} + k_{T3}} \quad (39a)$$

If the aspect ratio is a/b and the parameters are in terms of long length "b" then:

$$q_xb^2 = [q_xa^2] \times [b/a]^2$$

The exact deflection function for pure bending analyses of SSSS, CCCC, CCSS and CSCS plates after satisfying the boundary condition using Equations 27a in polynomial form are respectively:

$$w = A\,(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$$

$$W = \beta_1(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$$

$$W = \beta_1(1.5R^2 - 2.5R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4)$$

$$W = \beta_1(R^2 - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$$

The stiffness coefficients obtained using the polynomial deflection functions are shown on Table 1:

Table 1: Stiffness coefficients (k-values) for Different composite plates

	k_x	k_{xy}	k_y	k_{xxy}	k_{xyy}	k_q
SSSS	0.2362	0.2359	0.0239	0	0	0.04
CCCC	0.0136	0.00735	0.00127	0	0	0.00111
CCSS	0.0136	0.00735	0.01357	0	0	0.00563
CSCS	0.0394	0.00923	0.00762	0	0	0.00667

4.0 RESULTS AND DISCUSSIONS

The results for pure bending analysis are presented on Table 2 to Table 5. For any thin laminated composite plate considered, the orthotropic plate has the same pure bending coefficient, but the values increase with increase in the aspect ratio. When other orientations were considered, the change in orientation angles produces changes in the pure bending coefficient which increases with increase in aspect ratio

The maximum pure bending coefficient was obtained when SSSS thin laminated composite Plate was considered for two laminated composite plate with orientation $0^0 90^0$ as 0.0174 while the minimum pure bending coefficient was obtained when CCCC thin laminated composite plate was considered for orthotropic plate as 0.00153 for square plate.

The present work was compared with the work of Bhaskar & Kaushik (2004a and 2004b); The percentage differences recorded on Table 6 did not surprise us because the work presented by Bhaskar & Kaushik were based on Naiver and superimposition method which is based on assumed displacement function while the presented work was based on Euler-Bernoulli equilibrium equation.

Table 2: Effect of Orientation Angle on The Maximum Bending Coefficient for SSSS Plates

E1/E2 = 25; G12/E2 = 0.5**;** V12 = 0.25				
Orientations	Orthotropic	$0^0 90^0$	$0^0 90^0 0^0$	$0^0 90^0 90^0 0^0$
Aspect ratio	$W_{max} = \frac{qa^4}{E_0 t^3}$	$W_{max} = \frac{qa^4}{E_0 t^3}$	$W_{max} = \frac{qa^4}{E_0 t^3}$	$W_{max} = \frac{qa^4}{E_0 t^3}$
1	0.00695	0.0171	0.0070	0.00695
1.1	0.00714	0.0204	0.0072	0.00739
1.2	0.00727	0.0234	0.0074	0.00771
1.3	0.00738	0.0260	0.0075	0.00796
1.4	0.00746	0.0282	0.0077	0.00814
1.5	0.00753	0.0301	0.0077	0.00828
1.6	0.00758	0.0317	0.0078	0.00839
1.7	0.00762	0.0330	0.0079	0.00848
1.8	0.00765	0.0341	0.0079	0.00855
1.9	0.00768	0.0350	0.0079	0.00861
2.0	0.00771	0.0358	0.0080	0.00865

Table 3: Effect of Orientation Angle on The Maximum Bending Coefficient for CCCC Plate

E1/E2 = 25; G12/E2 = 0.5; V12 = 0.25

Orientations Aspect ratio	Orthotropic $W_{max} = \frac{qa^4}{E_0t^3}$	$0^0 90^0$ $W_{max} = \frac{qa^4}{E_0t^3}$	$0^0 90^0 0^0$ $W_{max} = \frac{qa^4}{E_0t^3}$	$0^0 90^0 90^0 0^0$ $W_{max} = \frac{qa^4}{E_0t^3}$
1	0.001532	0.00361	0.00153	0.00153
1.1	0.001557	0.0043	0.00157	0.00162
1.2	0.001575	0.00489	0.00160	0.00168
1.3	0.001587	0.00539	0.00162	0.00172
1.4	0.001596	0.00579	0.00164	0.00175
1.5	0.001603	0.00612	0.00165	0.00177
1.6	0.001609	0.00638	0.00166	0.00179
1.7	0.001613	0.00659	0.00166	0.00180
1.8	0.001616	0.00676	0.00166	0.00181
1.9	0.001618	0.00689	0.00167	0.00182
2.0	0.001621	0.007004	0.001676	0.00182

Table 4: Effect of Orientation Angle on The Maximum Bending Coefficient for CCSS Plates

E1/E2 = 25; G12/E2 = 0.5; V12 = 0.25

Orientations Aspect ratio	Orthotropic $W_{max} = \frac{qa^4}{E_0t^3}$	$0^0 90^0$ $W_{max} = \frac{qa^4}{E_0t^3}$	$0^0 90^0 0^0$ $W_{max} = \frac{qa^4}{E_0t^3}$	$0^0 90^0 90^0 0^0$ $W_{max} = \frac{qa^4}{E_0t^3}$
1	0.002834	0.006519	0.002834	0.002834
1.1	0.002893	0.007761	0.002923	0.002999
1.2	0.002934	0.008853	0.002986	0.003117
1.3	0.002965	0.009779	0.003032	0.003203
1.4	0.002987	0.010547	0.003065	0.003267
1.5	0.003005	0.011177	0.003090	0.003314
1.6	0.003019	0.011690	0.003110	0.003350
1.7	0.003030	0.012108	0.003125	0.003378
1.8	0.003039	0.012449	0.003138	0.003400
1.9	0.003046	0.012729	0.003147	0.003418
2.0	0.003052	0.012959	0.003156	0.003432

Table 5: Effect of Orientation Angle on The Maximum Bending Coefficient for CSCS and SCSC Plates

E1/E2 = 25; G12/E2 = 0.5; V12 = 0.25.

Orientations Aspect ratio	Orthotropic $W_{max} = \frac{qa^4}{E_0t^3}$	$0^0 90^0$ $W_{max} = \frac{qa^4}{E_0t^3}$	$0^0 90^0 0^0$ $W_{max} = \frac{qa^4}{E_0t^3}$	$0^0 90^0 90^0 0^0$ $W_{max} = \frac{qa^4}{E_0t^3}$
1	0.001536	0.00573	0.00158	0.00169
1.1	0.001545	0.00611	0.00159	0.00172
1.2	0.001553	0.00638	0.00160	0.00174
1.3	0.001558	0.00658	0.00161	0.00175
1.4	0.001562	0.00673	0.00162	0.00176
1.5	0.001565	0.00684	0.00162	0.00177
1.6	0.001567	0.00693	0.00162	0.00177
1.7	0.001569	0.00700	0.00162	0.00178
1.8	0.001571	0.00706	0.00163	0.00178
1.9	0.001572	0.00710	0.00162	0.00178
2.0	0.001574	0.00714	0.00163	0.00178

Table 6: Compering the present work with the work of Bhaskar & Kaushik (2004a and 2004b)

E1/E2 = 25; G12/E2 = 0.5; V12 = 0.25					
Bhaskar & Kaushik (2004a and 2004b) work				**% Diff. for Orthotropic**	**% Diff. for $0^0 90^0 0^0$**
Orientations Plate type	**Aspect ratio**	**Orthotropic** $W_{max} = \frac{qa^4}{E_0 t^3}$	**$0^0 90^0 0^0$** $W_{max} = \frac{qa^4}{E_0 t^3}$		
SSSS	1	0.006497	0.006660	4.17	6.51
CCCC	1	0.001308	0.001371	10.392	14.62

5.0 CONCLUSION

The Effect of Orientation Angles and Boundary Conditions on the Pure Bending Coefficient of Thin Laminated Composite Plate Using Euler-Bernoulli Equilibrium Equation were carried out considering the total potential energy function. Conclusively, no matter the number of laminas considered in orthotropic plate, the pure bending coefficients are the same but the values increases with increase in the aspect ratio. When the orientation changes, it causes changes in the pure bending coefficient. The pure bending coefficient increases with increase in the aspect ratio.

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