Nigerian Journal of Technology (NIJOTECH)
Vol. 31, No. 3, November, 2012, pp. 359–364.
 ISSN 1115-8443

STABILITY ANALYSIS OF A THREE-PHASE SOLID-STATE VAR COMPENSATOR (SSVC)

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Abstract

The problems associated with the flow of reactive power in transmission and distribution lines are well known. Several methods of var compensation have been applied in solving these problems. One of the modern devices employed in reactive power compensation is the three-phase solid-state var compensator (SSVC). The principal component of this device is a three-phase pulse-width-modulated voltage source inverter. A mathematical model of the inverter is derived in d-q reference frame and then used to examine the stability of the compensator in response to variations in circuit parameters.

Keywords: var compensation, inverter, stability, circuit parameters, root locus

1. Introduction

It is a well-known fact that poor power factor causes the flow of reactive power in transmission and distribution networks, resulting in (a) voltage drop at line ends, (b) a rise in temperature in the supply cables, producing losses of active power, (c) over-sizing of generation, transmission and distribution equipment, (d) over-sizing of load protection due to harmonic currents, and (e) transformer overloads.

In view of the aforementioned problems associated with poor power factor, it is necessary to improve the power factor of an installation. Before the advent of modern power electronics, shunt static capacitors/reactors and synchronous condensers were extensively used to reduce the level of reactive power flowing in transmission and distribution networks [1, 2] But these elements are costly, bulky and often relatively inefficient. As a result, extensive research developed line commutated thyristors converters in combination with some reactive components. But there is the problem of reliable controlled switching. Its effective use is only when it is force-commutated and it requires costly and complex external circuits that reduce circuit reliability. But the advent of fast self-commutating solid-state devices (bipolar junction transistor (BJT), insulated-gate bipolar transistor (IGBT), gate-turn-off thyristor (GTO) and power MOSFET has eliminated these problems. The voltage source inverter (VSI) employing any one of these devices is an efficient equipment for reactive power compensation or reduction of harmonic injection into ac mains - in order to improve power factor. They also reduce electromagnetic interference (EMI) without requiring bulky and lossy snubber circuits [3]. Equipment is available from 380V to 34.5kV.

2. The three-phase solid-state var compensator (SSVC)

2.1. Description

The power circuit of a three-phase solid-state var compensator (SSVC) is shown in figure 1 [4] It employs a pulse-width modulated (PWM) voltage source inverter (VSI). The inverter is connected to the ac mains through a reactor X_1. A dc capacitor is connected to the dc side of the var compensator. The capacitor maintains a ripple-free dc voltage at the input of the inverter as well as storing reactive power. The SSVC is connected to the load through a second-order low-pass filter, X_2 and X_C, which reduces the harmonic components of currents flowing into the utility grid.

2.2. Principle of operation

With reference to figure 1, the single-phase equivalent circuit and the phasor diagram of the var compensator at fundamental frequency are as shown in figure 2.

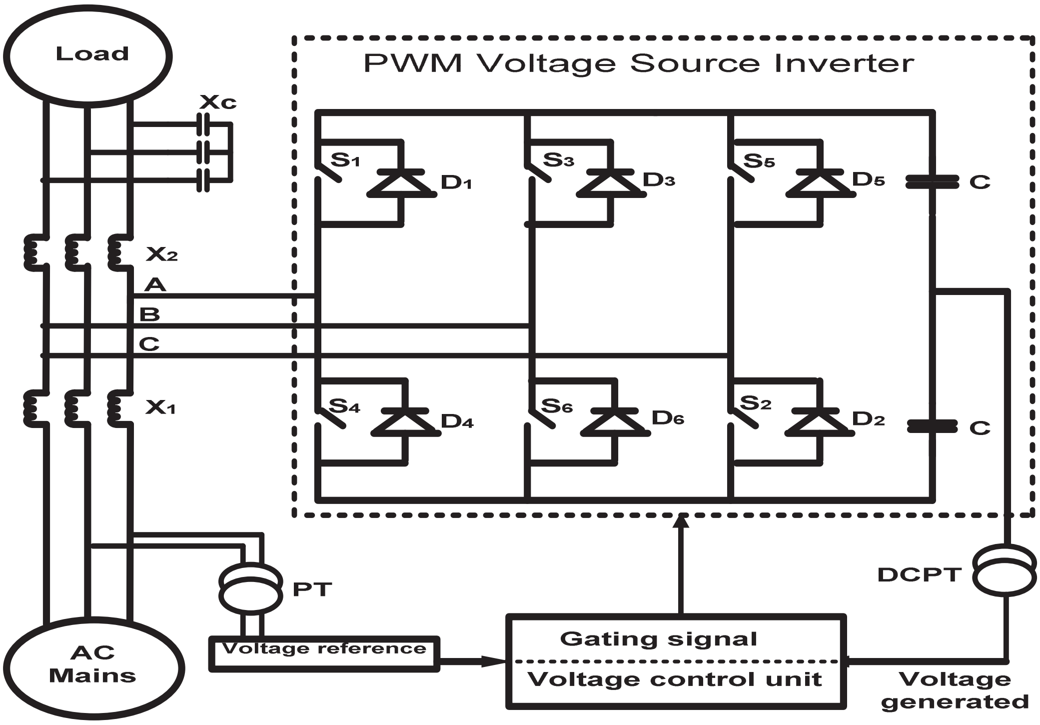
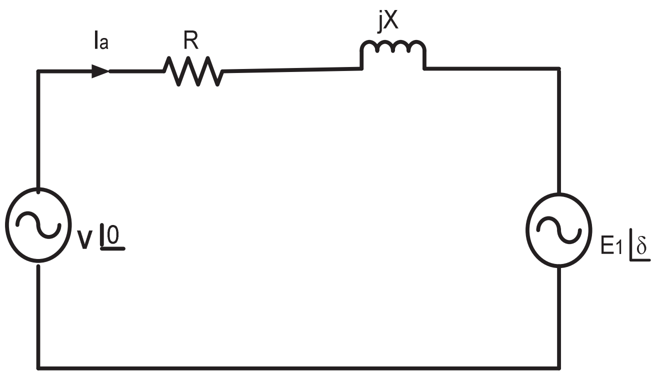
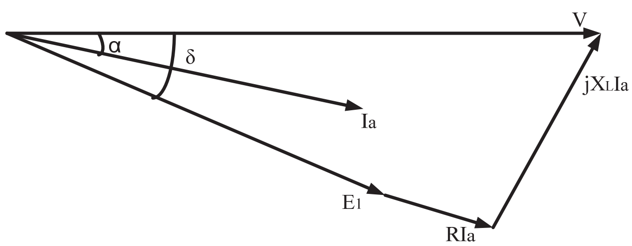


Figure 1: Solid-state var compensator configuration.



(a) Single-phase equivalent circuit.



(b) Phasor diagram.

Figure 2: V = Line-to-neutral voltage of the ac mains; E = Fundamental component of the inverter phase-to-neutral ac voltage; i = ac current; R = Losses of the system lumped; and L = Line inductor.

R is negligible compared to X $(2\pi f L)$ so the apparent power supplied by the ac source can be expressed as

$$\begin{aligned} S &= VI_a* = |V|\angle 0 I_a* \\ I_a* &= \frac{|V|\angle 0 - |E_1|\angle -\delta}{-jX} \\ S &= \frac{V^2\angle 90^\circ - VE_1\angle 90-\delta}{X} \\ S &= -\frac{VE_1}{X}\sin\delta + j\left\{\frac{V^2 - VE_1\cos\delta}{X}\right\} \end{aligned} \tag{1}$$

where, δ = phase-shift angle between the source voltage, V and the inverter ac voltage, E_1.

The real power supplied by the ac source is shared by the load and the inverter. The amplitude of the fundamental component of the inverter output ac voltage, E depends on the value of the dc bus voltage, V_{dc}. So, V increases or decreases if the capacitor is charged or discharged. The voltage drop across the inductor X_1 determines the source power factor. The voltage drop across X_1 can be minimized by equalizing V and E, thus maintaining near unity power factor. The var compensator responds to fluctuations in load power factor by providing extra power required by the load or absorbing excess power from the load. If the power factor of the load increases, the load draws more real power which is transiently supplied by the inverter. The capacitor is thus discharged, leading to decrease in E. The control system takes corrective measure to make E equal to the corresponding value of V and hence maintain the source power factor at near unity. This is done by increasing the phase-shift angle, δ and more active power will flow to the inverter to charge the capacitor. If the load power factor decreases, the load is taking less real power, and then the compensator absorbs the excess real power which charges the capacitor and then lead to higher output of E. To restore E to normal value, the capacitor is discharged by decreasing δ. By controlling the phase angle δ, the dc capacitor voltage levels can be changed, and thus the amplitude of the fundamental component of the inverter output voltage E can be controlled [5, 6]. Real power flows to the compensator if the load power factor is lagging (the load is drawing less active power); and the compensator supplies real power if load power factor is leading (load requires more active power).

3. Transient Analysis of the Var Compensator

3.1. Mathematical model

From Figure 2, R is the equivalent resistance representing the total compensator system losses. To derive mathematical model of the solid-state var compensator, we assume that (a) the ac source is a ripple-free, balanced, three-phase voltage, (b) only fundamental components of currents and voltages are represented by the equivalent circuit, (c) since the variations in the phase-shift angle, $\Delta\delta$ are small, the system is made linear [7].

From the equivalent circuit,

$$v(t) - e(t) = Ri(t) + L\frac{di(t)}{dt} \tag{2}$$

Let δ oscillate around a mean value δ_o between $\delta_o - \Delta\delta_o$ and $\delta_o + \Delta\delta_o$ with a frequency ω_d, then,

$$\delta(t) = \delta_{max}\cos(\omega_d t) = Re\lfloor \delta_{max} e^{j\omega_d t} \rfloor \tag{3}$$

Considering the inverter voltage oscillations, the voltages and current can be expressed as

$$\begin{aligned} v(t) &= Ve^{j(\omega t + \Delta\delta)} \\ e(t) &= Ee^{j(\omega t + \Delta\delta)} \\ i(t) &= Ie^{j(\omega t + \Delta\delta)} \end{aligned} \tag{4}$$

where $\omega = 2\pi f$ is the ac source frequency.

In d-q axis

$$\begin{aligned} v(t) &= (v_d + jv_q)[\cos(\omega t + \delta) + j\sin(\omega t + \delta)] \\ e(t) &= (e_d + je_q)[\cos(\omega t + \delta) + j\sin(\omega t + \delta)] \\ i(t) &= (i_d + ji_q)[\cos(\omega t + \delta) + j\sin(\omega t + \delta)] \end{aligned} \tag{5}$$

From equation (3)

$$\begin{aligned} v(t) &= \lfloor v_d\cos\omega t - v_q\sin\omega t \rfloor \cos(\delta) \\ e(t) &= \lfloor e_d\cos\omega t - e_q\sin\omega t \rfloor \cos(\delta) \\ i(t) &= \lfloor i_d\cos\omega t - i_q\sin\omega t \rfloor \cos(\delta) \end{aligned} \tag{6}$$

Equation (5) in (2) gives

$$\begin{aligned} (v_d - e_d)\cos\omega t - (v_q - e_q)\sin\omega t = \left(Ri_d + L\frac{di_d}{dt} - \omega Li_q\right) \\ \cos\omega t - \left(Ri_q + L\frac{di_q}{dt} + \omega Li_d\right)\sin\omega t \end{aligned} \tag{7}$$

And then, after grouping the steady-state equations are:

$$\begin{aligned} v_{do} - e_{do} &= Ri_{do} + L\frac{di_{do}}{dt} - \omega Li_{qo} \\ v_{qo} - e_{qo} &= Ri_{qo} + L\frac{di_{qo}}{dt} + \omega Li_{do} \end{aligned} \tag{8}$$

Applying small disturbances to variables in (7) around the operating point yields

$$\begin{aligned} v_d = v_{do} + \Delta v_d;\ \ e_d = e_{do} + \Delta e_d;\ \ i_d = i_{do} + \Delta i_d \\ v_q = v_{qo} + \Delta v_q;\ \ e_q = e_{qo} + \Delta e_q;\ \ i_q = i_{qo} + \Delta i_q \end{aligned} \tag{9}$$

Equation (7) becomes

$$\begin{aligned} v_{do} + \Delta v_d - e_{do} - \Delta e_d &= Ri_{do} + R\Delta i_d + L\frac{di_{do}}{dt} \\ &+ L\frac{d\Delta i_d}{dt} - \omega Li_{qo} - \omega L\Delta i_q \\ v_{qo} + \Delta v_q - e_{qo} - \Delta e_q &= Ri_{qo} + R\Delta i_q + L\frac{di_{qo}}{dt} \\ &+ L\frac{d\Delta i_q}{dt} + \omega Li_{do} + \omega L\Delta i_d \end{aligned} \tag{10}$$

Subtract (7) from (9),

$$\Delta v_d - \Delta e_d = R\Delta i_d + L\frac{d\Delta i_d}{dt} - \omega_o L\Delta i_q \tag{11}$$

$$\Delta v_q - \Delta e_q = R\Delta i_q + L\frac{d\Delta i_q}{dt} + \omega_o L\Delta i_d \tag{12}$$

Multiplying equation (7) by $\Delta\delta$

$$v_{do}\Delta\delta - e_{do}\Delta\delta = Ri_{do}\Delta\delta + L\frac{di_{do}}{dt}\Delta\delta - \omega Li_{qo}\Delta\delta \tag{13}$$

$$v_{qo}\Delta\delta - e_{qo}\Delta\delta = Ri_{qo}\Delta\delta + L\frac{di_{qo}}{dt}\Delta\delta + \omega Li_{do}\Delta\delta \tag{14}$$

Subtract (13) from (10), and sum (11) and (12)

$$\begin{aligned} \Delta v_d - \Delta e_d - v_{qo}\Delta\delta + e_{qo}\Delta\delta &= R\Delta i_d + L\frac{d\Delta i_d}{dt} \\ -\omega_o L\Delta i_q - Ri_{qo}\Delta\delta &- L\frac{di_{qo}}{dt}\Delta\delta - \omega Li_{do}\Delta\delta \\ \Delta v_q - \Delta e_q + v_{do}\Delta\delta - e_{do}\Delta\delta &= R\Delta i_q + L\frac{d\Delta i_q}{dt} \\ +\omega_o L\Delta i_d + Ri_{do}\Delta\delta &+ L\frac{di_{do}}{dt}\Delta\delta - \omega Li_{qo}\Delta\delta \end{aligned} \tag{15}$$

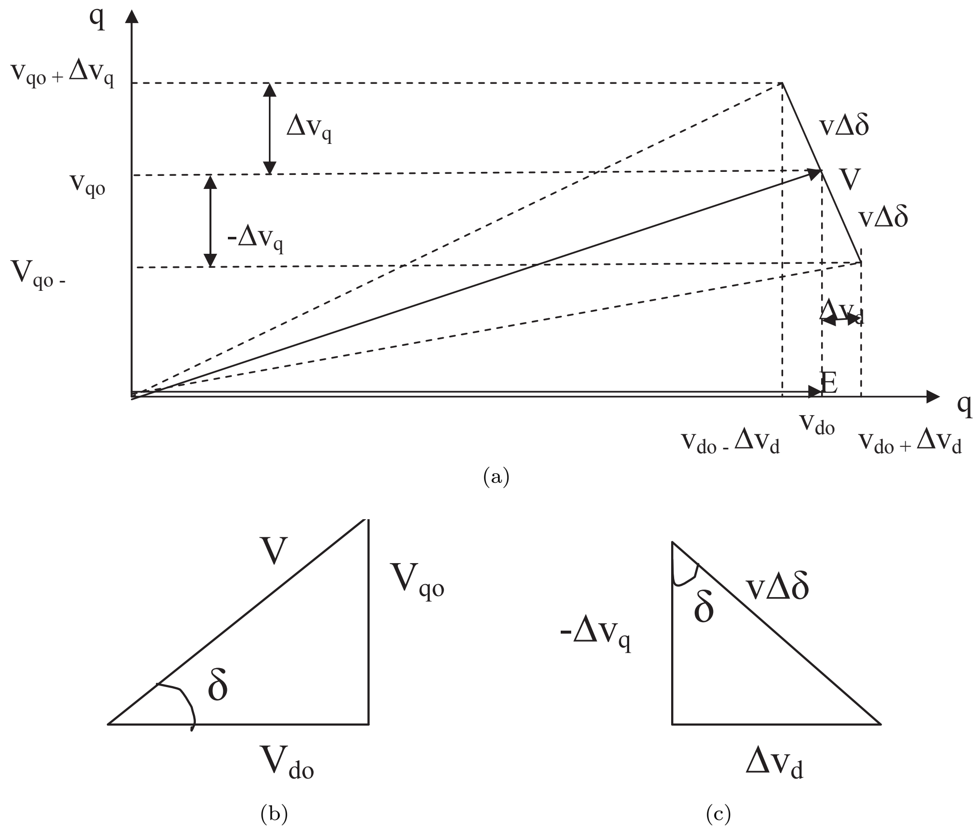


Figure 3: Phasor diagram of the perturbed system.

Applying the Laplace transform to equation (14),

$$\begin{aligned}\Delta v_d - \Delta e_d - v_{qo}\Delta\delta + e_{qo}\Delta\delta &= (R+sL)(\Delta i_d - i_{qo}\Delta\delta)\\ &\quad -\omega_o L(\Delta i_q - i_{do}\Delta\delta)\\ \Delta v_q - \Delta e_q + v_{do}\Delta\delta - e_{do}\Delta\delta &= (R+sL)(\Delta i_q + i_{do}\Delta\delta)\\ &\quad +\omega_o L(\Delta i_d - i_{qo}\Delta\delta)\end{aligned} \tag{16}$$

In matrix form, the final equations for the system are

$$\begin{gathered}\begin{bmatrix}\Delta v_d - v_{qo}\Delta\delta\\ \Delta v_q + v_{do}\Delta\delta\end{bmatrix} - \begin{bmatrix}\Delta e_d - e_{qo}\Delta\delta\\ \Delta e_q + e_{do}\Delta\delta\end{bmatrix} =\\ \begin{bmatrix}R+sL & -\omega_o L\\ \omega_o L & R+sL\end{bmatrix} \times \begin{bmatrix}\Delta i_d - i_{qo}\Delta\delta\\ \Delta i_q + i_{do}\Delta\delta\end{bmatrix}\end{gathered} \tag{17}$$

3.2. *Transfer function of the compensator*

Figure 3 is the phasor diagram of the perturbed system, where the inverter output voltage E is taken as the reference phasor with the ac voltage d-q components oscillating about their quiescent values, v_{do} and v_{qo} with amplitudes Δv_d and Δv_q and frequency ω_d.

From figure 3

$$\begin{aligned}v_{do} &= V\cos(\delta)\\ -\Delta v_q &= V\Delta\delta\cos(\delta)\end{aligned} \tag{18}$$

$$\begin{aligned}v_{qo} &= V\sin(\delta)\\ -\Delta v_d &= V\Delta\delta\sin(\delta)\end{aligned} \tag{19}$$

Equations (17) and (18) in (16) results in

$$\begin{aligned}\Delta v_d - v_{qo}\Delta\delta &= V\Delta\delta\sin(\delta) - V\Delta\delta\sin(\delta) = 0\\ \Delta v_q - v_{do}\Delta\delta &= -V\Delta\delta\cos(\delta) - V\Delta\delta\cos(\delta) = 0\end{aligned} \tag{20}$$

$$\begin{aligned}e_{do} = E;\ \ \Delta e_{do} &= \Delta E\\ e_{qo} = 0;\ \ \Delta e_{qo} &= 0\end{aligned} \tag{21}$$

If k is the modulation index of the inverter, then, the output voltage is related to the capacitor voltage as

$$\begin{aligned}e_{do} &= kV_{dco}\\ \Delta e_d &= k\Delta V_{dc}\end{aligned} \tag{22}$$

Equations (20) and (21) in (16) yields

$$\begin{aligned}\begin{bmatrix}\Delta i_d - i_{qo}\Delta\delta\\ \Delta i_q - i_{do}\Delta\delta\end{bmatrix} &= \begin{bmatrix}R+sL & -\omega L\\ \omega L & R+sL\end{bmatrix}^{-1}\\ &\quad \times \begin{bmatrix}k\Delta V_{dc}\\ -k\Delta V_{dco}\Delta\delta\end{bmatrix}\end{aligned} \tag{23}$$

$$\Delta i_d = \frac{-(R+sL)k\Delta V_{dc} + \omega_o LkV_{dco}\Delta\delta}{L^2s^2 + 2RLs + (\omega_o L)^2 + R^2} + i_{qo}\Delta\delta \tag{24}$$

Power balance equation of the inverter is

$$\frac{3}{2}(e_d i_d + e_q i_q) = V_{dc}C\frac{dV_{dc}}{dt} \tag{25}$$

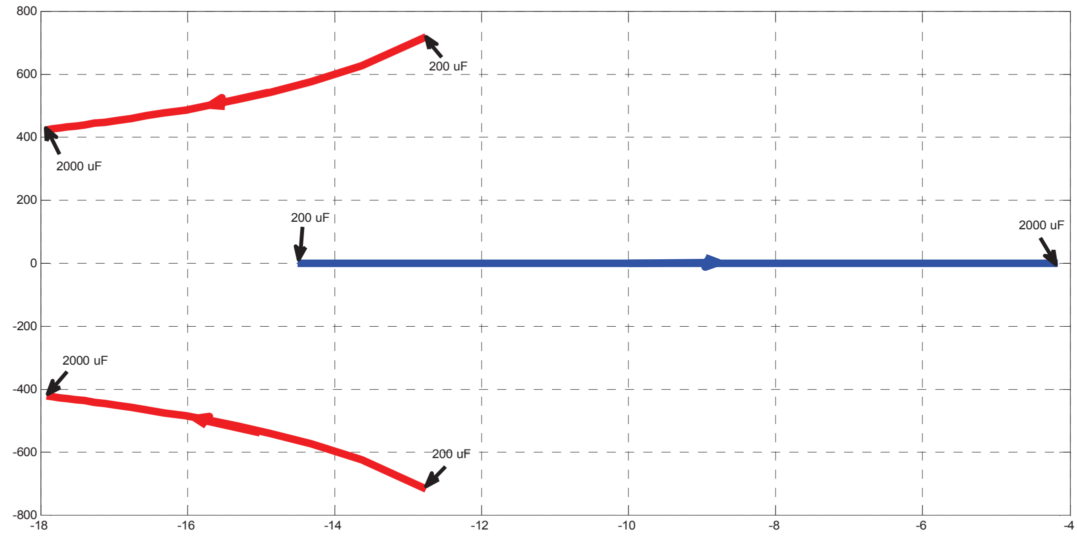


Figure 4: Varying values of Capacitance, C.

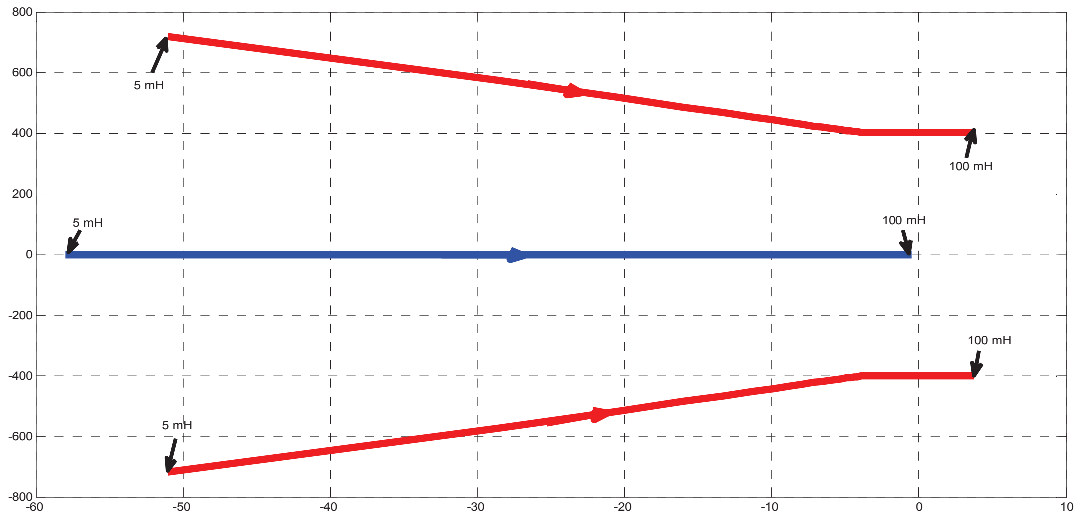


Figure 5: Varying values of Inductance, L.

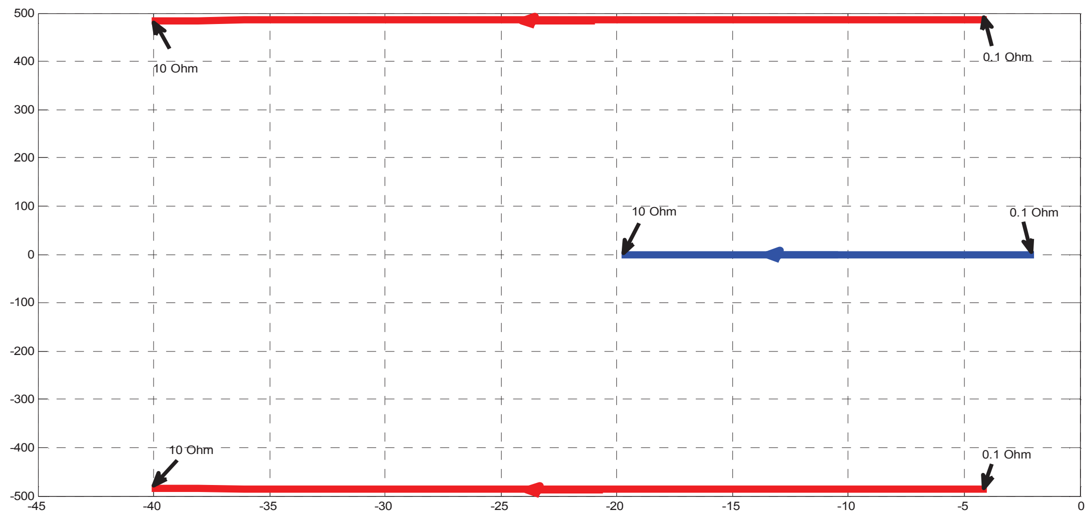


Figure 6: Varying values of Resistance, R.

C is the dc capacitor. Applying small perturbations around the steady-state operating point, $e_q = 0$,

$$\frac{3}{2}(e_{do}+\Delta e_d)(i_{do}+\Delta i_d) = (V_{dco}+\Delta V_{dc})C\frac{d}{dt}(V_{dco}+\Delta V_{dc}) \quad (26)$$

Subtracting the steady-state equation from equation (25) gives

$$\frac{3}{2}(e_{do}i_{do} + e_{do}\Delta i_d + \Delta e_d i_{do} + \Delta e_d \Delta i_d - e_{do}i_{do}) = V_{dco}C\frac{d}{dt}V_{dco} + V_{dco}C\frac{d}{dt}\Delta V_{dc} + \Delta V_{dc}C\frac{d}{dt}V_{dco} + \Delta V_{dc}C\frac{d}{dt}\Delta V_{dc} - V_{dco}C\frac{d}{dt}V_{dco} \quad (27)$$

Noting that, $\frac{d}{dt}V_{dco} = 0$, and neglecting second order terms, (27) becomes;

$$\frac{3}{2}(e_{do}\Delta i_d + \Delta e_d i_{do}) = V_{dco}C\frac{d}{dt}\Delta V_{dc} \quad (28)$$

i_{do} corresponds to the steady-state current component that provides the losses of the var compensator. Since the losses of the system are small, the product $\Delta e_d i_{do}$ can be neglected, therefore,

$$\frac{3}{2}e_{do}\Delta i_d = V_{dco}C\frac{d}{dt}\Delta V_{dc} \quad (29)$$

Recall, $e_{do} = kV_{dco}$. Hence, $\frac{3}{2}kV_{dco}\Delta i_d = V_{dco}C\frac{d}{dt}\Delta V_{dc}$

$$\frac{3}{2}k\Delta i_d = C\frac{d}{dt}\Delta V_{dc} \quad (30)$$

Applying Laplace transform,

$$\frac{3}{2}k\Delta i_d = Cs\Delta V_{dc}$$
$$\Delta i_d = \frac{2Cs\Delta V_{dc}}{3k} \quad (31)$$

From equations (23) and (29),

$$\frac{\Delta V_{dc}}{\Delta \delta} = \frac{3ki_{qo}A + 3k^2\omega_o LV_{dco}}{2CL^2s^3 + 4RCLs^2 + Bs + 3k^2R} \quad (32)$$

This is the transfer function of the var compensator system. Where, $A = L^2s^2 + 2RLs + \omega_o^2L^2 + R^2$ and $B = 2\omega_o^2L^2C + 2R^2C + 3k^2L$.

4. Analysis

Figure 4 is the root locus of the transfer function as the value of capacitance, C varies. It can be noted that as C increases, the real negative roots (being the dominant poles) of the characteristic equation of the transfer function approach the imaginary axis, making the speed of system response lower, and hence the system becomes less stable [8].

The root locus of the system transfer function for increasing values of the inductance, L is shown in Figure 5. It is shown that the real negative roots as well as the complex-conjugate roots move towards the imaginary axis. The compensator thus takes longer time to reach steady-state and consequently is less stable.

In Figure 6 it can be seen that increasing the value of resistance, R speeds up the system response, thus enhancing stability. R is therefore a damping component.

5. Conclusion

The features and operational principles of a var compensator employing PWM voltage source inverter with self-controlled dc bus, otherwise known as solid-state var compensator (SSVC) have been discussed in this paper. A model was derived, using *d-q* reference axis, for a var compensator operating with the δ phase-shift angle control. The model has been used to investigate the system response speed and the stability of the compensator in relation to its circuit parameters. Results show that the compensator under discussion is stable for wide variations of its circuit parameters. It is shown also that, with smaller circuit parameters, the compensator reaches steady-state faster and is therefore more stable and efficient.

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