HEAT FLOW IN A FINITE ISOLATED PULSED AVALANCHE SEMICONDUCTOR DIODE HEAT SINK FOR SHORT-TERM SYSTEM APPLICATIONS

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ABSTRACT

An analytical time solution of temperature rise of a finite heat sink under adiabatic boundary, conditions is proposed for partial system applications requiring short-term and low duty operation of pulsed high-power high-efficiency avalanche semiconductor devices. The temperature rise as a function of the heat sink size is computed, and useful practical design curves for a specified operation time presented.

INTRODUCITON

An appropriate heat-sinking is important for the investigation and application of high-power, high-efficiency semi-conductor devices. This calls for an accurate analytical model of heat flow in the diode heat-sink under given operating conditions.

Analytical time solutions of heat flow in semiconductor heat-sinks depend primarily on the required practical applications of the devices. For specified applications, the appropriate heat-sink model must be found if the ultimate capability of these devices in terms of the output power, reliability/failure rate, circuit miniaturization and integration and so on, is to be accurately determined. Numerous solutions [1] - [3] exist for the ideal cases where the heat-sink appears approximately semi-infinite in size. In many practical applications, however, the heat-sink is finite in size. Furthermore, for certain communications and pulsed radar systems applications requiring high-power, high-efficiency and short-term diode operation and compact system size, the size of the heat-sink is all the more important, and its dimensions are a critical parameter of the maximum allowable device junction temperature and the time duration of operation. For such short-term applications, substantially inaccurate results would be obtained if the heat-sink were approximated by a semi-infinite model. This paper, therefore, proposes an analytical time solution of heat flow a finite isolated high-power' high efficiency pulsed-diode heat-sink. The method of images [4] is employed to obtain the required solution of heat flow by extending the semi-infinite heat sink model to that of a finite heat sink with appropriate boundary conditions. The analytical technique described is believed to be also applicable to other semiconductor device heat-sinks under similar pulsed conditions.

It must be emphasized that the central practical importance of the present work is the fact that the "isolated" or insulated finite heat sink is subjected to a short term pulsed diode operation. Thus the model should also be applicable to a wide range of other semiconductor devices particularly those designed for airborne fuse-controlled systems.

2.1 THE PHYSICAL MODEL

Figure 1 shows a basic structure of a diode,-radius R, mounted on a finite heat-sink. The diode generates all the heat, and thus acts as a time varying heat source. The generated heat is assumed to enter the finite heat-sink (dimensions 2W x 2W x H) as a uniform flux of constant density throughout a region of circular symmetry ($0 < x^2 + y^2 < R^2$). The remainder of the § = 0 plane and all the remaining planes of the heat sink are adiabatic surfaces.

Because of the finite heat-sink size and the adiabatic boundary conditions, the heat-sink temperature for specified heat-sink material and dimensions, will increase indefinitely with time as heat is supplied to it by the heat source. Thus for the short-term applications considered, this variation of the heat-sink temperature as a function of operating time must be established.

2.2 HEAT FLOW ANALYSIS

Figure 2 shows a side view of the heat source - heat-sink geometry considered. The following pertinent assumptions apply:

i. A time varying circular input heat source, F(t),with constant average heat flux density, Fo, across the source area, and of radius R, equal to that of the diode, is incident on one end of the heat-sink only. (The assumption of a constant average heat flux density is obviously the case of most practical interest for the applications envisaged in this work).

ii. The heat sink (dimensions 2W x 2W x H) is considered to be a region of constant thermal conductivity, K_h, and diffusivity, α_h.

iii. The total heat reaching the heat sink is stored in the mass of the heat-sink.

iv. For spherical shells of radius, $\rho(x,y,z) = \sqrt{x^2+y^2+z^2} > R$, the heat source appears as a point source, that is an infinitesimally small circular source.

The required heat conduction equation is:

$$\nabla^2 T_h[\rho(x,y,z),t] = \frac{1}{\alpha_h}.\frac{\partial T_h}{\partial t}[\rho(x,y,z),t] \quad (1)$$

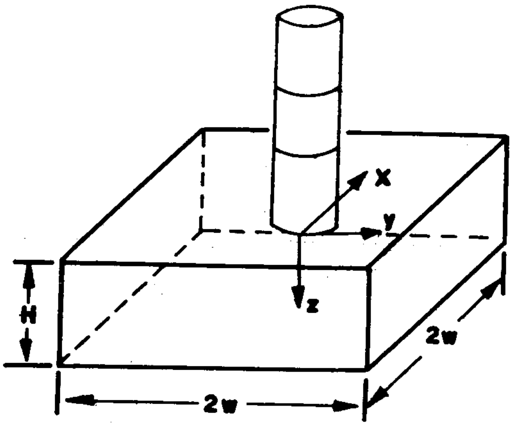

Fig.1. Basic configuration of a semiconductor device mounted on a finite heat sink.

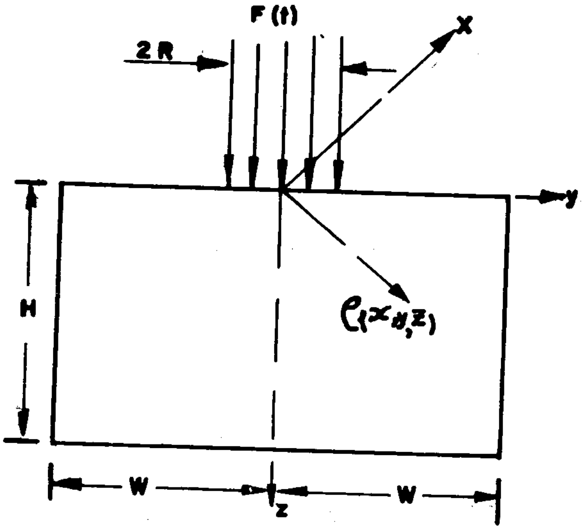

Fig.2. Side view of the heat source-heat sink geometry considered.

where

T_h is the heat-sink temperature above ambient at a point (x, y, z)
t = time

The boundary conditions may be mathematically written as follows:

At the heat source – heat-sink interface

$$\left.\frac{\partial T_h}{\partial t}\right|_{z=0} = \frac{-F_0}{K_h}, 0 < x^2 + y^2 < R^2$$
$$= 0, x^2 + y^2 > R^2$$
(2)

At the heat-sink edges:

$$\frac{\partial T_h}{\partial z} = 0, \quad z = H, -W < y < W;)$$
$$-W < x < W)$$
$$\frac{\partial T_h}{\partial x} = 0,\ x = \pm W, \quad 0 < z < W;) - (3)$$
$$W - < y < W)$$
$$\frac{\partial T_h}{\partial t} = 0, \quad t = \pm W, 0 < z < W;)$$
$$W - < x < W)$$

while the initial condition is:

$T_h[\rho(w, y, z), 0] = 0$ (4)

The solution of equation (1) for the case of a semi-infinite heat-sink is well known, and is given by Carslaw and Jaeger [4]. The axial temperature distribution is:

$$T_h(z,t) = \frac{2F_0\sqrt{\alpha_h t}}{K_h}\left[ierfc\left(\frac{z}{2\sqrt{\alpha_h t}}\right) - ierfc\left(\sqrt{\frac{z^2+R^2}{4\alpha_h t}}\right)\right. \quad (5)$$

For distances away from the heat source large compared to R, the isotherms in a semi-infinite sink are spherical, Therefore z→p(x,y,z), Thus because of assumptions (iii) and (iv), we can replace Z in equation (5) by ρ(x,y,z) without any loss of generality.

Hence

$$T_h(\rho(x,y,z),t) = \frac{2F_0\sqrt{\alpha_h t}}{K_h}\left[ierfc\left(\frac{\rho(x,y,z)}{2\sqrt{\alpha_h t}}\right) - ierfc\left(\sqrt{\frac{\rho^2(x,y,z)+R^2}{4\alpha_h t}}\right)\right. \quad (6)$$

The boundary conditions remain to be satisfied for the chosen model. By using the method of images [4], it can be easily shown that, a solution to equation (1) which also satisfies the given boundary conditions, is the summation of temperature contributions of form (6) due to an infinite number of sources at (±2X, ±2Z), plus the temperature contribution due to a single source at the centre, (0,0,0), of the heat-sink, where:

$X = iW;\ Y = jW;\ Z = kH$ and $\rho(x,y,z) = 2\sqrt{i^2W^2 + j^2W^2 + k^2H^2}$ (7)

Thus the required temperature distribution

$$T_h(\rho(x,y,z),t) = \frac{2DF_o\sqrt{\alpha_h t}}{\pi R^2 K_h}\left[T_{000} + \sum_{\infty}^{\infty}\sum_{\infty}^{\infty}\sum_{\infty}^{\infty} T_{ijk}\right] \quad (8)$$

Where

$$T_{000} = \frac{1}{\sqrt{\pi}} ierfc\left(\frac{R}{2\sqrt{\alpha_h t}}\right) \quad (9)$$

$$T_{ijk} = ierfc\left(\sqrt{\frac{\rho^2(x,y,z)}{4\alpha_h t}}\right) - ierfc\left(\sqrt{\frac{\rho^2(x,y,z)+R^2}{4\alpha_h t}}\right) \quad (10)$$

$D.P_0$ = total average power dissipated for a pulsed source of duty cycle D.
$= \pi R^2 F_o$ (11)

Equation (8) gives the required heat-sink time average temperature distribution. For computational purposes, it is convenient to expand this equati0n as:

$$T_h(\rho(x,y,z),t) = \frac{2DP_O\sqrt{\alpha_h t}}{\pi R^2 K_h}\left[T_{000} + 2\sum_{i=1}^{\infty} T_{i00} + 2\sum_{j=1}^{\infty} T_{0j0} + 2\sum_{k=1}^{\infty} T_{00k} + 4\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} T_{ij0} + 4\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} T_{i0k} + 4\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} T_{0jk} + 8\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} T_{ijk}\right] \quad (12)$$

3.0 NUMERICAL RESULTS

3.1 Choice of Heat Sink Material

It is evident from equations (8) and (10) that, in order to minimize the heat sink temperature rise, both the material diffusivity α_h and the parametric ratio $\sqrt{\alpha_h}/K_h$ should be made as small as possible.

This would require heat sink materials with high thermal conductivity and low material diffusivity. Table 1 shows the thermal conductivity and diffusivity [5] of some of the most commonly used heat sink materials together with their respective parametric ratios. Copper is seen from this table to be the optimum material to minimize the temperature rise of the heat sink. That copper is the superior heat sink material is shown by observing the computed temperature rise at the end of 10 seconds operation for a pulsed source of 1 percent duty obtained for each heat sink material for a constant heat sink volume of $0.5cm^3$. Copper is seen to have the minimum temperature rise, and hence in the following calculations, the heat sink material used will be copper. Omitted from Table 1 is Type IIa Diamond (α_h = 1.56, K_h = 9.0, $\sqrt{\alpha_h}/K_h$ = 0.139) which is currently under intensive study as a heat sink material. Under the above conditions, the temperature rise of Type-IIa Diamond would be only 63^0C (approximately). Thus Type-IIa Diamond heat sink would provide a factor of approximately 2 lower thermal resistance than copper. However, its processing technology is not yet fully established, and its manufacturing cost is much higher than that of copper.

TABLE 1: A Comparison of Typical Heat Sink Materials.

Material	Diffusivity α_h (cm^2/sec)	Conductivity K_h W/cm °C)	Parametric Ratio $\sqrt{\alpha_h}/K_h$	Temperature Rise °c t = 10secs H = W =0.5cm; D = 1%
Cu	1.140	3.94	0.270	107.78
Ag	1. 700	4.08	0.319	149.80
Diamond	3.700	6.60	0.291	193.60
Au	1.220	2.96	0.370	151.70
Si	0.472	0.84	0.818	235.90

3.2 Temperature Profiles

Figure 3 shows typical computed temperature profiles as a function of the operating time. Copper heat sink with a fixed height to half-width (H/W) ratio of one-half is assumed. The heat source has a diameter of 500 microns and is pulsed at one percent duty. The solution for the case of a semi-infinite copper heat sink is also plotted for comparison. It is evident from Figure 3 that approximation of the finite heat sink requirement by a semi-infinite model would grossly underestimate the actual temperature rise.

It is often necessary in practical heat sink structure design to be able to predict the heat sink dimensions to yield a predetermined temperature for a given constant power density at the end of specified system operating time. Figure 4 shows the design curves for constant temperature surfaces at the end of a typical system operating time considered and for a heat source power density of 1. 52mW/(micron)2 pulsed at one percent duty. It is seen from the distribution that for a specified time and power density, a trade-off between the heat-sink height and width is needed to maintain a specified operating temperature.

4.0. CONCLUSIONS

An analytical time solution of the temperature rise of a finite heat sink under adiabatic boundary conditions has been proposed for specialized pulsed radar and certain communication systems applications requiring short-term and low duty operation of pulsed high-power, high -efficiency semi-conductor devices. The temperature rise is found to be critically dependent on the heat sink dimensions and material. It is shown that the approximation of the finite heat sink model by a semi-infinite one would grossly underestimate the actual temperature rise. Practical design curves are presented. The present analytical model offers a significant and powerful tool for the design and miniaturization of heat sink structures for the specified system applications. It is believed that the analysis described is also equally applicable to several other solid state device heat sinks under similar pulsed conditions.

The analytical approach discussed in the present paper has obvious advantages over the corresponding numerical approach namely: it is exact, can produce generalized results and is relatively easy to evaluate.

The total junction temperature of a given semiconductor device and heat sink assembly is the summation of the temperature contribution of the heat sink as presented in this work, and the temperature contribution of the bulk of the semi- conductor diode. This total junction temperature is currently under investigation.

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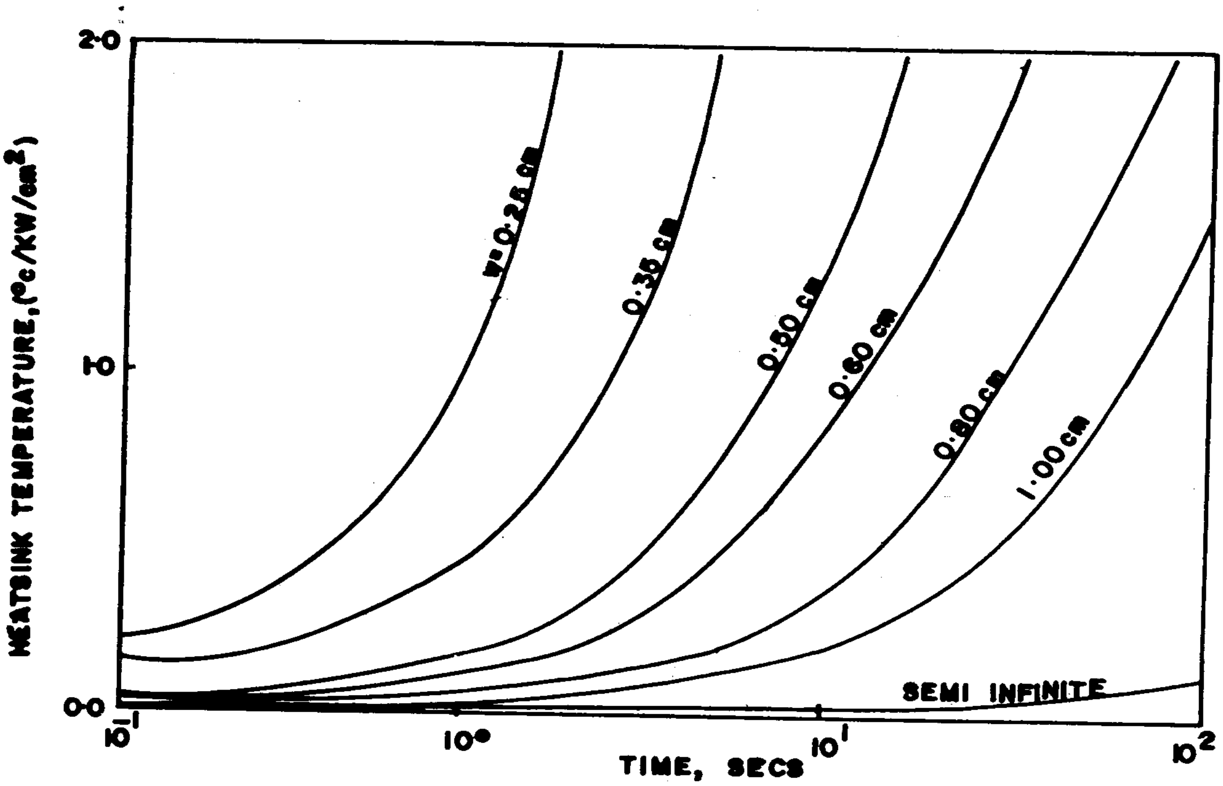

Fig.3. Heat sink temperature rise Vs time (H/W = 0.5, D = 1%)

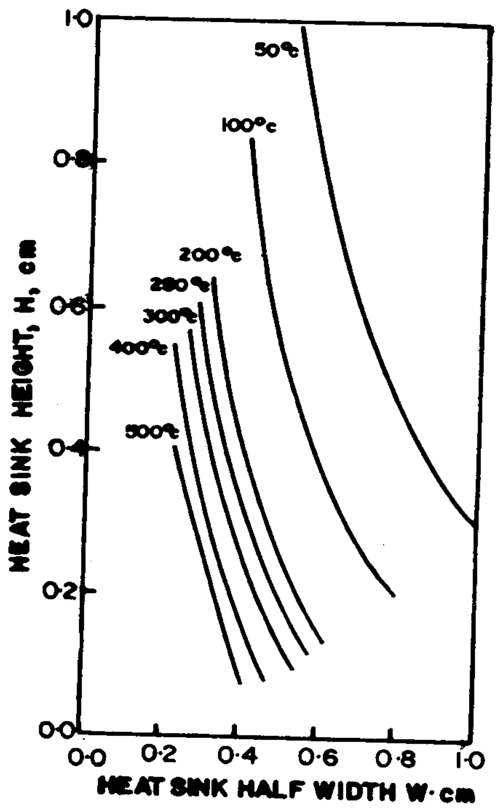

Fig.4. Constant temperature surfaces (F_o= 1.528 m W/(Micron)^2,
t - 10 secs.).