EXPERIMENTAL VISCOELASTIC CHARACTERIZATION OF CORN COB COMPOSITES UNDER RADIAL COMPRESSION

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ABSTRACT
The nature of viscoelasticity in biomaterials and the techniques for characterizing their rheological properties were reviewed. Relaxation tests were performed with cylindrical samples of corn cob composites which were initially subjected to radial compression. It was found that a Maxwell model composed of two simple elements is sufficient to represent the viscoelastic behavior of the corn cob. The effect of moisture content and rates of loading on the mechanical model determined were investigated.

1. INTRODUCTION

The subject of viscoelasticity deals with the mechanical response of those materials which have both elastic and viscuous properties. In general, an elastic material can be considered as one which has the capacity to store energy under the application of a load so that upon removal of the load the material returns to its initial (under- formed) state. This is the case of a completely reversible process. On the other hand, a purely viscous material is one which dissipates energy under the application of a load thereby giving rise to a steady-state flow. This is the case of an irreversible process characterized by the fact that the material does not return to its initial state upon the removal of the load. Since viscoelastic materials combine the properties of elastic and viscuous materials they exhibit the capacity to store and dissipate energy.

The dissipation of energy in viscoelastic materials is characterized in terms of two distinct phenomena, creep and relaxation. Creep is associated with the gradual increase of deformation or strain beyond the elastic yield point under the application of a constant load. Relaxation is associated with the gradual decrease of load so that the deformation or strain may be kept constant as the time progresses.

The behaviour of viscoelastic materials depends on stress, time, and temperature histories and thus they are said to posses memories. There are two broad types of viscoelastic behaviours. A material behaves under load in a linear viscoelastic manner if, under isothermal conditions, the stress is linearly proportional to strain but only dependent on time. If stress is non-linearly proportional to strain and if such a relationship is also dependent on both time and stress magnitude, then the material is said to behave in a non- linear viscoelastic manner.

The elastic properties of biological materials as illustrated with those of the corn cob under radial ompression were discussed in the first two articles of the series on force-deformation analysis of the behaviours of radically compressed biomaterials by the author[1,2]. The objective of this third article is to describe the force relaxation behaviour of corn cobs under quasi static radial compression, determine a viscoelastic or rheological model for the tested corn cob samples, and investigate the effects of moisture content and rate of loading on the viscoelastic constants that characterize the rheological behaviour of corn cobs. The effects of these factors, as well as those due to changes in corn variety and cultural practice, on the elastic properties of corn cobs were already investigated[3-5].

The practical importance and applications of rheological studies of agricultural products have been discussed by several workers[6–9]. A knowledge of the mechanical behaviour and properties of the corn cob will be of valuble assistance in the design and performance of the corn combine, particularly its cylinder-concave shelling device.

2. LITERATURE REVIEW

2.1 Simple One-Dimensional Viscoelastic Models.

For the phenomenological description of the material behaviour under small stresses simple mechanical models are often employed, which consist of parallel or series arrangement of springs and dashpots[10] The springs are analogous to the Hookean elastic element while the dashpots represent the Newtonian viscous element. These two basic elements are then combined either in series or in parallel to give simple models known as the Maxwell and Kelvin elements, respectively. Some of the more commonly used models are illustrated in Figure 1, together with the corresponding stress-strain relations.

Viscoelastic, rheological or mechanical models(as they are often variously called) are not meant to be exact structural models of the biological material being studied. Rather they should be visualized in this way: if the spring and the dashpot symbolized storage of potential energy and dissipation of energy respectively on the molecular scale, the mass stands for storage and exchange of kinetic energy on that scale. As any other model, mechanical models give at most part of the picture. They lead to some understanding of after-effects by analogy, but they do not explain much[10]

There is no relation between the structure of a mechanical model and the molecular structure of the material. This is evident from the simple facts that a multitude of different mechanical models can be constructed which have the same relaxation and retardation functions[10]•

2.2 Characterization of Viscoelastic Behaviour of Materials.

In general, the properties of viscoelastic materials are an exceedingly complex combination of permanent deformation, reversible flow, and instantaneous elasticity. Furthermore, all of these components are modified to an extent by previous stress history. Therefore, it is usually difficult to establish an absolute behaviour constants and even the best analytic techniques are usually only comparative in nature.

Figure 2 gives a summary of the various techniques employed by several researchers [11-19] in characterizing the behaviour of viscoelastic materials. The techniques are classified under three broad groups, namely, analytic, semi- analytic and empirical approaches. In each case, a solution of the viscoelastic problem requires an assumption of a viscoelastic model. The choice of which approach and technique to use depends on the simplicity of the body geometry of the material to be tested, its mechanical behaviour under load and on the loading conditions.

The method of successive residuals was used in the present study. This is a graphical method based on the empirical approach to the ·characterization of the viscoelastic equation. The steps involved in this technique are as follows:

(i) Plot the logarithm of stress (or force) against time, using experimental data obtained from the relaxation test,
(ii) The inverse of the slope of the straight portion of the curve gives the first relaxation time constant, T_I
(iii) the intercept of the straight line gives the first exponential coefficient, C_1,
(iv) obtain the first residual curve by plotting on the same semi-log paper the difference between the straight line and the original curve;
(v) treat the slope and intercept of the straight portion of the first residual curve as for the original curve;
(vi) obtain the second residual curve and treat as for the previous curves;

(vii) continue until a residual curve approximates to a straight line;

(viii) write down the viscoelastic equation using the assumed model and visoelastic constants determined from steps (i) to(iii).

In the case of the generalized Maxwell model, the stress relaxation equation may be written as

$\sigma(t) = c_1 e^{-t/T_1} + c_2 e^{-t/T_2} + ... + c_n e^{-t/T_n} + E_e/\varepsilon_o$

Where

σ = stress,

ε_o = instantaneous strain applied at time zero.

t = time.

n = number of elements in the mechanical model

E_e = equilibrium modulus or modulus after infinite time.

$T_1, T_2, ... T_n$ = relaxing times

$C_1, C_2, ... C_n$ = coefficients of the exponential terms

= E_{d1}/ε_0, E_{d2}/ε_0, ... E_{dn}/ε_0, and $E_{d1}. E_{d2}, ... E_{dn}$ = decay moduli.

The relaxation time of the ith element (where i=1,2,. . . .,n) is calculated from the straight line segment as follows:

$$T_n \frac{t_b - t_a}{\ln(\sigma_a) - \ln(\sigma_b)} = \frac{t_b - t_a}{2.3(\log(\sigma_a) - \log(\sigma_b))} \quad (2)$$

where a and b are any conveniently chosen points on the straight line segment of the original curve or a residual curve corresponding to the ith element.

2.3 Viscoelastic Nature of Plant Tissues

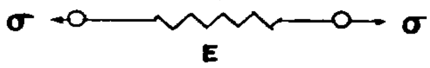
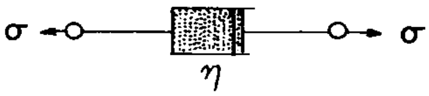
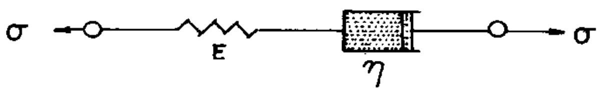
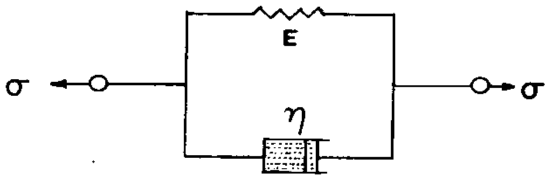
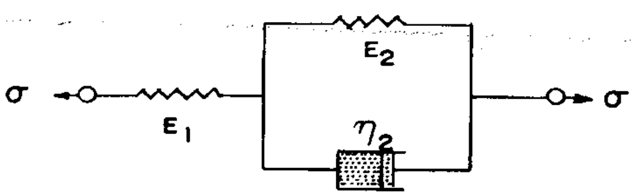
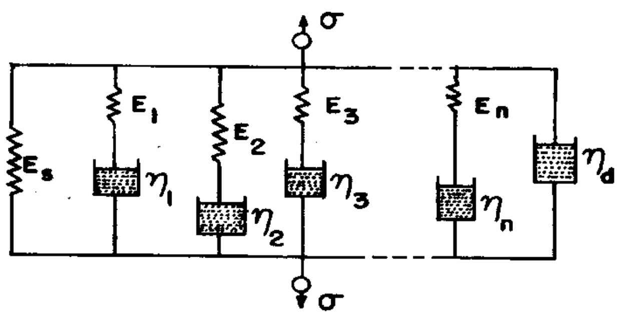
From microscopic considerations of the structures of plant tissues[20],[21], it is generally agreed that the mechanical properties of cell walls in plant materials reflect the mechanical properties of the plant tissues. The cell walls are composed of cellulose molecules which exhibit a high degree of elasticity and flexibility. In addition to the elasticity of the cell walls, the cytoplasm within the cell appears to show both elastic and viscous properties[21]. The cells of plant tissues also contain protoplasm which is composed chiefly of water. Thus. physiological active protoplasm may be expected to exhibit a predominant vacuous effect in response to applied load. Due to high sensitivity to swelling with water by pectin's hemicelluloses and amorphous cellulose, all of which also constitute the plant cells, the elasticity and plasticity of plant tissues depend markedly on moisture content.

Morrow and Mohsenin (1966)[22] used the experimental or empirical approach to lay the foundation of viscoelastic characterization of agricultural products. The elastic solutions used in relating stress and strain were based on the classical contact stress theories of Hertz and boussical[23]. Since then, several workers [11, 14,24-30], have determined viscoelastic models for many agricultural products, some of which are summarized in Table 1. Stress relaxation tests and Maxwell models were more frequently employed in previous studies. This practice was adopted in the present study.

3. EXPERIMENTAL WORK

Cylindrical samples of corn cob composites, 30mm in length, were prepared as described elsewhere[1,4,.5] The radial compression device used in this study was the Instron universal testing machine as previously described by the author[1].

The two important factors in any relaxation test are the initial deformation applied on the specimen and the duration of the relaxation test for each sample tested. The initial deformation must be less than the deformation at failure of the tested specimen but high enough to impose considerable strain on the specimen. An initial total deformation of about 5.2mm was applied in this study which from previous tests with corn cobs was about 65 percent of the total deformation at pith cracking for the same variety.

MODEL	REPRESENTATION	STRESS-STRAIN RELATION
SPRING		$\sigma = E\ \epsilon$
DASHPOT		$\sigma = \eta \mathring{\epsilon}$
MAXWELL UNIT		$\sigma \left[\frac{1}{E} + \frac{t}{\eta}\right] = E$
KELVIN UNIT		$\sigma = E\epsilon + \eta\dot{\epsilon}$
STANDARD LINEAR SOLID		$\sigma + \left[\frac{\eta_2}{E_1 + E_2}\right]\dot{\sigma} = \left[\frac{E_1 E_2}{E_1+E_2}\right]\epsilon + \left[\frac{\eta_2 E_1}{E_1+E_2}\right]\dot{\epsilon}$
GENERALIZED MAXWELL		For $\epsilon = \epsilon_o$ at $t \geq 0$; $\sigma = \epsilon_o\left[E_s + \eta_d \dot{\epsilon}_o + \sum_{i=1}^{n} E_i\ e^{-t/\tau_i}\right]$ where: $\tau_i = \eta_i / E_i$

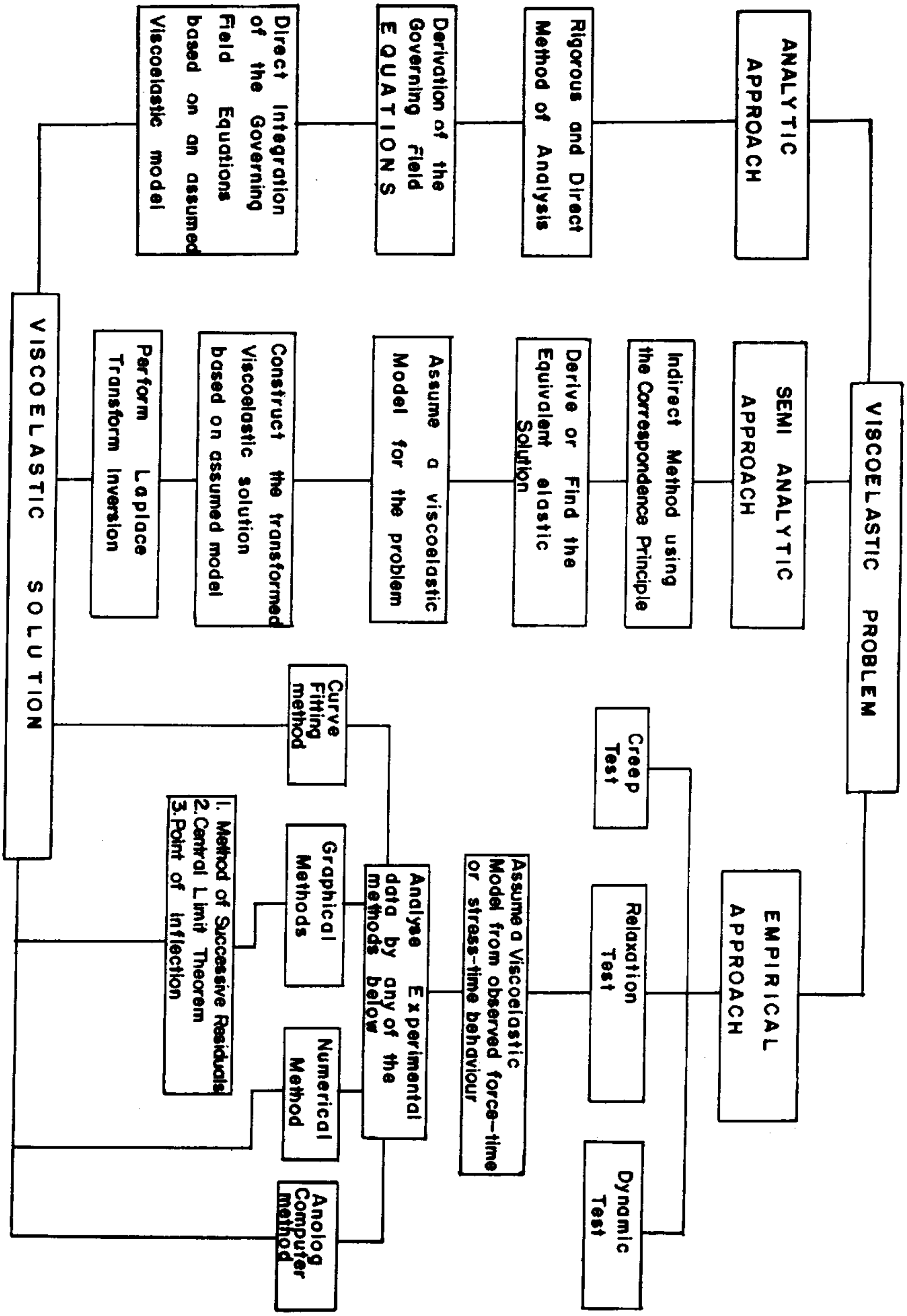

VISCOELASTIC PROBLEM
ANALYTIC APPROACH
Rigorous and Direct Method of Analysis
Derivation of the Governing Field EQUATIONS
Direct Integration of the Governing Field Equations based on an assumed Viscoelastic model
SEMI ANALYTIC APPROACH
Indirect Method using the Correspondence Principle
Derive or Find the Equivalent elastic Solution
Assume a viscoelastic Model for the problem
Construct the transformed Viscoelastic solution based on assumed model
Perform Laplace Transform Inversion
EMPIRICAL APPROACH
Creep Test
Relaxation Test
Dynamic Test
Assume a Viscoelastic Model from observed force–time or stress-time behaviour
Analyse Experimental data by any of the methods below
Curve Fitting method
Graphical Methods
1. Method of Successive Residuals
2. Central Limit Theorem
3. Point of inflection
Numerical Method
Analog Computer method
VISCOELASTIC SOLUTION

TABLE 1:viscoelastic models determined for some agricultural products

PRODUCT	VISCOELASTIC MODEL	TEST	REFERENCE
Pea beans	Two-element Maxwell Model	Relaxation	Zoerb and Hall (1960)[30]
Fruits	A Maxwell Model in series with a Kelvin Model	Creep	Mohsenin et al (1963)[26]
Potato tuber	Four-element Maxwell Model in Parallel i) One vacuous + two simple Maxwell elements in parallel for compression loading	Relaxation	Finney et al (1964)[25]
Wheat Plant stem	ii) Two simple Maxwell elements in parallel for Flexure- loading		Moustafa (1967)[27]
Cotton Seed	Kelvin Model	Relaxation and Creep	Clark et al (1968)[14]
Pear	Six-element Maxwell Model	Relaxation	Chen and Fridley (1972)[11]
Soybean	Three-Maxwell Elements And one spring in parallel	Relaxation	Saxena (1972)[28]
Apple flesh	Generalized Maxwell Model	Relaxation	Baerdemaeker and Segerlind (1976)[24]
Apple flesh	Three-element Maxwell Model	Modulus Master Curve from Force-Defor-mation Curve	Silberstein and Rao (1977)[29]

Of corn at comparable ranges of moisture content. The duration of the relaxation test for each specimen could be as long as possible since the theoretical time limit is infinity. In the present study, the duration of each relaxation test was about ten minutes.

The tests were performed at three different (initial) rates of loading and two levels of moisture content. The variety of corn used in this study was Coop 265.

Loading and unloading tests were performed with representative samples of corn cob composites to determine the degree of elasticity. This was needed in the computation of the initial elastic strain, E , as well as the initial contact surfa8e area, 2bl, where 2b is the width of the area of contact of the radically compressed cob composite and 1 is its length. Loading and unloading tests were performed at rates of loading corresponding to those for the initial radial compression of the corn cobs during the relaxation tests.

4. DATA ANALYSIS AND RESULTS

Figure 3 shows the force-relaxation and the loading and unloading curves of cylindrical samples of corn cob composites as directly traced from the Instron $^{x-y}$ plots. Figure 3(a) and others similar to it were reported as, shown in Figure 4 by condensing the time scales and converting the force recorded in kilogram-forceto

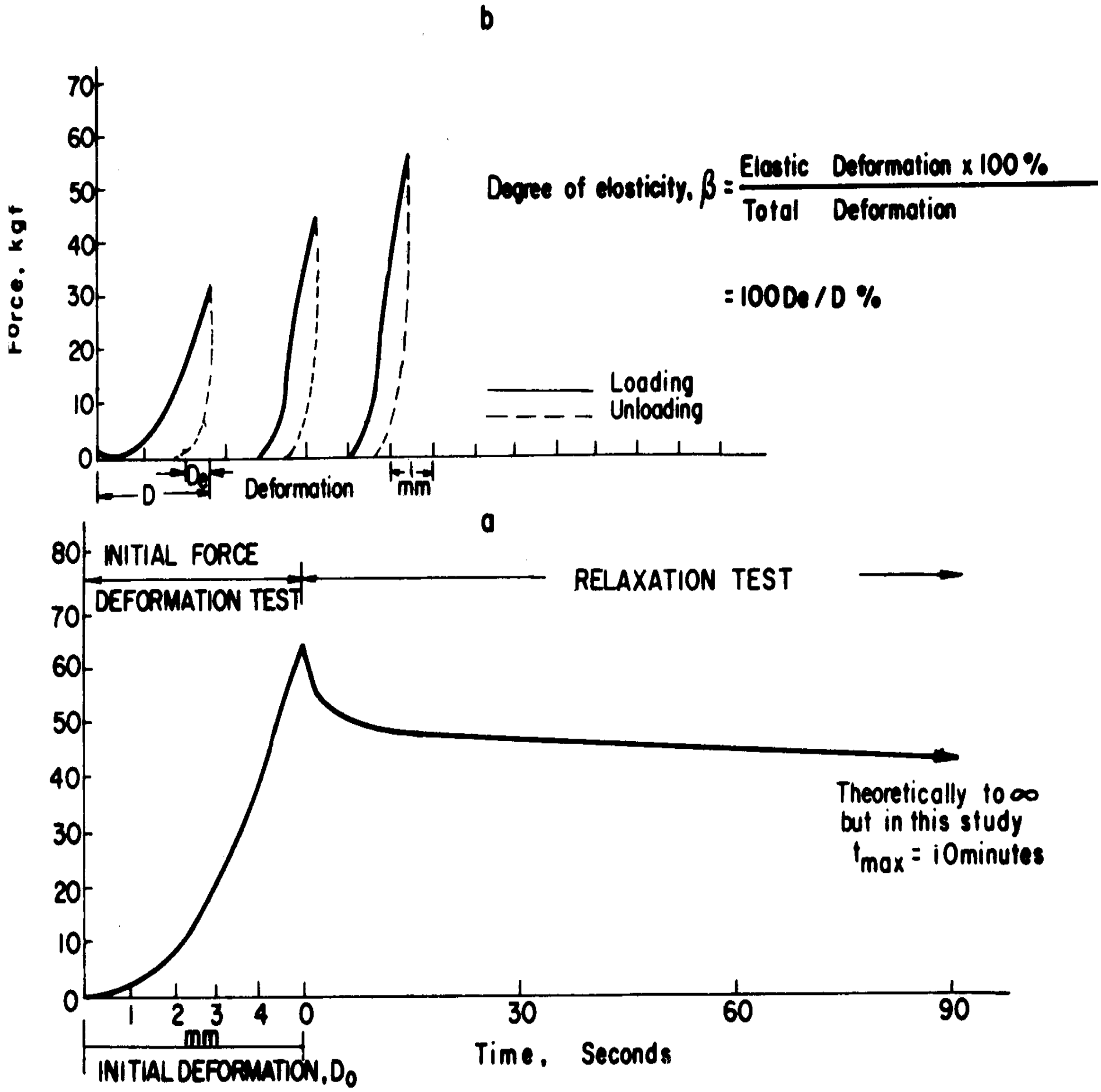


Figure 3.Force-relaxation (a) and unloading (b) curves of corn cob sanole as traced from the chart-recarded under instron radial compression test.

to Newtons.

The converted force in Newtons was then transformed into stresses by using the approximate elastic equation.

$\sigma_{xy} = F/2b\vartheta = F/(2Rz)\,\lambda$

where

F = applied force or residual force

σ_{xy} = contact stress, assumed to be uniformly distributed at the two surfaces of contact of the de-formed cob with the two parallel loading steel plates;

2b = contact width, assumed to be fairly constant during the relaxation test;

R = radius of the cylindrical sample of the corn cob.

The term Z, as presented in greater details by the author elsewhere[1], was defined as equal to R/b and determined from the expression:

$D_e/2R = (1/(2Z^2))(\ln(2Z) + 1/2)$ (4)

Where D_e = elastic deformation.

The computed stresses were plotted against time in a semi-log. Paper and the analysis of the stress relaxation performed graphically using the method of successive residuals as previously outlined under the literature review. The result of this analysis is shown in Figure 5 and Table 2. It was found that a two- element Maxwell model represented the viscoelastic behaviour of the corn cob composite at the three rates of loading and the two moisture content levels tested. Both the first and second coefficients of the exponential terms C_1 and C_2, increased in magnitude for the dry cob samples compared with the wet cob samples. C_1 and C_2 also increased as the rate of loading, during the initial radial compression, was increased for the dry cob samples. While the relaxation time of the first element, T_1 , appeared to be higher for the dry cob samples; T_2 remained essentially of the same order of magnitude for the three rates of loading and two levels of moisture content tested.

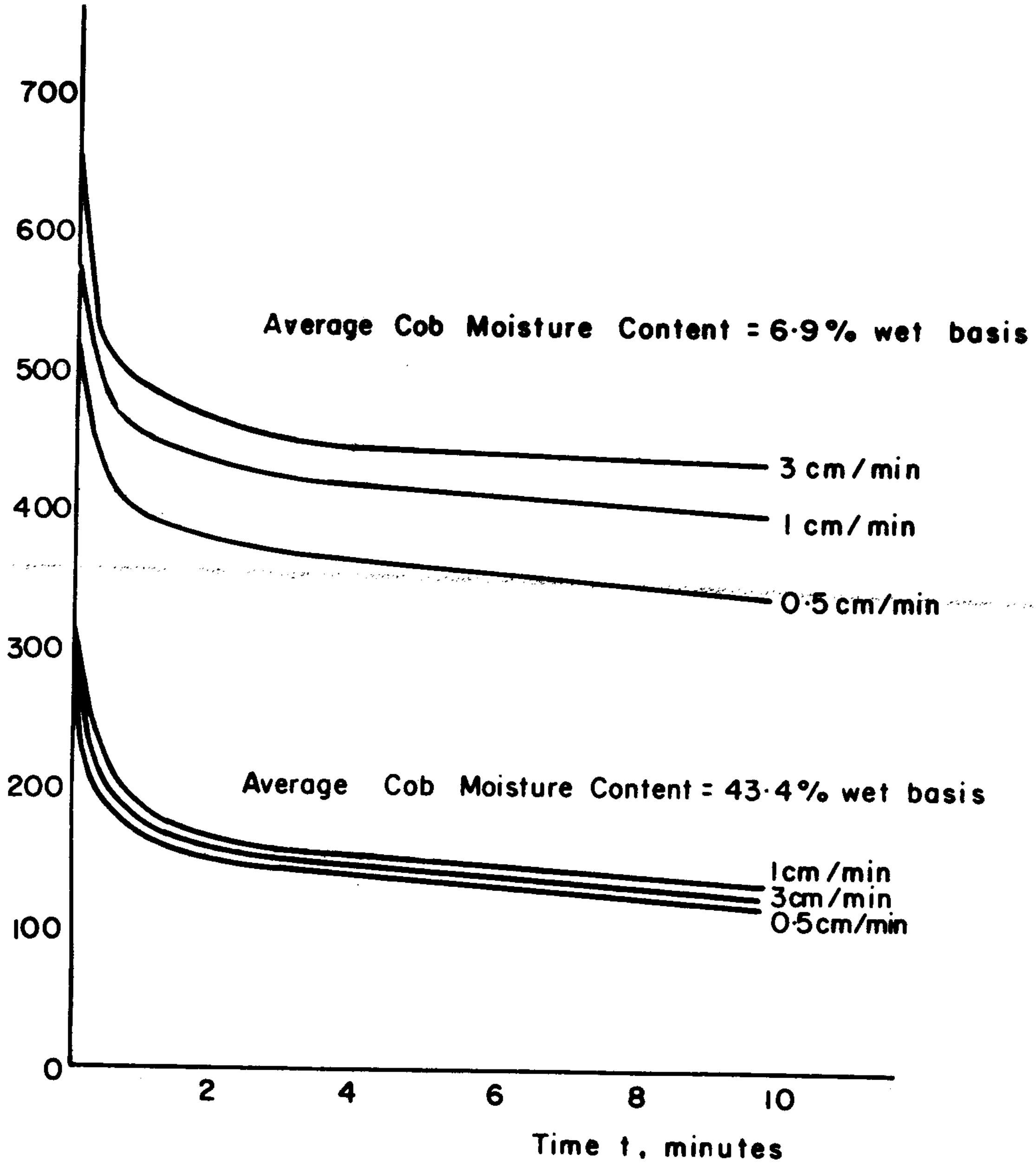


Figure 4. Force - relaxation behaviour of corn cob composite at three rates of loading and two moisture content levels

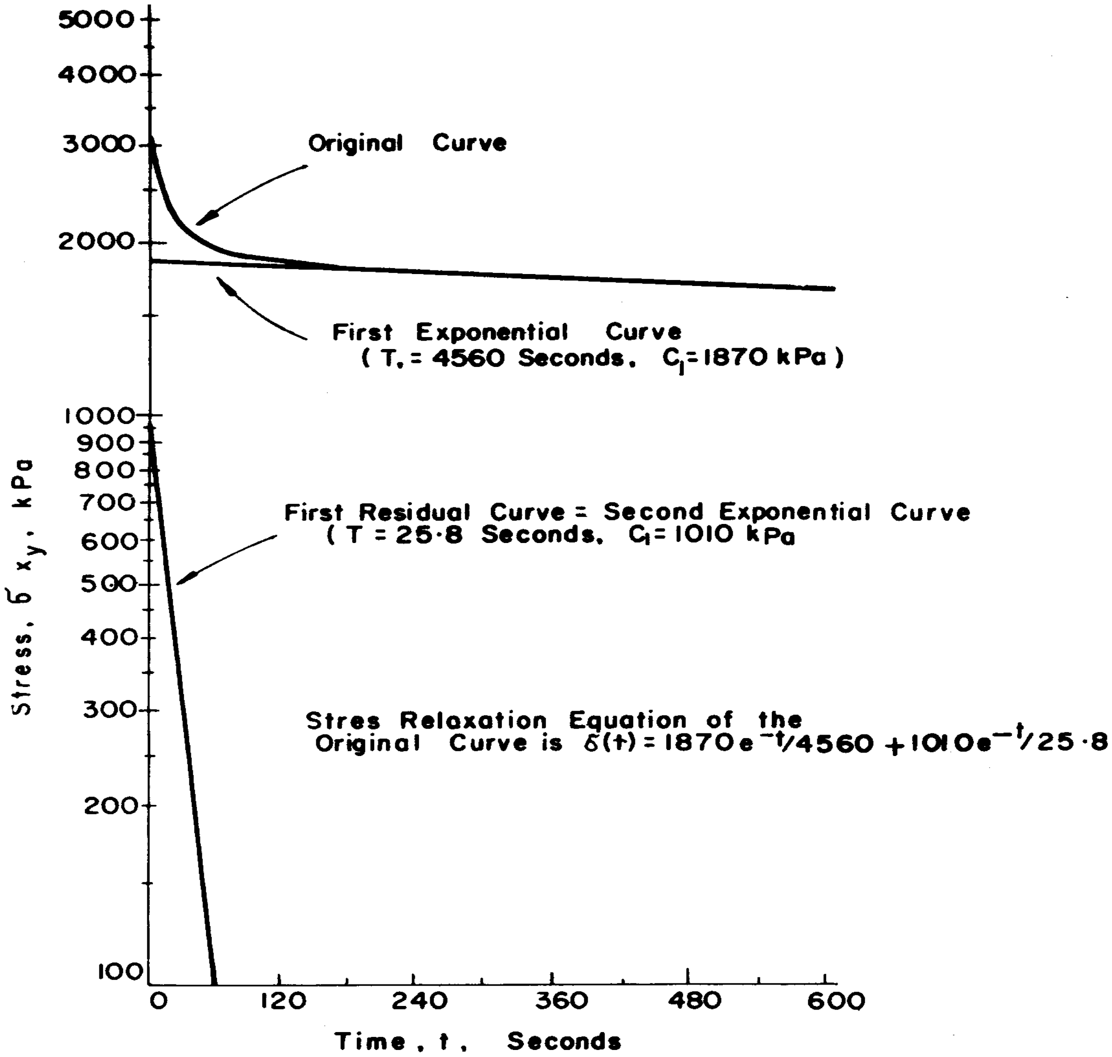


_Figure 5. graphical analysis of strss-relaxation for rthe corn cob composite under radial compression.

From the result presented in Table 2, the relaxation modulus of the corn cob composite subjected to an initial radial compression can be computed from the equation:

$E(t) = \sigma(t)/\varepsilon_0$

Where $\sigma(t)$ = stress relaxation as give in table 2,

$\varepsilon_0 = D_e/2R = \beta D_0/(2R(2R(100))$

D_0 = initial deformation (Fig.3)

β = degree of elasticity (Fig. 3).

The units of both $\sigma(t)$ and $E(t)$ are in kilo-Pascals (kPa) and the time, t, is in seconds. The actual computation of $E(t)$ was not undertaken in the present study.

5. CONCLUSIONS

The viscoelastic behaviour of the corn cob composite can be represented

TABLE 2: SUMMARY OF STRESS RELAXATION EQUATIONS DETERMINED

Average Cob Moisture content % wet basis	Loading Rate cm/min	Initial Strain	Stress		Relaxation		Equations
41.4	0.5	0.0951	$\sigma(t)$	=	$720\ e^{-t}/2088$		$710_e^{-t}/25.8$

					+		
44.1	1.0	0.1172	σ(t)	=	$640\ e^{-t/3428}$	+	$870\ e^{-t/26.5}$
44.7	3.0	0.1094	σ(t)	=	$740\ e^{-t/1714}$	+	$780\ e^{-t/19.3}$
6.9	0.5	0.909	σ(t)	=	$1870\ e^{-t/4565}$	+	$1010\ e^{-t/25.8}$
6.8	1.0	0.0968	σ(t)	=	$2100\ e^{-t/5000}$	+	$1120\ e^{-t/24.8}$
7.0	3.0	0.0891	σ(t)	=	$2340\ e^{-t/4800}$	+	$1180\ e^{-t/27.1}$
∴ A general stress relaxation equation for the corn cob is			σ(t)	=	$C_1 e^{-t/T_1}$	+	$C_2 e^{-t/T_2}$

by a Maxwell model composed of two simple elements in parallel. Dry cob samples have higher values of the decay modulus for each of the two elements in the mechanical model when compared to wet cob samples. Only the relaxation time of the first element seems to increase with decrease in cob moisture content.

The initial rates of loading, within the low range tested, do not appear to have pronounced effects on the magnitudes of the decay modulus and the relaxation time.

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