



APPLICATION OF CHRISTER'S INSPECTION MODEL FOR BUILDING MAINTENANCE

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Abstract

This work applies the Christer's inspection model in building maintenance. The model estimates the effect of inspection intervals on total maintenance costs, which include inspection, material and down time cost. An experimental collection of data designed to test tradesmen's ability to comprehend with what was needed to provide the required subjective information was conducted chiefly within the maintenance Department of Benue State Ministry of Works Makurdi, and Maintenance section of the Ministry of Environment. Both organizations operated a contingency system. Information was collected by means of forms and 100 were completed for trades including painter, plumber, electrician, slaterer, plasterer and others. Tradesmen were requested to select a lapse time from within a discrete set and not on a continuum, as supposed so far. The cost of repairs was measured in terms of time and material which tradesmen felt able to readily state or estimate. Numerical results show that the optimal maintenance policy for the housing estate is inspection policy, as opposed to the contingency policy which is in use. It was also observed that the inspection frequency of six (6) months is the best for the housing estate as it gave cost savings of three million one hundred and twenty thousand naira (₦3,120,000.00) in a period of 5 years.

Keywords: Christer inspection model, maintenance, building, housing estate, cost

1. Introduction

The two most important concepts in this paper are model and maintenance and are therefore defined thus; a model is an abstraction of reality or a representation of a real object or situation. In other words, a model presents a simplified version of something. Secondly, maintenance is defined as work undertaken in order to keep, restore or improve every facility, i.e. every part of the building, its services and surroundings to a currently acceptable standard and to sustain the utility and value of the facility. BS 3811 defines maintenance as a combination of any actions carried out to retain an item in, or restore it to an acceptable condition [1].

It is most people's dream to have a safe and comfortable home. Such an aspiration stands in stark contrast with the strong reluctance of many building owners to take proper care of their buildings. For a lot of people, this is their biggest life-time investment and it is a pity to see some owners allowing their most valuable asset to fall into disrepair. The problem we face of building maintenance neglect is a result of a prevailing

weak building care culture and lack of appreciation of the threat poorly maintained buildings pose to public safety [2].

2. Literature Review

In the domain of construction infrastructure, the models for rationalizing building maintenance, developed by Christer and Redmond [3] were used at the strategic level for the planning of maintenance for buildings owned by a building society. The value of the model as a management instrument in estimating and allocating maintenance budgets was demonstrated in a pilot case of four (4) building elements, namely: masonry, roofing, window frames and painting. Modelling the deterioration of these components permitted determining the maintenance policy that ensured a specified average quality level at minimal cost.

Building owners are increasingly faced with having to maintain their buildings assets more efficiently whilst reducing the short and long-term cost of main-

tenance and rehabilitation. Over the years, building organizations have relied solely on contingency maintenance method, while carrying out their services [4]. The method, however, in the recent times, has received concerns as it concerns its efficiency and cost effectiveness. Researchers have continued to explore better ways of maintenance services in the maintenance industries. Over the last few decades, numerous papers have appeared in the literatures which deal with the problem of finding optimal inspection policies for systems which are subject to failures. This phenomenon is indicated in various surveys of maintenance models [5 – 12]. The models address various aspects of inspection problems in such systems as an industrial production plant, a vehicle fleet, a housing estate, or a motor way system. The complexity of the models varies from a very simple deterministic model of a single-unit system to a very complex model of a stochastically failing multi-unit system. In general, the basic type of decision problem involved in an inspection system concentrates on determining the inspection schedules which minimizes the cost per unit time or tile downtime per unit time [13].

It was observed that a maintenance strategy (or concept or policy) describes what events (e.g. failure or passage of time) trigger what type of maintenance (e.g. inspection or repair/replacement) [14]. The inspection policy considered in this setting involves regular inspections dictated by the passage of time. The specified inspection interval for this policy represents the fixed amount of time between two inspections.

Building inspection is one of the key components of building maintenance [13]. The primary purpose of performing a building inspection is to evaluate the buildings condition. Without inspection, it is difficult to determine a built asset's current condition, so failure to inspect can contribute to the asset's future failure. Traditionally, a longhand survey description has been widely used for property condition reports. Surveys that employ ratings instead of descriptions are gaining wide acceptance in the industry because they cater to the need for numerical analysis output. The inspection is a key means of identifying a building's defects. Defects usually display their symptoms before getting worse and causing building failure. It is therefore crucial for building inspections to be performed many times in an asset's life cycle.

A step-by-step process for successfully building a useful model was proposed thus [15]:

1. Define the problem, decision, situation, or scenario and the factors that influence it.
2. Select criteria to guide the decision, and establish objectives. A perfect example of this is the use of heuristics in assembly-line balancing to guide the decision and the criteria of maximizing efficiency/minimizing idle time as an objective.

3. Formulate a model that helps management to understand the relationships between the influential factors and the objectives the firm is trying to achieve.
4. Collect relevant data while trying to avoid the incorporation of superfluous information into the model.
5. Identify and evaluate alternatives. Once again, the example of assembly-line balancing is appropriate. The user can manipulate the model by changing the heuristics and comparing the final results, ultimately finding an optimal solution through trial-and-error. However, the production of alternatives may not be necessary if the model in use initially finds an optimal solution.
6. Select the best alternative
7. Implement the alternative or re-evaluate.

In the book, Operations Management [16], the following nine benefits of models were listed:

1. Models generally are easy to use and less expensive than dealing with the actual situation.
2. Models require users to organize and sometimes quantify information and, in the process, often indicate areas where additional information is needed.
3. Models provide a systematic approach to problem solving.
4. Models increase understanding of the problem.
5. Models enable managers to analyze "what if" questions.
6. Models require users to be very specific about objectives.
7. Models serve as a consistent tool for evaluation.
8. Models enable users to bring the power of mathematics to bear on a problem.
9. Models provide a standardized format for analyzing a problem.

Christer [7] proposed an inspection policy models for maintenance organisation, as opposed to the conventional contingency system. Christer, in his inspection models, suggested that for any specific repair model, the most important data required are

- (1) Date of repair, *d*.
- (2) Trade or trades involved,
- (3) Man-hours expended by trade and material used.
- (4) Cost of repair, *c*.
- (5) Brief description of job.

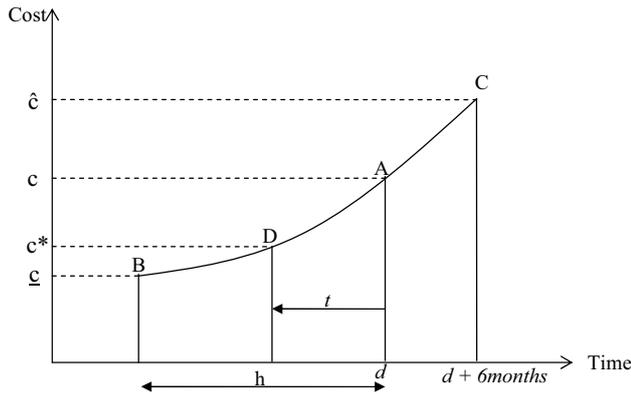


Figure 1: Cost of Repairs vs Time of Repairs.

He continued that, apart from the above five pieces of information, tradesman engaged in a particular repair were asked the following questions relating to that repair: (1) How long ago could the defect have reasonably been expected to be initially noticed in an inspection, this time is called the lapse time h . for the specific repair. (2) What would the likely repair in terms of time and material, have entailed had it been repaired at the initial stage, and what would it have cost, c ? (3) What would the likely repair be in terms of time and material if the defect was currently left unattended for a further period of, say six months, and the consequential cost c ?

Let the defect in question be reported at time d and then rectified at cost, c . This information is shown by point (A) of Figure 1, and this is the only certain point in the figure, which otherwise represents a notional variation of repair cost with time of repair. If the defect had been reported and repaired at an earlier time, $d - t$ say, the cost of repair would be estimated to have cost* say which is likely to be less than c . In short, provided there are no sudden developments of innovatory repair techniques or tooling, and expected inequality to hold in most instances. Points (B) and (C) which fixed by the tradesmens opinion along with the known point (D) are the only three points available for this curve.

2.1. Distribution of lapse time $H(h)$

For any particular organization and trade, data of the above type could be generated to enable the distribution $H(h)$ of an inspection scheme, since one expects a non-zero probability that a lapse time would be greater than the order of magnitude of the envisaged inspection period [3]. In the data collection experiment for example, about of the repairs had lapse times in excess of six months.

Considering now a defect which is reported under a contingency system at time d (Fig. 2). This defect is supposed to have first arisen at time $0 = (d - h)$. Here

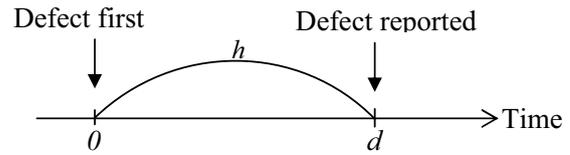


Figure 2: Building of the Christer's model.

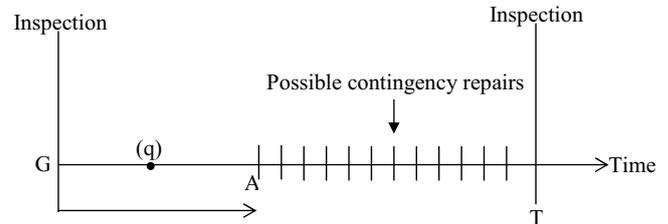


Figure 3: The expected number of contingency repairs arising in the course of the total period of length T .

we estimate the clustering effect of repairs due to an inspection system under the following assumptions:

- (1) The epoch 0 is independent to time.
- (2) h is independent of 0;
- (3) An inspection is perfect: and
- (4) Defects are reported at a constant rate of k per unit time.

Suppose an inspection takes place at regular intervals T units. Any defect with lapse time $h > T$ will always be identified upon inspection and never arise as a contingency repair. Consider now defects with lapse time $h < T$ will always be identified upon inspection and never arise as a contingency repair. Consider now defects with lapse time $h < T$. for defects with a lapse time in $(h, h + dh)$. No contingency repair can be reported in period GA. Although a defect can arise at any point, q say, in the inspection cycle GT. From Figure 3, we have therefore that $N(T)$, the expected number of contingency repairs arising during the course of the total period of length T is given by

$$N(T) = k \int_0^T (T - h)H(h)dh \quad (1)$$

The proportion of defects identified at an inspection which would otherwise be contingency repairs is

$$\frac{TK - N(T)}{TK} = 1 - \frac{1}{T} \int_0^T (T - h)H(h)dh \quad (2)$$

Upon switching from a contingency system to an inspection system for well established property, a surge of defects were found at the first inspection, after which the Equation (1) would apply. An estimate of the number M , say of repairs in this surge is given by.

$$M = \int_0^T khH(h)dh = k\bar{h} \quad (3)$$

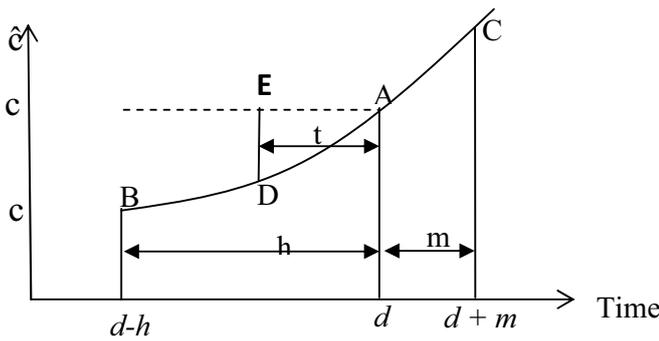


Figure 4: Average expected saving by repairing a defect a time t unit before the reported stage (Christer, 1982).

In Christer's model, his opinion was that the financial advantage $\beta(T)$ arising because of the clustering of $N(T)$ repairs at discrete points in time and location arises from an ease in organization and supervision coupled with a reduction in the time spent travelling between repairs. An order of magnitude calculation for $\beta(T)$ is given below:

Measured over normal working days of e hour (usually $e = 8$) let $u\%$ and $v\%$ denote respectively the proportion of time of tradesman spent in actually working on repair jobs and the proportion of time spent both in travelling between jobs and in general organization. For medium to large scale organization values of u and v around 40 and 60 are typical. If an $r\%$ reduction is made in v , the consequential increase in u is $(vr/u)\%$ which represents an amplification a most cases where $v > u$. For the present case, it will be assumed that r can be supplied in an estimate by the manager once he is familiar with $N(T)$.

Suppose each job takes on average a total of J hours or equivalently, that on average e/j jobs are completed per man per day under a contingency system. By clustering repairs each man would have available an extra $v/100.r/100.e$ hours per day, which represents a saving of $v.r.j/10,000$ hours per job. If e_1 is the hourly rate of payment for a tradesman, the saving to be expected from $N(T)$ jobs is of the order,

$$\beta(T) = \frac{N(T).e_1.v.r.J}{10,000} \tag{4}$$

To formulate $\gamma(T)$, an estimate of the expected saving to be made at an inspection epoch by the early rectification of defects which would otherwise arise as contingency repairs. Christer explained that such an estimate would, of course, depend upon indicators from the statistical analysis of the sample survey data such as, amongst other quantities, the distribution of costs c and its dependence upon h . Here he formulated the basic expected value model whilst accepting that the need could arise for a stochastic one.

Using a sufficiently large sample size, all defects

with a particular delay time could be analyzed to give the information depicted in Figure 4, which is an averaged form of Figure 1. As before, point A is known and points B and C are averages of subjective estimates. Let $S_h(t)$ denote the average expected saving to be made by repairing a defect t time units before the reported stage, as opposed to the reported stage, when the lapse time is h units. $S_h(t)$ corresponds to the distance ED of Figure 4. There are one or two immediate estimates of $S_h(t)$ available, depending upon whether or not a meaningful point C can be obtained from the analysis. The first estimate is based upon a linear representation of the curve B using points B and A, namely

$$S_h(t) = (c - c_0)0th. \tag{5}$$

And the second estimate is based upon a quadratic fit through point B, A and C and leads to

$$S_h(t) = \frac{(\hat{c} - c)ht(h - t) + (c - \underline{c})mt(t + m)}{mh(h + m)} \tag{6}$$

It is necessary that the survey sample size be sufficient enough to enable $S_h(t)$ to be obtained for ranges of h pertinent to the properly complex and trade or components being studied.

Considering all defects with a specific delay time $h < T$. Figure 3, and suppose an inspection takes place on a regular basis with period T . By inspecting at point G a saving of $S_h(q)$ will accrue on a defect which would otherwise have been reported at point qh . The total saving due to inspection where $h < T$ is

$$\int_0^T \int_0^H K S_h(q) H(h) dq dh \tag{7}$$

It has already been observed that no defect should arise as a contingency for the case $h > T$ but be observed at an inspection. An inspection at time G will observe all defects which would otherwise have been reported between times $h - T$ and h . We have, therefore, the saving to be made from defects with $h > T$ by operating an inspection system as,

$$\int_{h=T}^{\infty} \int_{q=h-T}^h K S_h(q) H(h) dq dh \tag{8}$$

Therefore, estimate of the expected savings is expressed as,

$$\gamma(T) = \int_0^T \int_0^H K S_h(q) H(h) dq dh + \int_{h=T}^{\infty} \int_{q=h-T}^h K S_h(q) H(h) dq dh \tag{9}$$

For $\alpha(T)$ the total expected savings function per unit time from both sources, namely clustering and early repair, Christer compounded the respective expected

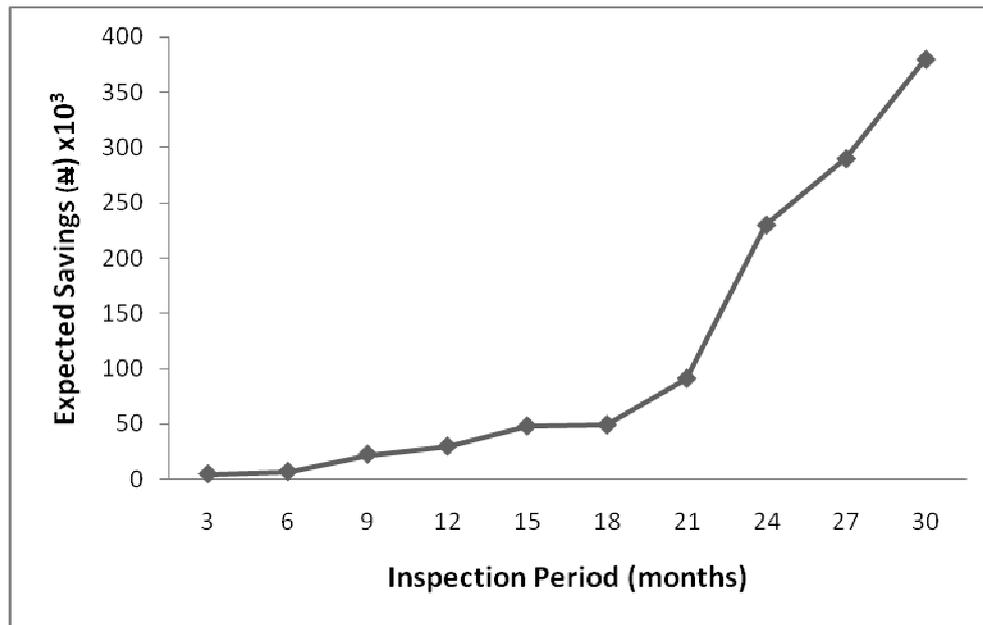


Figure 5: Expected Savings from Inspection frequency of 3 months.

savings at an inspection epoch, less the cost of inspecting I and divided by the time period T . Thus,

$$\alpha(T) = \frac{\beta(T) + \gamma(T) - I}{T} \quad (10)$$

This research is therefore important as it applies the inspection maintenance policy and estimates the savings that will result from the optimised model. The intent of the study provides some estimate of the maintenance optimization which includes the determination of the optimal system of maintenance and prioritization of inspection system of maintenance that satisfies: (i) minimization of maintenance and repairs costs; (ii) maximization of building performance.

This research seeks to obtain the optimal maintenance policy using the Benue State Housing Estate (Ankpa quarters) as a case study.

3. Materials and Methods

An experimental collection of data, designed to test tradesmen's ability to comprehend with what was needed to provide the required subjective information was conducted, chiefly within the maintenance Department of Benue State Ministry of works Makurdi, but also within the local maintenance section of the Ministry of Environment. Both organizations operated a contingency system. Information was collected by means of a form and 100 were completed for trades including painter, plumber, electrician, slater, plasterer and others. Tradesmen were requested to select a lapse time from within a discrete set and not on a continuum, as supposed so far. A discrete scale was

used to simplify the task of the tradesman and approximated by a continuum. The cost of a repair was measured in terms of time and material which tradesmen felt able to readily state or estimate.

The parameters that are relevant for the decision making are summarized in this section: T = inspection period, H = lapse time, k = rate of reports of defects, $\beta(T)$ = financial advantage of cluster, $S_h(t)$ = average expected saving made by repairing a defect at a time t unit before the reported stage, $N(T)$ = expected number of contingency repairs, $H(h)$ = distribution of lapse time, J = total manhours for each job, $u\%$ = time tradesmen spent in actually working on repair jobs, $v\%$ = time tradesmen spent travelling in jobs and in general Organisation, $r\%$ = reduction in u .

3.1. Model assumptions

The model developed is based upon the following assumptions;

- i) The epoch 0 is independent of time.
- ii) h is independent of 0:
- iii) An inspection is perfect; and
- iv) Defects are reported at a constant rate of k per unit time.

The expected number of contingency repairs arising during the course of the total period of length T was obtained using Equation (1) as the model that is drawn up is expected to take into account parameters that are relevant to making the decision how often to inspect building.

Let the expected number of contingency repairs arising, during the course of the total period of length

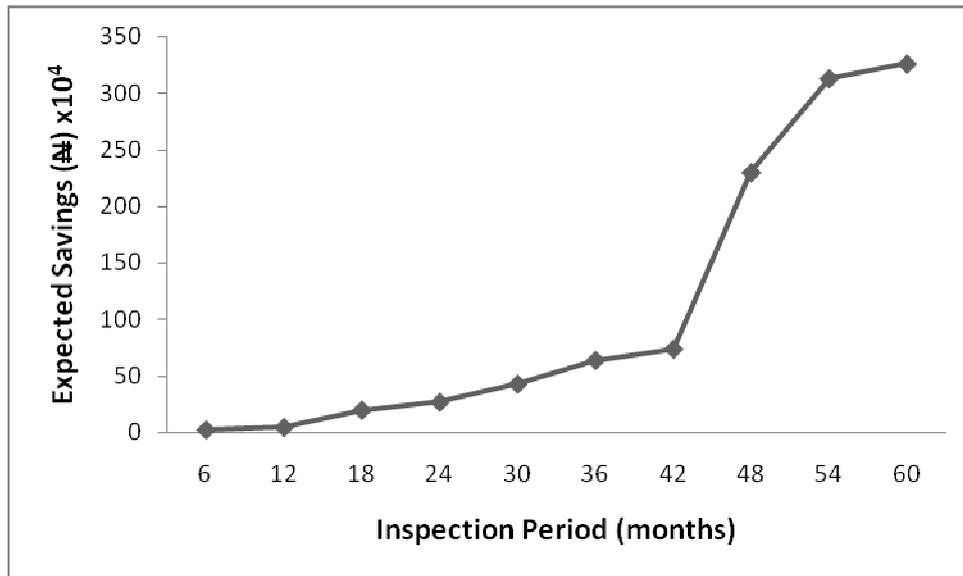


Figure 6: Expected Savings from Inspection frequency of 6 months.

T , is;

$$N(T) = k \int_0^T (T+h)H(h) dh \quad (11)$$

Let, the distribution of lapse time, $H(h)$ be given as

$$H(h) = \frac{2}{(1+h)^3}, \quad 0 \leq h \leq T \quad (12)$$

Putting equation (12) into (11),

$$\begin{aligned} N(T) &= K \int_0^T (T-h) \frac{2}{(1+h)^3} dh \\ &= 2K \int_0^T \frac{(T-h)}{(1+h)^3} dh \end{aligned} \quad (13)$$

But, $\frac{(T-h)}{(1+h)^3} = \frac{A}{(1+h)} + \frac{B}{(1+h)^2} + \frac{C}{(1+h)^3}$ Resolving the L.H.S. of the equation into partial fraction we obtain

$$\frac{(T-h)}{(1+h)^3} = \frac{-1}{(1+h)^3} + \frac{-2}{(1+h)^2} + \frac{T+1}{(1+h)^3} \quad (14)$$

Substituting in equation (13) and evaluating

$$N(T) = \left[\frac{3}{T-1} - 2 \ln(T+1) - (T+5) \right] \quad (15)$$

Substituting equation (15) into equation (4) the financial advantage of cluster $\beta(T)$, becomes

$$\beta(T) = K \left[\frac{3}{T-1} - 2 \ln(T+1) - (T+5) \right] \frac{c_1 vr J}{10000} \quad (16)$$

3.2. Direct cost savings

We have from equation 5 that by inspecting at any other point say q , such that $q \leq h$ at cost say c^* ,

$$S_h(t)S_h(q) = \frac{t}{q}(c - c^*)$$

Then, savings at $h < T$, from equation 7 is

$$\begin{aligned} \int_0^T \int_0^H k S_h(q) H(h) dq dh &= \left[t(c - c^*) \ln q k \frac{(1+h)^{-2}}{-2} \right]_{0,0}^{T,h} \\ &= \left[\frac{kt(c - c^*) \ln q}{2(1+h)^2} \right]_{0,0}^{T,h} \\ &= \left[\frac{-kt(c - c^*) \ln h}{2(1+T)^2} \right] \end{aligned}$$

When $h > T$, no contingency would be observed. Then the savings made is

$$\begin{aligned} \int_0^T \int_0^H k S_h(q) H(h) dq dh &= \left[\frac{-kt(c - c^*) \ln q}{2(1+h)^2} \right]_{hq=h-T}^{\infty,h} \\ &= \frac{-kt(c - c^*) \ln q}{2(1+\infty)^2} - \frac{-kt(c - c^*) \ln(h-T)}{2(1+T)^2} \\ &= \frac{kt(c - c^*) \ln(h-T)}{2(1+T)^2} \end{aligned}$$

But, total expected savings from cluster and early repair from equation (10) will be

$$\alpha(T) = \frac{\beta(T) + \gamma(T) - I}{T} \quad (17)$$

Where

$$\beta(T) = K \left[\frac{3}{T-1} - 2 \ln(T+1) - (T+5) \right] \frac{c_1 vr J}{10000}$$

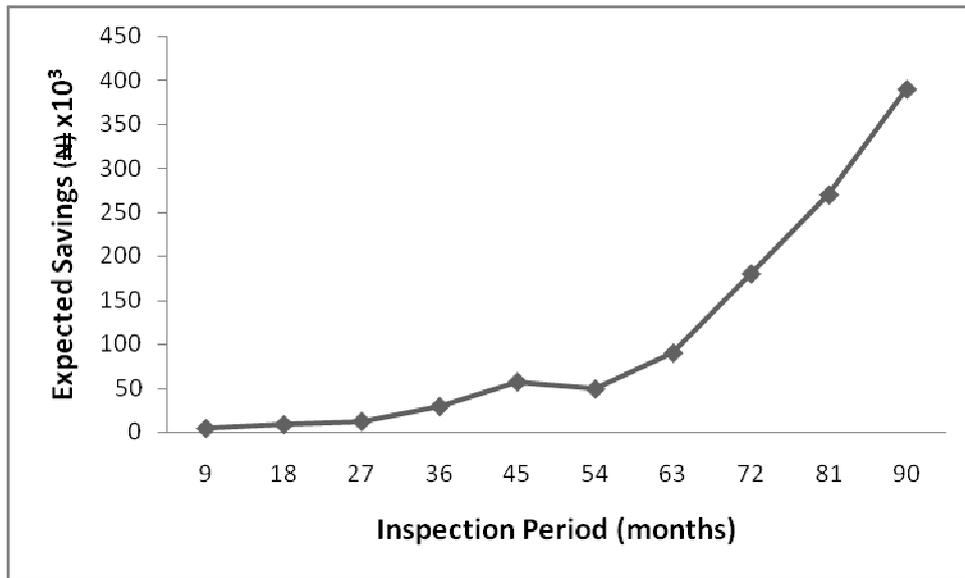


Figure 7: Expected Savings obtained from Inspection frequency of 9 months.

$$\gamma(T) = \text{equation A} + \text{equation B}$$

$$\gamma(T) = \frac{-kt(c - c^*) \ln h}{2(1 + T)^2} + \frac{kt(c - c^*) \ln(h - T)}{2(1 + h)^2} \quad (18)$$

Substituting equations (16) and (18) into (17) we have that

$$\alpha(T) = \frac{1}{T} \left[K \left\{ \frac{3}{T-1} - 2 \ln(T+1) - (T+5) \right\} \frac{c_1 vr J}{10000} + \left\{ \left(\frac{-kt(c - c^*)}{2(1 + T)^2} \right) (\ln(h - T) - \ln h) \right\} - I \right] \quad (19)$$

4. Results and Discussion

The results of the study show that inspection policy attracts some savings for the maintenance organizations. These results are verified in Figure 6, where the expected saving cost runs up to ₦3,100,000.00 in the 5th year as the lapse time is increased from 3 to 6 months. This is also confirmed in the plots of expected savings, obtained from inspection frequency of 3months, 9months, and 12months for 60% manhours per day during repair jobs, as presented in Figures 5-9, and Tables 1-5 in the appendix.

From the plots of expected savings from inspection frequency of 3 months for 60% of manhours per day, during repairs shown in Figures 5, it is obvious that, even though the policy attracts no losses for the maintenance organization, savings made are meagre as compared to the 6months inspection frequency in Fig.6

The results of Figure 7 show a constant increase in the expected savings starting from the first inspection

interval. It means any increase in manhours for repair jobs enhances reduction in maintenance cost, thereby increasing the expected savings steadily.

The savings in Figure 8 were stagnant for the first 3years as the cluster of effects of defects steadily increased. In the 5th year, serious decrease in the savings was observed. This is as a result of the effect of cluster within this month. As the maintenance organization continued to work on the buildings at the regular inspection interval, the cluster effect of the reported defects reduced steadily giving rise to steady increase in expected savings.

In Figure 9, it was clear to see that inspection policy attracts serious loss for the management since the manhours were not efficiently utilised, the cost of inspection went so high that, after compounding the financial advantage of the cluster and direct cost savings of the early repair then subtracting the maintenance cost (I), the negative results indicate losses.

4.1. Numerical study

For the Benue State Housing Estate, data of maintenance works carried out on buildings for a period of 5 years was used to verify the model in equation 19. The results of the study are shown in Figures 5-9. These results show variations of cost, or cost - savings as the inspection model developed is used.

5. Conclusion and Recommendations

In this research, the Christer's Inspection Model has been used to optimize the inspection intervals for

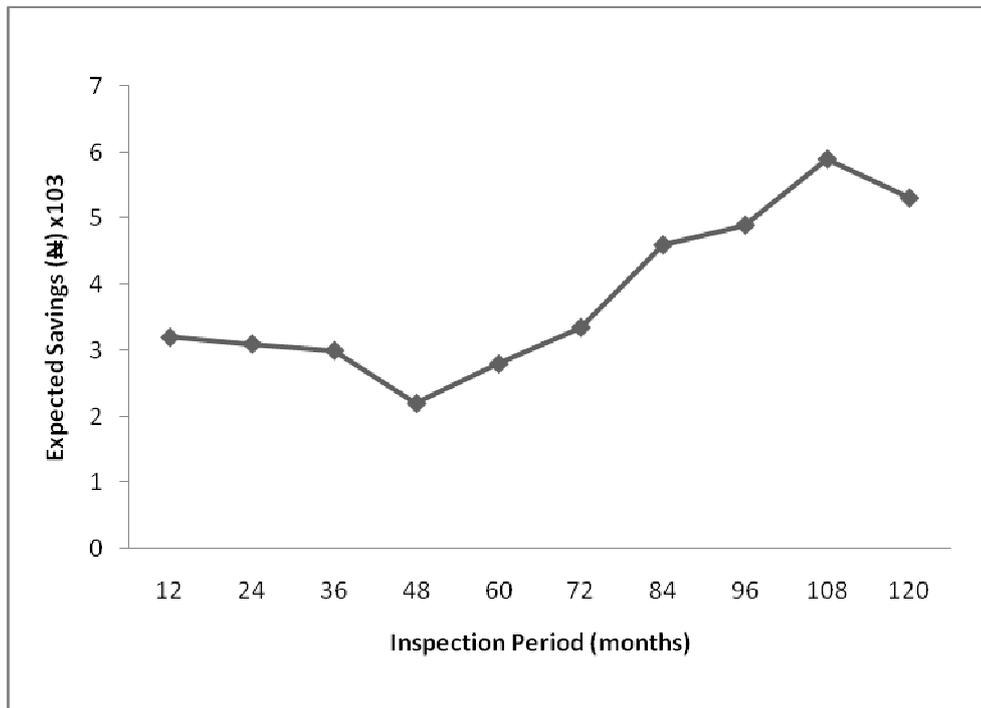


Figure 8: Expected Savings obtained from Inspection frequency of 12 months.

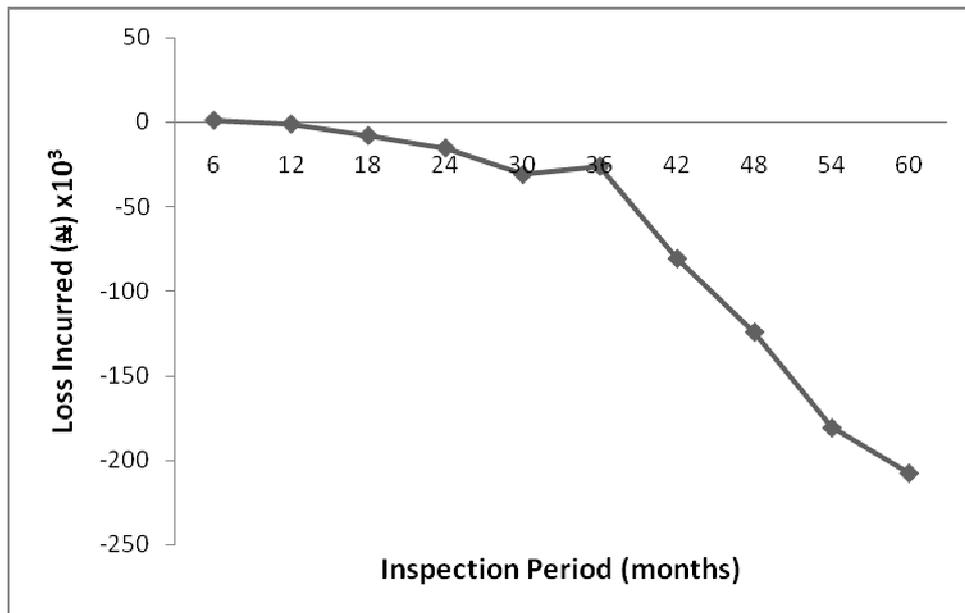


Figure 9: Expected Savings obtained from Inspection frequency of 6 Months when the Manhours spent on Repair Jobs are Less 60% of Workings Hours per Day.

Maintenance of buildings, for which an inspection policy is an effective way of dealing with the failure behaviour of buildings in the Benue State housing Estate (Ankpa Quarters). Total expected maintenance costs (ETMC) as a performance measure was an important indicator, which was influenced by changing inspection intervals and manhours spent on repair jobs. The model used maximizes the total expected savings by choosing an optimal inspection interval expressed in number in terms of six (6) month inspections.

It is therefore recommended that, in adopting an inspection system of maintenance, an inspection frequency of six months should be employed with emphasis on manhours on repair jobs, as it controls cost savings of the system. Also, the Benue State government should adopt inspection maintenance policy as opposed to the contingency system of maintenance, as it attracts some savings and other benefits like eliminating collapse of buildings thereby enhancing safety of occupants.

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