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A Multivariate Model to Predicting Vibration Features for Equipment Prognosis

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Abstract

Vibration analysis, a vital tool in the scheduling of equipment for maintenance is used to assess the useful life of equipment for allocation of resources to mitigate downtime. Compared to previous approaches of univariate prediction, this study presents a more practical model by employing vibration analysis data as a multivariate problem in predicting the remaining useful life (RUL) of an equipment. Applying the model, Multiple Linear Regression (MLR) and Linear Programming (LP) were explored to determine the deterioration rate and the RUL of the equipment. The results showed that the MLR had a high predictive accuracy on the data sets. Furthermore, a p-value of 1.546e-06 and Multiple R-squared value of 0.8215 were obtained showing that the MLR appears to be a good prediction model. From the solution of the LP formulation, the RUL of the equipment was 181 days. These results closely matched the historical data of the equipment which implied this model could be used for planning of maintenance activity for this equipment and any similar equipment.

Keywords: Multivariate Modelling; Predictive Maintenance; Prognosis; Remaining Useful Life; Vibration Analysis.

1.0 INTRODUCTION

In any typical industrial scenarios, one of the main goals is to increase the business profit margins. To achieve this, several approaches could be utilised, which may be by increasing the service delivery charges to be paid by the customers or by implementing some cost reduction strategies to reduce the operational cost [1-2]. As companies strive to meet the mark of a near zero unwanted cost which may be termed as waste, there has been an increasing need for a right implementation of a cost-effective maintenance strategy. Proper maintenance results in the decrease of depreciation costs (resulting from longer economic life) and consequently leads to increased profitability while improper planning of maintenance of structures will give rise to uneconomic management practices that lead to the overspending of budgeted finances and a negative outcome in productivity [3].

Predictive maintenance (PdM) has gained much ground with regards to a cost-effective based maintenance policy since it is efficient in early detection of impending equipment failure which in turn reduces unplanned downtime [4-5]. For an effective early fault detection strategy, PdM implements predictive tools in Condition based Maintenance (CBM) programme to provide robust information about equipment's state at a future period [6]. The CBM uses many tools for checking state of equipment among which vibration-based monitoring is a most applied technique in the industry [7].

The aim of any vibration analysis is to ascertain the vibration severity of an equipment and the trend of the vibration over time, which tells on the equipment degradation pattern to avoid equipment breakdown [8]. In a sense, vibration analysis gives view of the happenings in an equipment, from the running condition of the shaft to the motor, blades, and other components [9]. This results in a set of vibration data representing vibration magnitude. Generally, readings are taken from the vertical, horizontal, and axial (VHA) direction due to its mode of installation and active forces causing the vibration of the equipment as represented by Figure 1 [10].

By conducting vibration analysis, single value peak readings are gotten which are trended with previous readings to ascertain the degradation pattern of such equipment. The severity of the vibration reading which would regard the equipment as failed is governed by some ISO standards. ISO 10817 and ISO 7919 are most adhered

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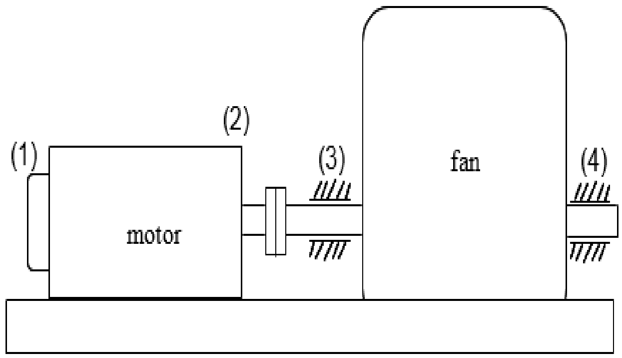
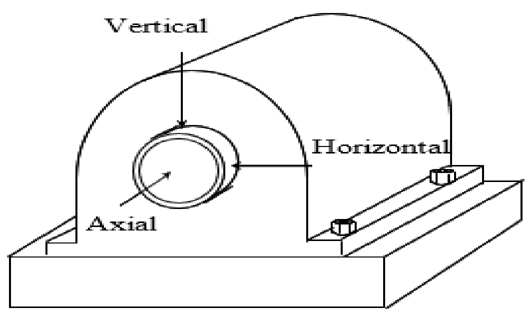
to generally [7-8][11]. When equipment condition degrades over time, the vibration severity tends to show an upward trend



(a) Locations of measurement



(b) Directions of measurement

Figure 1: Locations and Directions of Measurements (Alsalaet 2012)

i.e., vibration increases overtime under normal working conditions [12]. This is the general rule of thumb for which several literatures have applied various forecasting techniques towards equipment prognosis. For example, Chukwuekwe *et al.* [7], worked on predicting future vibration severity level of one of the gearbox bearings of an industrial machine, such that readings within a six-month period interval were used to predict the severity level for the next six months with the Autoregressive Moving Average (ARMA) model for which the results proved to be better-off compared to when the root mean square (RMS) values of the time waveform were trended over time. Similarly, Chen *et al.* [13] worked on fault prediction of a steam turbine for a power plant based on a full vector spectrum fusion of vibration amplitudes from the horizontal and vertical planes of the equipment and then using an ARMA model to determine its prediction model.

Gangsar and Tiwari [14] investigated equipment condition monitoring by vibrations as well as electrical current for effective fault prediction on the electrical and mechanical components of an induction motor. This was achieved by training a one-versus-one multi class support vector machine (MSVM) with data obtained at different equipment running conditions. The prediction results performance was investigated for a large set of radial basis function (RBF) kernel parameter and an optimal result was selected for each case. In a study by Xu *et. al.*, [15], it was noted that previous research had considered CBM for predictive maintenance as single-component problems and as such usually made predictions based on one variable. The study of Wang *et al.* [16] captured the fluctuations in equipment degradation patterns utilized a dynamic RUL technique with the aid of Gamma process model. While previous research has introduced various methods for degradation and RUL prediction, as [15] stated, these studies have utilized only univariate modelling approaches for degradation predictors which take away some of the complexities akin to real life scenarios. In this work, degradation predictor (i.e., vibration features) was taken as multivariate components to estimate the RUL for a real-life manufacturing equipment.

One challenge that could surface in using a multivariate approach to trending vibration feature magnitude over time is finding an optimum RUL since each of the features trend differently. In tackling this, maintenance model fuses a deterioration model and a decision model to reach an optimum policy defining the RUL in line with Kallen [17] definition of a proper maintenance system as seen in Figure 2.

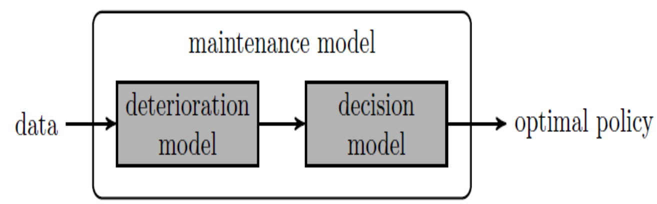


Figure 2: Deterioration and decision models as elements of a maintenance model (Kallen 2009)

2.0 METHODOLOGY

2.1 Materials and Methods

A motor for a cement mill industrial fan which sucks air into the combustion chamber of a boiler and produces water at superheated temperatures was used for prognosis study. The status quo maintenance practice involved the utilization of vibration analysis for equipment fault diagnosis or for the determination of the current health status of the equipment to make maintenance decision i.e., to schedule maintenance or not. In this context, scheduling maintenance is done based on the intuition of the vibration analyst.

For this equipment, in addition to the use of velocity readings for vibration magnitude at the VHA axes, shock pulse and thermography readings were used to enhance maintenance decision based on deduced health status. Having observed the obtained data from the equipment, this study proposes a forecast model using the framework given in Figure 3.

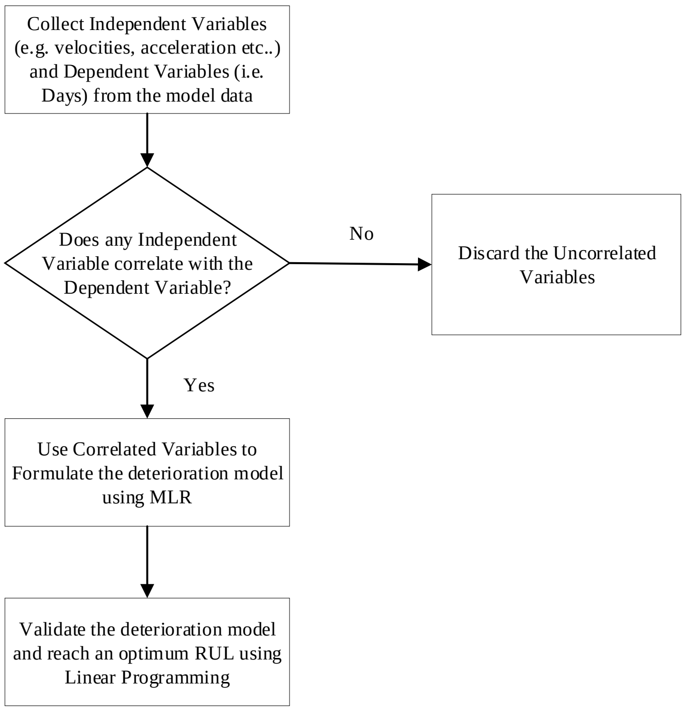


Figure 3: Framework for the Proposed Maintenance Model

2.2 *Multiple Linear Regression (MLR)*

In cases where there are more than one independent variable affecting an outcome, Multiple Linear Regression (MLR) is often applied. MLR is a generalized regression model for the simple linear regression in cases where there are multiple independent variables and a single dependent variable. The basic form of the MLR is given by Equation 1 for discrete observations *i* to *n*:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_n X_{in} + \varepsilon \quad (1)$$

In Equation 1, β represents regressor parameters to be estimated with the regressor coefficient β_0 called the intercept. Y_i is the dependent variable and X_{in} stands for all independent variables for all i observations in the model while ε represents normally distributed error term.

The estimation of parameters is determined based on the least square method (LSM) with the ordinary least square (OLS) being the most widely used LSM [18-19].

To assess the goodness of fit of the model, the coefficient of determination R^2 as given in Equation 2 was used to describe the ratio of the explained variance to the total variance of the dependent variable Y_i

$$R^2 = 1 - \frac{SSR}{SST} = \frac{SSE}{SST} \quad (2)$$

where SST is the total sum of squares, SSR is the residual sum of squares, SSE is the sum of squares if the regression contains a constant. R^2 assumes values from 0 to 1 such that higher values depict better goodness of fit and explains how much of the deviations of Y_i is explained by X_{in}.

The t-statistic and F-statistic are also computed for significance testing of each of the predictor variables on the explanatory variables.

The statistic to test the significance of regression is the F-statistic, given by:

$$F_{statistic} = \frac{MS_R}{MS_E} \quad (3)$$

where: MS_R denotes the regression mean square and MS_E denotes mean square error.

To check the significance of individual regressors in the MLR model, t-test was conducted. More regressors may or may not affect the effectiveness of the MLR model. In carrying out the test, statistical software revealed p-values for all coefficients in the model. Each p-value was based on a t-statistic calculated as:

$$t_{statistic} = \frac{(sample\ coefficient\ -\ hypothesized\ value)}{standard\ error\ of\ coefficient} \quad (4)$$

To prevent the likely problem of over fitting that might arise through MLR, and to improve model accuracy, only variables that contributed significantly were considered. These variables were selected through determination of Correlation coefficient.

2.3 *Pearson's Correlation Coefficient*

Correlation coefficient is a measure used in statistics to show the strength of the relationship between two variables. i.e., the degree to which one variable affects the outcome of the other. There are several methods used for computing correlation coefficients. However, the Pearson's correlation coefficient, otherwise referred to as the Pearson's – r, is very widely used. The Pearson's – r is given by:

$$r = \frac{N\sum xy - (\sum x)(\sum y)}{\sqrt{(N\sum x^2 - (\sum x)^2) \times (N\sum y^2 - (\sum y)^2)}} \quad (5)$$

where:
N is number of sample data points
X is independent variable(s)
y is dependent variable(s)
r normally takes values ranging from -1 to 1 where absolute values greater than 0.5 shows some strong correlation.

3.0 MODEL FORMULATION

As an overview, to account for deterioration over time, the deterioration was modelled as a multiple linear regression (MLR) model. Thereafter, to ascertain the remaining useful life (RUL) of the equipment, the MLR deterioration model was used as the objective function of a Linear Programming (LP) problem where the constraints follow industry standards.

3.1 Formulation of Deterioration Model

The deterioration model for the equipment was formulated using the following parameters in depicting the equipment's health status:

- MV_{NDEV}: Motor non-drive-end vertical velocity readings in *mm/s*
- MV_{NDEH}: Motor non-drive-end horizontal velocity readings in *mm/s*
- MV_{NDEA}: Motor non-drive-end axial velocity readings in *mm/s*
- MV_{DEV}: Motor drive-end vertical velocity readings in *mm/s*
- MV_{DEH}: Motor drive-end horizontal velocity readings in *mm/s*
- SV_{NDEV}: Shaft non-drive-end vertical velocity readings in *mm/s*
- SV_{NDEH}: Shaft non-drive-end horizontal velocity readings in *mm/s*
- SV_{NDEA}: Shaft non-drive-end axial velocity readings in *mm/s*
- SV_{DEV}: Shaft drive-end vertical velocity readings in *mm/s*
- SV_{DEH}: Shaft drive-end horizontal velocity readings in *mm/s*
- SV_{DEA}: Shaft drive-end axial velocity readings in *mm/s*
- MU_{NDD}: The difference between shaft non-drive-end decibel carpet value and decibel maximum reading measured in *dB*
- MU_{DD}: The difference between shaft drive-end decibel carpet value and decibel maximum reading measured in *dB*
- SU_{NDEC}: Shaft non-drive-end decibel carpet value reading measured in *dB*
- SU_{NDEM}: Shaft non-drive-end decibel maximum value reading measured in *dB*
- SU_{DEC}: Shaft drive-end decibel carpet value reading measured in *dB*
- SU_{DEM}: Shaft drive-end decibel maximum value reading measured in *dB*
- MT_{NDE}: Shaft nondrive-end temperature reading measured in ^{0}C
- MT_{DE}: Shaft nondrive-end temperature reading measured in ^{0}C
- ST_{NDE}: Shaft drive-end temperature reading measured in ^{0}C
- ST_{DE}: Shaft drive-end temperature reading measured in ^{0}C
- T_i: Time associated with the readings given in *days*

Hence, the deterioration model following the MLR is given by:

$$\begin{aligned} T_1 = \beta_0 &+ \beta_1 MV_{NDEV} + \beta_2 MV_{NDEH} + \beta_3 MV_{NDEA} \\ &+ \beta_4 MV_{DEV} + \beta_5 MV_{DEH} + \beta_6 MV_{DEA} \\ &+ \beta_7 SV_{NDEV} + \beta_8 SV_{NDEH} + \beta_9 SV_{NDEA} \\ &+ \beta_{10} SV_{DEV} + \beta_{11} SV_{DEH} + \beta_{12} SV_{DEA} \\ &+ \beta_{13} MU_{NDD} + \beta_{14} MU_{DD} + \beta_{15} SU_{NDD} \\ &+ \beta_{16} SU_{DD} + \beta_{17} MT_{NDE} + \beta_{18} MT_{DE} \\ &+ \beta_{19} ST_{NDE} + \beta_{20} ST_{DE} + \varepsilon \end{aligned} \quad (6)$$

3.2 Model Validation

To ascertain the validity of the model developed, vibration data for the motor of a cement mill of an industrial fan of a manufacturing company was collected for one year. The data obtained are shown in the Table 1.

Table 1: Sensor data for a cement mill industrial fan for a period of 1 year

S/N	Day	MV_NDEV (m/s)	MV_NDEH (m/s)	MV_NDEA (m/s)	MV_DEV (m/s)	MV_DEH (m/s)	MV_DEA (m/s)	SV_NDEV (m/s)	SV_NDEH (m/s)	SV_NDEA (m/s)	SV_DEV (m/s)	SV_DEH (m/s)	SV_DEA (m/s)	MU_NDD (dB)	MU_DD (dB)	SU_NDD (dB)	SU_DD (dB)	MT_NDE (^{0}C)	MT_DE (^{0}C)	ST_NDE (^{0}C)	ST_DE (^{0}C)

S/N	Day	MV_NDEV (m/s)	MV_NDEH (m/s)	MV_NDEA (m/s)	MV_DEV (m/s)	MV_DEH (m/s)	MV_DEA (m/s)	SV_NDEV (m/s)	SV_NDEH (m/s)	SV_NDEA (m/s)	SV_DEV (m/s)	SV_DEH (m/s)	SV_DEA (m/s)	MU_NDD (dB)	MU_DD (dB)	SU_NDD (dB)	SU_DD (dB)	MT_NDE (^{0}C)	MT_DE (^{0}C)	ST_NDE (^{0}C)	ST_DE (^{0}C)
1	0	0.5	0.9	0.5	0.9	1.1	0.5	0.8	1.3	1.9	1.6	2.4	2.5	10	7	12	16	47.21	65.14	56.2	64.21
2	18	0.5	1	0.62	0.87	1.23	0.46	0.85	1.34	1.98	1.56	2.59	2.45	11	8	13	8	48.1	66.2	56.1	62.2
3	38	0.6	0.8	0.7	0.7	0.8	0.5	1.1	1	1.9	1.4	2.6	3.2	11	5	15	16	-	-	-	-
4	46	0.63	0.85	0.69	0.59	0.79	0.6	0.65	1.5	1.66	0.67	1.4	2.46	11	17	14	11	-	-	-	-
5	53	0.63	0.92	0.57	0.26	0.86	0.61	0.89	1.55	2.19	0.69	2.08	0.98	9	6	11	22	39.1	46.2	-	49.5
6	57	0.5	0.9	0.6	0.6	0.9	0.6	1.6	1.2	1.6	1.6	2.4	2.6	10	8	7	12	46.5	65.9	54.8	63.2
7	61	0.56	0.68	1.76	0.58	0.92	0.57	0.45	1.09	1.58	0.65	0.37	1.46	9	16	12	12	44.2	60	60.8	76
8	66	0.56	0.68	0.53	0.57	0.9	0.52	0.84	1.07	2.1	0.53	0.95	1.46	9	16	12	12	44.2	60	60.8	76
9	65	0.32	0.74	0.64	0.43	0.62	0.58	0.56	0.86	1.77	0.77	1.2	1.67	10	19	8	10	36	47	48	53
10	72	0.56	0.68	1.76	0.58	0.92	0.57	0.45	1.09	1.58	0.65	0.37	1.46	9	16	12	12	44.2	60	60.8	76
11	79	0.9	1.5	1.1	0.8	1.7	1	1.8	2.1	2.1	2	2.7	2.5	8	3	8	12	46.4	67.1	54.8	63.1
12	85	0.8	1.2	1.1	1.1	1.6	1.1	1.6	1.3	1.7	1.3	3	3.5	11	6	10	16	46.2	56.2	52.1	65.2
13	96	0.64	1.07	1.03	0.56	1.19	0.73	0.85	1.03	1.77	0.8	1.86	2.7	11	5	10	11	-	-	-	-
14	100	0.7	1.1	0.6	1	1.7	1.1	1.8	1.4	2.2	2.2	2.4	3.4	10	8	6	8	42.6	52.6	57.6	65
15	113	0.6	1.33	0.59	0.74	1.66	0.67	0.67	1.56	2.35	0.73	2.6	2.81	10	11	8	13	40.1	59.5	56.1	62.2
16	123	0.8	1.3	1.4	0.74	1.66	0.69	1.7	1.58	2.2	1.2	3.1	3.12	10	59	6	13	43.1	59.4	57.5	65.5
17	123	0.8	1.2	1	1	1.5	0.8	1.5	1.6	2.5	1.4	4.7	4.3	11	9	8	13	42.1	65.4	62.3	64.8
18	127	0.73	1.16	0.6	1.1	1.4	0.6	0.7	1.12	1.8	1.3	2.8	4.2	-	-	-	-	-	-	-	-
19	189	1	1.4	1.2	1.6	1.7	0.9	1.4	1.3	2.2	1.1	4.5	4.4	-	-	-	-	-	-	-	-
20	282	0.8	1.2	0.7	1.06	1.58	0.71	0.69	1.18	2	1.03	3.17	4.74	9	24	13	24	43.2	62.5	65.2	63.2
21	314	0.82	1.2	0.73	1.03	1.51	0.67	0.55	1.16	1.97	1.24	3.05	4.61	9	7	12	14	45.1	63.5	66.8	64.2
22	345	0.79	1.16	0.79	0.9	1.36	0.62	0.96	1.11	1.98	1.34	2.95	4.44	10	10	11	19	50.1	65.4	65.4	64.8

3.3 Determination of the Remaining Useful Life (RUL)

Having determined the deterioration model denoted by Equation 6, the RUL was evaluated based on some decision rules. Ordinarily, in the traditional usage of MLR, the dependent variable is ascertained by inputting the conditional values of the independent variables for which the dependent variable is to be derived. However, considering that in an MLR model, individual variables trend at different rate (i.e., when considered as distinct Linear Models), whereas in vibration monitoring, an extreme value reading in any of the axes could indicate a failure, it is not a plausible approach to input extreme vibration magnitude values in the MLR as it does not depict reality. Hence, a linear programming approach is used based on several criteria utilized to judge equipment status based on vibration readings.

The decision criteria used follows the ISO 10816 and 7919 specifications which is in line with the company's maintenance policies regarding vibration analysis. The criteria are as follows:

1. Motor non-drive-end velocities i.e., MV_{NDEV}, MV_{NDEH} and MV_{NDEA}, by exceeding a threshold value ϕ_1 for a range of different equipment running speed indicates a warning on the equipment and should be scheduled for maintenance.
2. Motor drive-end velocities i.e., MV_{DEV}, MV_{DEH}, and MV_{DEA}, by exceeding a threshold value ϕ_2 for a range of different equipment running speed indicates a warning on the equipment and should be scheduled for maintenance.
3. Shaft non-drive-end velocities i.e., SV_{NDEV}, SV_{NDEH}, and SV_{NDEA}, by exceeding a threshold value ϕ_3 for a range of different equipment running speed indicates a warning on the equipment and should be scheduled for maintenance.
4. Shaft drive-end velocities i.e., SV_{DEV}, SV_{DEH}, and SV_{DEA}, by exceeding a threshold value ϕ_4 for a range of different equipment running speed indicates a warning on the equipment and should be scheduled for maintenance.
5. The difference between Motor non-drive-end decibel carpet value and decibel maximum reading i.e. $MU_{NDD,}$by exceeding a threshold value ϕ_5 for a range of different equipment running speed indicates a warning on the equipment and should be scheduled for maintenance.
6. The difference between Motor drive-end decibel carpet value and decibel maximum reading i.e. MU_{DD}, by exceeding a threshold value ϕ_6 for a range of different equipment running speed indicates a warning on the equipment and should be scheduled for maintenance.
7. The difference between shaft non-drive-end decibel carpet value and decibel maximum reading i.e. SU_{NDD}, by exceeding a threshold value ϕ_7 for a range of different equipment running speed indicates a warning on the equipment and should be scheduled for maintenance.
8. The difference between shaft drive-end decibel carpet value and decibel maximum reading i.e. SU_{DD}, by exceeding a threshold value ϕ_8 for a range of different equipment running speed indicates a warning on the equipment and should be scheduled for maintenance
9. Shaft and Motor, non-drive and drive ends i.e., MT_{NDE}, $MT_{DE}, ST_{NDE}, ST_{DE}$ by exceeding a temperature δ_1, is a warning indication for which the equipment should be scheduled for maintenance within the next available period.
10. Motor non-drive-end velocities i.e., MV_{NDEV}, MV_{NDEH} and MV_{NDEA}, by exceeding a threshold value ψ_1 for a range of different equipment running speed indicates a criticality on the equipment and should be shut down immediately for maintenance.
11. Motor drive-end velocities i.e., MV_{DEV}, MV_{DEH}, and MV_{DEA}, by exceeding a threshold value ψ_2 for a range of different equipment running speed indicates a criticality on the equipment and should be shut down immediately for maintenance.
12. Shaft non-drive-end velocities i.e., SV_{NDEV}, SV_{NDEH}, and SV_{NDEA}, by exceeding a threshold value ψ_3 for a range of different equipment running speed indicates a criticality on the equipment and should be shut down immediately for maintenance.
13. Shaft drive-end velocities i.e., SV_{DEV}, SV_{DEH}, and SV_{DEA}, by exceeding a threshold value ψ_4 for a range of different equipment running speed indicates a criticality on the equipment and should be shut down immediately for maintenance.
14. The difference between Motor non-drive-end decibel carpet value and decibel maximum reading i.e. MU_{NDD}, by exceeding a threshold value ψ_5 for a range of different equipment running speed indicates a criticality on the equipment and should be shut down immediately for maintenance.
15. The difference between Motor drive-end decibel carpet value and decibel maximum reading i.e. MU_{DD}, by exceeding a threshold value ψ_6 for a range of different equipment running speed indicates a criticality on the equipment and should be shut down immediately for maintenance

16. The difference between shaft non-drive-end decibel carpet value and decibel maximum reading i.e. SU_{NDD}, by exceeding a threshold value ψ_7 for a range of different equipment running speed indicates a criticality on the equipment and should be shut down immediately for maintenance
17. The difference between shaft drive-end decibel carpet value and decibel maximum reading i.e. SU_{DD}, by exceeding a threshold value ψ_8 for a range of different equipment running speed indicates a criticality on the equipment and should be shut down immediately for maintenance
18. Shaft and Motor, non-drive and drive ends i.e., MT_{NDE}, $MT_{DE}, ST_{NDE}, ST_{DE}$ not exceed the temperature δ_2, otherwise it is regarded as a critical state for which the equipment must be maintained immediately.

By considering the above boundaries, the RUL for the Lower Bound can be denoted as:

$$
\begin{aligned}
&maximize: \\
&T_1 = \beta_0 + \beta_1 MV_{NDEV} + \beta_2 MV_{NDEH} + \beta_3 MV_{NDEA} \\
&\quad + \beta_4 MV_{DEV} + \beta_5 MV_{DEH} + \beta_6 MV_{DEA} \\
&\quad + \beta_7 SV_{NDEV} + \beta_8 SV_{NDEH} + \beta_9 SV_{NDEA} \\
&\quad + \beta_{10} SV_{DEV} + \beta_{11} SV_{DEH} + \beta_{12} SV_{DEA} \\
&\quad + \beta_{13} MU_{NDD} + \beta_{14} MU_{DD} + \beta_{15} SU_{NDD} \\
&\quad + \beta_{16} SU_{DD} + \beta_{17} MT_{NDE} + \beta_{18} MT_{DE} \\
&\quad + \beta_{19} ST_{NDE} + \beta_{20} ST_{DE} + \varepsilon \\
&Subject\ to: \\
&\quad MV_{NDEV}, MV_{NDEH}, MV_{NDEA} \le \phi_1 \\
&\quad MV_{DEV}, MV_{DEH,} MV_{DEA} \le \phi_2 \\
&\quad SV_{NDEV}, SV_{NDEH}, SV_{NDEA} \le \phi_3 \\
&\quad SV_{DEV}, SV_{DEH}, SV_{DEA}, \le \phi_4 \\
&\quad MU_{NDD} \le \phi_5 \\
&\quad MU_{DD} \le \phi_6 \\
&\quad SU_{NDD} \le \phi_7 \\
&\quad SU_{DD} \le \phi_8 \\
&\quad MT_{NDE}, MT_{DE}, ST_{NDE}, ST_{DE} \le \delta_1 \\
&MV_{NDEH}, MV_{NDEV}, MV_{NDEA}, MV_{DEH,} MV_{DEA}, MV_{DEV}, \ge 0 \\
&\quad SV_{NDEV}, SV_{NDEH}, SV_{NDEA}, SV_{DEV}, SV_{DEH}, SV_{DEA}, \ge 0 \\
&\quad SU_{NDD}, SU_{DD} \ge 0 \\
&\quad MU_{NDD}, MU_{DD} \ge 0 \\
&\quad MT_{NDE}, MT_{DE}, ST_{NDE}, ST_{DE} \ge 0
\end{aligned} \qquad 7
$$

Similarly, for the upper bound, the RUL model formulation is given by:

$$
\begin{aligned}
&maximize: \\
&T_2 = \beta_0 + \beta_1 MV_{NDEV} + \beta_2 MV_{NDEH} + \beta_3 MV_{NDEA} \\
&\quad + \beta_4 MV_{DEV} + \beta_5 MV_{DEH} + \beta_6 MV_{DEA} \\
&\quad + \beta_7 SV_{NDEV} + \beta_8 SV_{NDEH} + \beta_9 SV_{NDEA} \\
&\quad + \beta_{10} SV_{DEV} + \beta_{11} SV_{DEH} + \beta_{12} SV_{DEA} \\
&\quad + \beta_{13} MU_{NDD} + \beta_{14} MU_{DD} + \beta_{15} SU_{NDD} \\
&\quad + \beta_{16} SU_{DD} + \beta_{17} MT_{NDE} + \beta_{18} MT_{DE} \\
&\quad + \beta_{19} ST_{NDE} + \beta_{20} ST_{DE} + \varepsilon \\
&Subject\ to: \\
&\quad MV_{NDEV}, MV_{NDEH}, MV_{NDEA} \le \psi_1 \\
&\quad MV_{DEV}, MV_{DEH,} MV_{DEA} \le \psi_2 \\
&\quad SV_{NDEV}, SV_{NDEH}, SV_{NDEA} \le \psi_3 \\
&\quad SV_{DEV}, SV_{DEH}, SV_{DEA}, \le \psi_4 \\
&\quad MU_{NDD} \le \psi_5 \\
&\quad MU_{DD} \le \psi_6 \\
&\quad SU_{NDD} \le \psi_7 \\
&\quad SU_{DD} \le \psi_8 \\
&\quad MT_{NDE}, MT_{DE}, ST_{NDE}, ST_{DE} \le \delta_2 \\
&MV_{NDEH}, MV_{NDEV}, MV_{NDEA}, MV_{DEH,} MV_{DEA}, MV_{DEV}, \ge 0 \\
&\quad SV_{NDEV}, SV_{NDEH}, SV_{NDEA}, SV_{DEV}, SV_{DEH}, SV_{DEA}, \ge 0 \\
&\quad SU_{NDD}, SU_{DD} \ge 0 \\
&\quad MU_{NDD}, MU_{DD} \ge 0 MT_{NDE}, MT_{DE}, ST_{NDE}, ST_{DE} \ge 0
\end{aligned} \qquad (8)
$$

By solving the optimization problem of Equations 7 and 8 through linear programming and determining RUL to give Lower and Upper bound respectively, the plausible time to schedule maintenance would be known.

4.0 RESULTS AND DISCUSSION

4.1 Results

From analysis of the data collected and solving for the RUL at lower and upper bound, the summary of the results is displayed in Tables 2 to 6.

Table 2: Pearson's Correlation Coefficient for the Independent Variables

Independent Variables	Pearson's R-value
M_{NDEV}***	0.587813
MV_{NDEH}	0.406206
MV_{NDEA}	-0.089466
MV_{DEV}	0.453869
MV_{DEH}	0.395642

Independent Variables	Pearson's R-value
MV_{DEA}	0.092645
SV_{NDEV}	-0.146301
SV_{NDEH}	-0.138284
SV_{NDEA}	0.195799
SV_{DEV}	-0.041291
SV_{DEH}	0.394019
SV_{DEA}***	0.782795
MU_{NDD}	-0.190366
MU_{DD}	0.078332
SU_{NDD}	0.176138
SU_{DD}***	0.636801
MT_{NDE}	0.151755
MT_{DE}	0.167939
ST_{NDE}***	0.717093
ST_{DE}	-0.110101

Key: * ** Variables that have strong correlation

Table 3: Summary of MLR Result for Equipment

Coef of:	Estimate	Std. Error	t value	Pr(>|t|)
MV_{NDEV}	2.869	130.391	0.022	0.98269
SV_{DEA}	51.921	17.652	2.941	0.00873
SU_{DD}	4.755	4.066	1.170	0.25742
ST_{NDE}	-1.820	1.400	-1.300	0.21006

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 66.96 on 18 degrees of freedom

Multiple R-squared 0.8215,

F-statistic: 20.71 on 4 and 18 DF, p-value: 1.546e-06

Table 4: Boundary Values for RUL Model

Parameters	Remark	Values
ϕ_1	Warning motor non-drive-end velocity	4 mm/s
ϕ_2	Warning motor drive-end velocity	4 mm/s
ϕ_3	Warning shaft non-drive-end velocity	4 mm/s
ϕ_4	Warning shaft drive-end velocity	4 mm/s
ϕ_5	Warning decibel carpet and maximum difference for motor non-drive-end	16 dB
ϕ_6	Warning decibel carpet and maximum difference for motor drive-end	16 dB
ϕ_7	Warning decibel carpet and maximum difference for shaft non-drive-end	16 dB

Parameters	Remark	Values
ϕ_8	Warning decibel carpet and maximum difference for shaft drive-end	16 dB
δ_1	Warning temperature reading for shaft and motor	60 ^{0}C
ψ_1	Critical motor non-drive-end velocity	7 mm/s
ψ_2	Critical motor drive-end velocity	7 mm/s
ψ_3	Critical shaft non-drive-end velocity	7 mm/s
ψ_4	Critical shaft drive-end velocity	7 mm/s
ψ_5	Critical decibel carpet and maximum difference for motor non-drive-end	20 dB
ψ_6	Critical decibel carpet and maximum difference for motor drive-end	20 dB
ψ_7	Critical decibel carpet and maximum difference for shaft non-drive-end	20 dB
ψ_8	Critical decibel carpet and maximum difference for shaft drive-end	20 dB
δ_2	Critical temperature reading for shaft and motor	70^{0}C

Table 5: Lower Bound Solution for RUL Linear Program

Variable	Value
Objective function value	324.7146
MV_{NDEV}	0.0000
SV_{DEA}	4.0000
SU_{DD}	16.0000
ST_{NDE}	0.0000

Table 6: Upper Bound Solution for RUL Linear Program

Variable	Value
Objective function value	526.8925
MV_{NDEV}	0.0000
SV_{DEA}	7.0000
SU_{DD}	20.0000
ST_{NDE}	0.0000

4.2 Discussion of Results

From Table 2, the Pearson's R-value of the independent variables ranges from -0.190366 to 0.782795. When Pearson's R-value is near ± 1, then it said to be a perfect correlation as one variable increases, the other variable tends to also increase (if positive) or decrease (if negative). If the coefficient value lies between ± 0.50 and ± 1, then it is said to be a strong correlation [19]. From the results indicated in Table 2, it could be seen that 4 variables out of the initial 20 variables showed strong correlation with the dependent variables RUL (depicted as T_i in the model formulation). This is because these 4 variables have their absolute Pearson's R-value greater than or equal to 0.5. The variables that are strongly correlated in this case are M_{NDEV}, SV_{DEA}, SU_{DD}, and ST_{NDE} implying they are the one critical for the continuous functioning of the equipment.

For the MLR results (Table 3), though the Pr(>|t|) value for each variable are quite large proving some insignificance, the combined model p-value of 1.546e-06, a Multiple R-squared value of 0.8215 results to

$$T_i = 2.869MV_{NDEV} + 51.921SV_{DEA} + 4.755SU_{DD} - 1.82ST_{NDE} \quad (9)$$

Equation 9 is a good prediction model that is statistically significant and can be applied for failure prediction of the equipment.

Having determined the multiple linear regression as regarding the deterioration function for the equipment, the result of the MLR was then applied to the RUL Model. The boundary conditions for the RUL formulation are given in Table 4 from which the Lower and Upper Bound RUL value for the equipment were gotten after solving the Linear Programme problem. The results for the RUL model for the equipment as shown by Table 5 and Table 6 reveal a Lower Bound value of 324.7146 and an Upper bound value of 526.8925 days. By subtracting 345 (since the last reading for the equipment was taken on day 345) from both values which is the day the most recent reading was taken, the Lower bound and Upper bound RUL for Equipment is -20.29 and 181.89 days respectively. This implies that the equipment should be scheduled for maintenance not later than 181.89 days (into the future) to avoid failure.

5.0 CONCLUSION

Appropriate scheduling maintenance is very essential to ensure a smooth running of any operational system, hence should be well planned. In this study MLR was explored to model the vibration features of any equipment with the aim of predicting time to maintain such machine. For the equipment used as a case study, twenty (20) independent variables were found to be associated with the equipment, from the procedure adopted, four (4) of the variables were identified as critical. And with the analysis of the vibration data collected for the equipment, a deterioration model to predict time to failure of the machines was developed. The model developed aided in determining the remaining useful life (RUL) of the equipment. With a p-value of 1.546e-06 and Multiple R-squared value of 0.8215 gotten these results show that the model has a good reliability in forecasting time for equipment maintenance. The procedure described in this study if implemented, could aid in planning and scheduling effective maintenance system for any operational set-up.

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