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VOLTAGE STABILITY ANALYSIS OF THE NIGERIAN POWER SYSTEM USING ANNEALING OPTIMIZATION TECHNIQUE

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ABSTRACT
The paper addresses the means of overcoming the challenge posed by voltage collapse to the stability of the Nigerian electric power system. The technique applied is based on time identification algorithm elaborating, at a given grid bus, the local phasor measurements at fast sampling rate. Elements such as adjustable shunt compensation devices, generator reactive generation, transformer tap settings are optimally adjusted at each operating point to reach the objective of minimizing the voltage stability index at each individual bus as well as minimizing the global voltage stability indicator. The control elements setting were optimized and the maximum possible MVA voltage stable loading has been achieved and a best voltage profile was obtained. Results of tests conducted on a 6-bus IEEE system and a typical 28-bus Nigerian power distribution network are presented and discussed.

Keywords: *Special protection systems, voltage stability analysis, voltage stability limit, voltage collapse mechanism, system security*

1. INTRODUCTION

As regards the record available from the recent deregulation of the generation and distribution segment of many electric power systems in the world including Nigeria's, it has become evidently necessary to investigate the best operating conditions under which the transmission system is expected to perform with respect to the high level of transactions expected to always take place. Moreover, it is quite devastating that voltage collapse has remained one of the unresolved riddles that are currently plaguing the power system operation and performance under the recent electricity regime leading to blackout or abnormally low voltages in a significant part of the power system. Special protection Schemes (SPS) have been widely used to increase the transfer capability of the network by assisting system operators in administering fast corrective actions. The cause of this instability can be categorized into technical and non-technical. The technical causes may be due to tripping of lines on account of faulty equipment or increase in load than the available supply [1].

The power system ability to maintain constantly acceptable bus voltage at each node under normal operating conditions, after load increase, following system configuration changes or when the system is being subjected to a disturbance is a very important characteristic of the system. The non-optimized control of VAR resources may lead to progressive and uncontrollable drop in voltage resulting in an eventual wide spread voltage collapse.

The phenomenon of voltage instability is attributed to the power system operation at its maximum transmissible power limit, shortage of reactive power resources and inadequacy of reactive power compensation tools. A non-optimized setting of the level or control of the reactive resources plays an effective role to expedite the voltage instability and to speed up reaching the maximum loading limit. The main factors contributing to the voltage collapse are the generators reactive power limit, load characteristics, voltage control limits, reactive

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compensation devices characteristics and their reactions.

Voltage stability estimation techniques based on load flow Jacobian analysis such as, singular value decomposition, eigen value calculations, sensitivity factories, and modal analysis are time consuming for a large power system [2- 6]. Several indices based methods such as Voltage Instability Proximity Index (VIPI) and Voltage Collapse Proximity Indicator (VCPI) are used to evaluate voltage instability. These are based on multiple load flow solutions and give only global picture [7, 8]. The transmission proximity index that specifies the weakest transmission part of the system based on voltage phasor approach necessitate the scanning of the whole power system structure for several times which is the time consuming approach [9].

The strong tie of the voltage stability problem with the reactive power resources and flow in the system raise the interest in optimizing the rescheduling of the VAR control tools [10]. An optimum VAR picture would maintain a good voltage profile and extend the maximum loading capability of the power network [11]. Many researchers reported on several approaches for optimal reactive power, dc-based techniques and features of accurate voltage-based contingency selection algorithm [12], in literature. Methods such as linear programming and nonlinear programming algorithms were applied. They are complex, time consuming and require considerable amount of memory [13 -19].

The non-incremental quadratic programming model used for optimal reactive power control, though the technique is relatively accurate and shows a satisfactory convergence characteristic but as the system gets bigger the number of variables to be evaluated would rise sharply [20].

This paper proposes an optimized fast voltage stability indicator dedicated for evaluation and monitoring. The optimized index gives information covering the whole power system and evaluated at each individual bus, this is calculated at every operating point. The used indicator is simple to derive and fast to calculate. In order to enhance the voltage stability profile throughout the whole power network, simulated annealing (SA) optimization technique [21, 22] is applied to control the power elements of major influence on the voltage stability profile. Elements such as generator reactive generation, adjustable shunt compensation devices, transformer tap settings are optimally adjusted at each operating point to reach the objective of increasing the distance from an unstable system state and therefore to increase the maximum possible system safe loading. The objective is achieved through minimizing the L-index values at every bus of the system and consequently the global power system L-index.

2. FAST VOLTAGE STABILITY INDICATORS

For voltage stability bus evaluation in connection with transmission path, an indicator L-index is used [23]. The indicator value varies in the range between 0 (at no load case) and 1 which corresponds to magnitude and phase along with the power network information provided by the load flow program.

For a multi-node system, for example:

$$I_{bus} = Y_{bus} \times V_{bus} \tag{1}$$

Segregating the load buses (PQ) from generator buses (PV), equation (1) can write as:

$$\begin{bmatrix} I_L \\ I_G \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix} \begin{bmatrix} V_L \\ V_G \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \tag{3}$$

V_L, I_L: Voltages and Currents for PQ buses

V_G, I_G: Voltages and Currents for PV buses

Where, B_1, B_2, B_3, B_4: submatrices generated from Y_{bus} partial inversion.

Let

$$\bar{V}_{0k} = \sum_{i=1}^{n_G} B_{ki} \bar{V}_i \tag{4}$$

n_G: number of generators

$$B_2 = -Y_1 \times Y_2 \tag{5}$$

$$L_K = \left| 1 + \frac{V_{0k}}{V_K} \right| \tag{6}$$

L_K: L-index voltage stability indicator for bus k [23, 24]

Stability requires that $L_K < 1$ and must not be violated on a continuous basis. Hence a global system indicator L, describing the stability of the complete system is $= L_{max}(L_k)$, where in (L_K) all L bus indexes are listed.

In practice L_{max} must be lower than a threshold value.

The predetermined threshold value is specified at the planning stage depending on the system configuration and on the utility policy regarding the quality of service and the level of system decided allowable margin. In practice, the calculation of the complex vector V_{0k} never uses the inversion of Y_1.

$$[-Y_1]V_{0k} = [Y_2]V_G \qquad (7)$$

Sparse factorization vector methods have been used to solve the linear system in equation (7) and make from L-index a potential candidate for real-time performance [23, 24]

3. SIMULATED TECHNIQUES

3.1 Brief Introduction

Simulated annealing (SA) is an optimization technique that simulates the physical annealing process in the field that combine optimization. Annealing is the physical process of heating up a solid until it melts, followed by slow cooling it down by decreasing the temperature of the environment in steps. At each step, the temperature is maintained constant for a period of time sufficient for the solid to reach thermal equilibrium.

The research work [22] proposed a Monte Carlo method to simulate the process of reaching thermal equilibrium at a fixed value of the temperature *T*. In this method, a randomly generated perturbation of the current configuration of the solid is applied so that a trial configuration is obtained. This trial configuration is accepted and becomes the current configuration if it satisfies an acceptance criterion. The process continues until the thermal equilibrium is achieved after a large number of perturbations. By gradually decreasing the temperature *T* and repeating [2, 22] simulation, new lower energy levels become achievable. As *T* approaches zero least energy configurations will have a positive probability of occurring.

3.2 Simulated Annealing Algorithm

At first, the analogy between a physical annealing process and a combinatorial optimization problem is based on the following [21]:

- solutions in an optimization problem are equivalent to configurations of a physical system.
- the cost of a solution is equivalent to the energy of a configuration.

In addition, a control parameter C_p is introduced to play the role of the temperature *T*.

The basic elements of SA are defined as follows:-

- ***Current, trial, and best solutions:*** $x_{current}$, x_{trial}, and x_{best}: these solutions are sets of the optimized parameter values at any iteration.
- ***Acceptance criterion***: at any iteration, the trial solution can be accepted as the current solution if it meets one of the following criteria:

(a) $J_{(x_{trial})} < J_{(x_{current})}$; **(b)** $J_{(x_{trial})} > J_{(x_{current})}$ and $exp\left(-\left(J_{(x_{trial})} - J_{(x_{current})}\right)/C_p\right) \geq rand(0,\ 1)$.

Where, $rand(0,\ 1)$ is a random number with domain (0, 1) and $J_{(x_{trial})}$ and $J_{(x_{current})}$ are the objective function values associated with x_{trial} and $x_{current}$ respectively. Criterion (b) indicates that the trial solution is not necessarily rejected if its objective function is not as good as that of the current solution with hoping that a much better solution becomes reachable.

- ***Acceptance ratio***: at a given value of C_p, an n_1 trial solution can be randomly generated. Based on the acceptance criterion, an n_2 of these solutions can be accepted. The acceptance ratio is defined as $^{n_2}/_{n_1}$.
- ***Cooling schedule***: it specifies a set of parameters that governs the convergence of the algorithm. This set includes an initial value of control parameter C_{0p}, a decrement function for decreasing the value of C_p, and a finite number of iterations or transitions at each value of C_p, that is, the length of each homogeneous Markov chain. The initial value of C_p should be large enough to allow virtually all transitions to be accepted.

However, this can be achieved by starting off at a small value of C_{0p} and multiplying it with a constant larger than 1, α that is $C_{0p} = \alpha C_{0p}$. This process continues until the acceptance ratio is close to 1. This is equivalent to heating up process in physical systems.

The decrement function for decreasing the value of C_p is given by $C_p = \mu C_p$

Where, μ is a constant smaller than but close to 1. Typical values lie between 0.8 and 0.99 [21].

- ***Equilibrium condition***: it occurs when the current solution does not change for a certain number of iterations at a given value of C_p. It can be achieved by generating many transitions at that value of C_p.
- ***Stopping Criteria***: these are the conditions under which the search process will terminate.

In this study, the search will terminate if one of the following criteria is satisfied:

(a) the number of Markov chains since the last change of the best solution is greater than a prespecified number; or,

(b) the number of Markov chains reaches the maximum allowable number.

The SA algorithm can be described in steps as follows:

Step 1: Set the initial value of C_{0p} and randomly generate an initial solution $x_{initial}$ and calculate its objective function. Set this solution as the current solution as well as the best solution, i.e. $x_{initial} = x_{current} = x_{best}$.

Step 2: Randomly generate an n_1 of trial solutions about the current solution.

Step 3: Check the acceptance criterion of these trial solutions and calculate the acceptance ratio. If acceptance ratio is close to 1 go to step 4; else set $C_{0p} = \alpha C_{0p}$, $\alpha > 1$, and go back to step 2.

Step 4: Set the chain counter $K_{ch} = 0$.

Step 5: Generate a trial solution x_{trial}. If x_{trial} satisfies the acceptance criterion set $x_{current} = x_{trial}$, $J_{(x_{current})} = J_{(x_{trial})}$, and go to step 6; else go to step 6.

Step 6: Check the equilibrium condition. If it is satisfied go to step 7; else go to step 5.

Step 7: Check the stopping criteria. If one of them is satisfied then stop; else set $K_{ch} = K_{ch} + 1$ and $C_p = \mu C_p$, $\mu < 1$, and go back to Step 5.

4. TEST RESULTS AND DISCUSSION

The demonstrated is applied on IEEE 6-bus test system [25] which has two voltage sources and four load buses. Bus-1 is the swing bus, bus 2 is PV bus and buses 3-6 are PQ load buses. It is also tested on a typical practical Nigerian 28-bus Network (Nsukka 11kV Campus Feeder Network).

4.1 Case 1: 6 Bus Network

The single –line diagram of the 6-bus IEEE network is shown in Figure 1 with line data and bus data are given in Tables 1 and 2 respectively.

The voltage stability index is evaluated at every operating point and for every bus in the system along the system overall index L_{max}. The simulated annealing optimization is activated of every operating in order to adjust the available VAR control tools for the objective of minimizing the value of L-index at every bus in the system and consequently the system overall voltage stability indicator L_{max}. Hence, the optimization problem can be written as

$$Minimize\ \left(max\left(L_k;\ k = 1,\ 2,\ \dots,\ n\right)\right) \qquad (8)$$

n: number of buses

The problem constraints are the control variable bounds as given in Table 3.

The optimal values of control variables are given in Table 3 which also shows the load flow solution with the initial settings and the proposed optimal settings of the control variables. It is clear that the voltage profile is greatly improved and the real power loss is reduced by 11.32%.

Table 4 shows a comparative list of results using both voltage stability evaluation of L-index with and without optimization.

It can be seen that the values of L-index at load buses are reduced; therefore, the voltage stability of the system is enhanced and improved.

The test was carried out for a different load level starting from 40% of the base load with a step increase of all loads in the system till voltage collapse.

The voltages of load buses versus load factor without and with optimization are as shown in Figure 3.

In addition, L-index values at load buses versus load factor are as shown in Figure 4.

It is clear that the application of the SA algorithm has significantly reduced the values of L-index all over the system. Consequently, the voltage stability distance from collapse has increased. The gain in power system MVA loading was found to be 23%. The above positive results demonstrate the potential of the proposed approach to improve and enhance the system voltage stability.

4.2 Case 2: Typical 28 Bus Nigerian Networks

The line diagram of a typical 28-bus Nigerian network (Nsukka 11kV Campus Feeder) [26] is as shown in Figure 5 with network bus details and line indications presented in Table 5 and 6 respectively. The objective function convergence rate is shown in Figure 2 and this shows the fast convergence of the proposed technique.

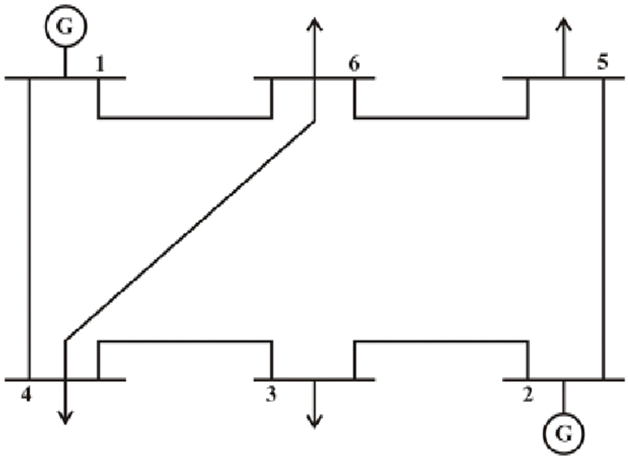


Fig. 1: Line Diagram of IEEE 6-Bus System

Table 1: Line Data on 100 MVA base

Line number	From	To	R (pu)	X (pu)
1	1	6	0.1230	0.518
2	1	4	0.0800	0.370
3	2	3	0.0723	1.050
4	2	5	0.2820	0.064
5	3	4	0.0000	0.133
6	4	6	0.0970	0.407
7	6	5	0.0000	0.300

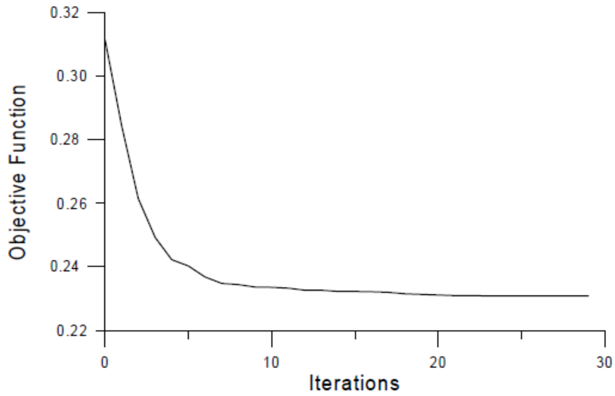


Fig. 2: Objective function convergence

Table 2: IEEE 6-Bus Data on 100 MVA base

Bus number	Voltage (pu)		P_g (pu)	Q_g (pu)	P_L (pu)	Q_L (pu)
	V	Q				
1	1.0500	0.0000	0.9662	0.3792	0.0000	0.0000
2	1.1000	-06.1494	0.5000	0.3499	0.0000	0.0000
3	0.8563	-13.8236	----	----	0.5500	0.1300
4	0.9528	-09.9245	----	----	0.0000	0.0000
5	0.8992	-13.4205	----	----	0.3000	0.1800
6	0.9338	-12.6485	----	----	0.5000	0.0500

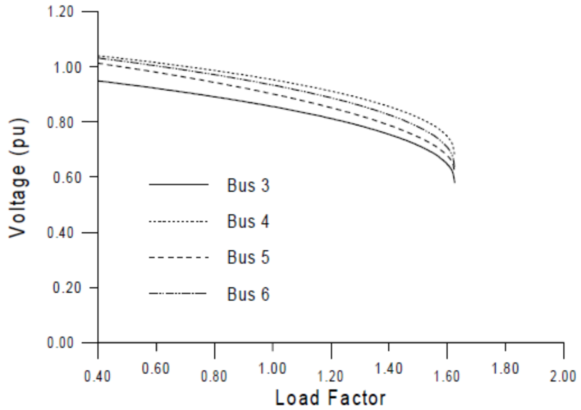
Table 3: Load flow results without and with optimization

Variable	Limits		Without Optimization	With Optimization
	Low	High		
Control Variables				
Transformer Taps T_4;	0.90	1.10	1.025	0.958
T_6	0.90	1.10	1.100	0.984
Generator voltage (pu) V_1;	1.00	1.10	1.050	1.092
V_2	1.10	1.15	1.100	1.150
Dependent Variables				
Generator MVAR Q_{g1};	-20.0	100.0	38.11	35.82
Q_{g2}	-20.0	100.0	34.80	19.35
Voltages at load Buses (pu) V_3;	0.90	1.00	0.855	1.001
V_4;	0.90	1.00	0.953	1.001
V_5;	0.90	1.00	0.901	1.000
V_6	0.90	1.00	0.933	0.984
System Losses (MW)	----	----	11.61	8.880

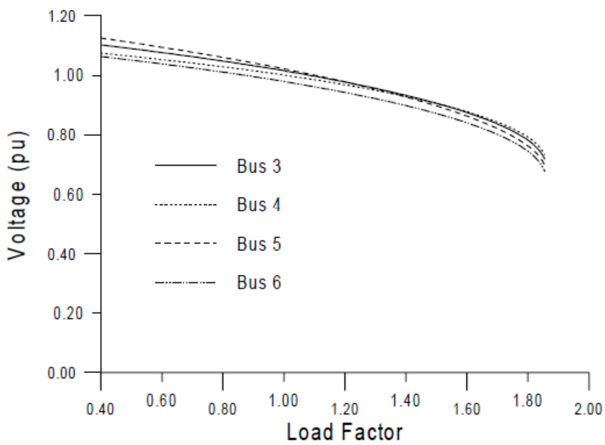
Table 4: Behavior of L-index with and without optimization

Bus number	L-index without optimization	L-index with optimization
3	0.288	0.234
4	0.211	0.178
5	0.278	0.234
6	0.258	0.218

(a) Without optimization
(b) With Optimization

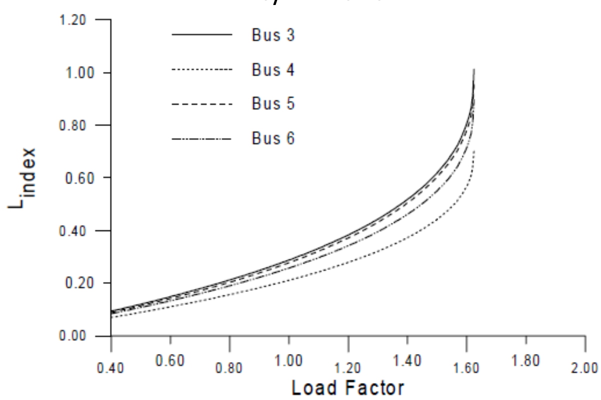


(a)

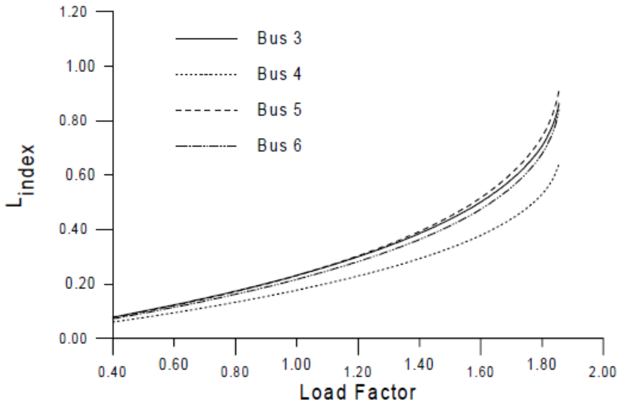


(b)

Fig. 3: Load buses voltages Without optimization With optimization



(a)



(b)

Fig. 4: Load buses L_{index} values (a) Without optimization (b) With optimization

The optimal values of control variables are given in Table 3 as well as Table 7, which also shows the load flow solution with the initial settings and the proposed optimal settings of the control variables. It is clear that the voltage profile is greatly improved and the real power loss is reduced by 11.32% also. Table 4 as well as Table 8 also shows a comparative list of results using both voltage stability evaluation of L-index with and without optimization from the existing and the proposed.

The Table 5 shows the bus loading report of the existing 11kV Campus Feeder.

The Table 6 shows the Cable Loading Report of the Existing 11kV Campus Feeder.

Similarly, the voltage stability index was also evaluated at every operating point and for every bus in the system along the system overall index L_{max}. The simulated annealing optimization was activated of every operating in order to adjust the available VAR control tools for the objective of minimizing the value of L-index at every bus in the system and consequently the system overall voltage stability indicator L_{max}.

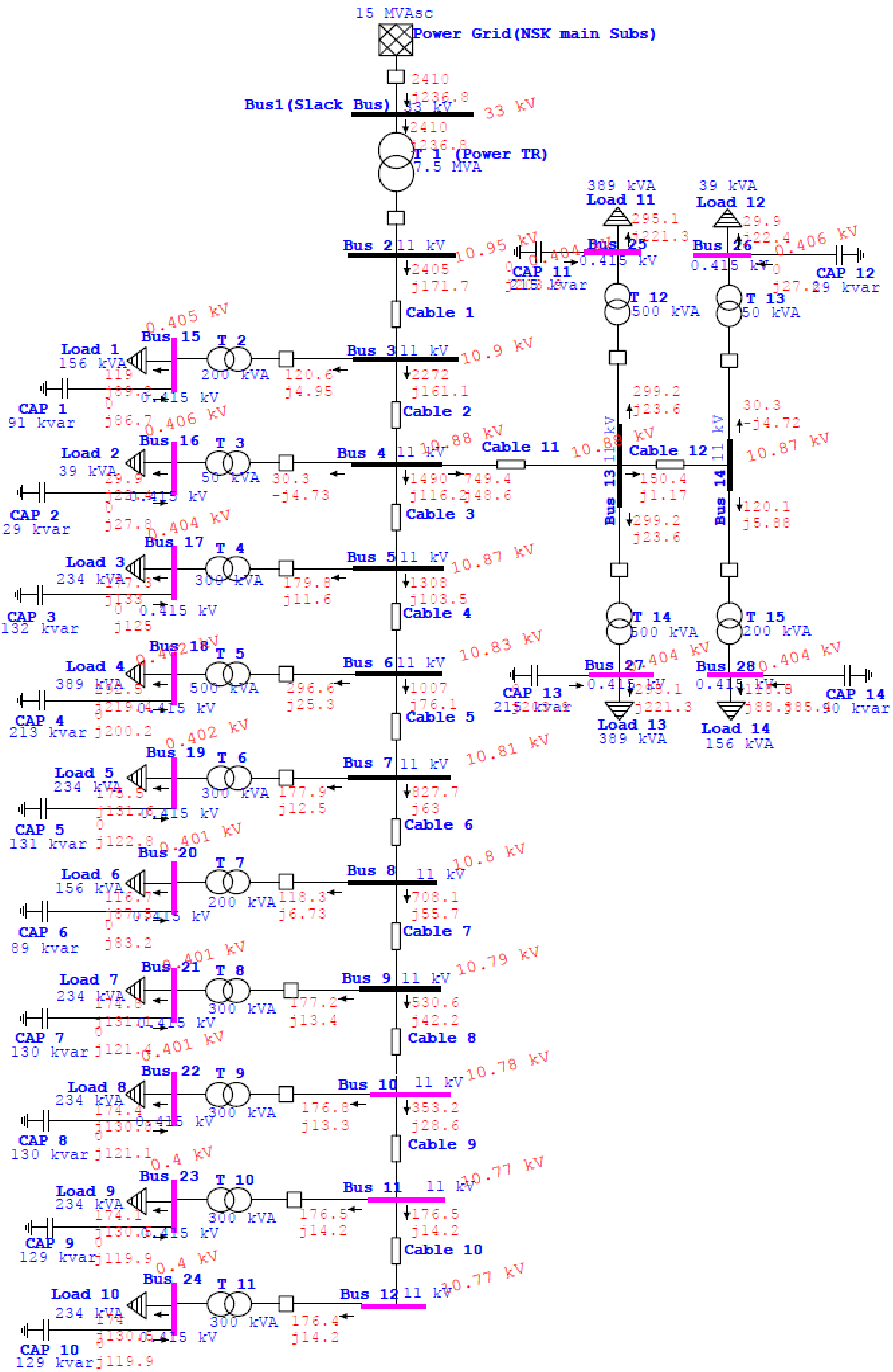


Fig. 5: Nsukka 11kV Campus Network System

Table 5: Bus Loading Report of the Existing 11kV Campus Feeder

Bus ID	Nominal Voltage (kV)	Operating Voltage (kV)	Operating Voltage (P.U)	Total Bus Load		
				MVA	% PF	Amp
Bus 1	33	33	1.0000	2.290	78.1	51.1
Bus 2	11	10.759	0.9781	2.856	79.6	153.2
Bus 3	11	10.687	0.9715	2.837	79.5	153.2
Bus 4	11	10.673	0.9703	2.690	79.5	145.5
Bus 5	11	10.651	0.9683	1.762	79.5	95.5
Bus 6	11	10.603	0.9639	1.542	79.4	83.9
Bus 7	11	10.582	0.9620	1.188	79.4	64.8
Bus 8	11	10.562	0.9602	0.976	79.4	53.4
Bus 9	11	10.555	0.9595	0.836	79.4	45.7
Bus 10	11	10.541	0.9583	0.626	79.4	34.3
Bus 11	11	10.529	0.9572	0.416	79.4	22.8
Bus 12	11	10.527	0.9570	0.208	79.4	11.4
Bus 13	11	10.660	0.9691	0.888	79.4	48.1
Bus 14	11	10.622	0.9656	0.178	79.4	9.6
Bus 15	0.415	0.392	0.9446	0.139	80.0	204.8
Bus 16	0.415	0.391	0.9422	0.035	80.0	51.1
Bus 17	0.415	0.390	0.9398	0.207	80.0	306.2
Bus 18	0.415	0.389	0.9373	0.341	80.0	506.8
Bus 19	0.415	0.388	0.9349	0.204	80.0	304.2
Bus 20	0.415	0.387	0.9325	0.136	80.0	202.4
Bus 21	0.415	0.387	0.9325	0.203	80.0	303.4
Bus 22	0.415	0.386	0.9301	0.203	80.0	303.0
Bus 23	0.415	0.386	0.9301	0.202	80.0	302.7
Bus 24	0.415	0.386	0.9301	0.202	80.0	302.6
Bus 25	0.415	0.391	0.9422	0.345	80.0	509.6
Bus 26	0.415	0.391	0.9422	0.035	80.0	51.1
Bus 27	0.415	0.391	0.9422	0.345	80.0	509.6
Bus 28	0.415	0.391	0.9422	0.138	80.0	204.3

Table 6: Cable Loading Report of the Existing 11kV Campus Feeder

Cable ID	Ampacity (Amps)	Loading (Amps)	% Loading
Cable 1	270.43	153.24	56.67
Cable 2	270.43	145.51	53.81
Cable 3	270.43	95.50	35.32
Cable 4	270.43	83.95	31.04
Cable 5	270.43	64.83	23.97
Cable 6	270.43	53.35	19.73
Cable 7	270.43	45.72	16.91
Cable 8	270.43	34.27	12.67
Cable 9	270.43	22.84	8.44
Cable 10	270.43	11.42	4.22
Cable 11	270.43	48.08	17.78
Cable 12	270.43	9.63	3.56

5. CONCLUSIONS

This paper has proposed an optimized voltage stability index using fast voltage stability indicator minimized by the simulated annealing optimization technique. The developed systems, both IEEE 6-bus and the typical 28-bus Nigerian distribution systems, has shown accurate results, success in convergence to optimal solution. The results are obtained fast and direct. The conducted application on standard system has satisfactory results for optimal voltage stability level as well as for extending the loading level of the system.

Table 7: Bus Loading Report of the Proposed 11kV Campus Feeder

Bus ID	Nominal Voltage (kV)	Operating Voltage (kV)	Operating Voltage (P.U)	Total Bus Load		
				MVA	% PF	Amp
Bus 1	33	33	1.000	2.422	99.5	42.4
Bus 2	11	10.952	0.9956	2.411	99.7	127.1
Bus 3	11	10.895	0.9905	2.399	99.8	127.1
Bus 4	11	10.884	0.9894	2.276	99.7	120.7
Bus 5	11	10.866	0.9878	1.492	99.7	79.3
Bus 6	11	10.828	0.9844	1.308	99.7	69.7
Bus 7	11	10.811	0.9828	1.008	99.7	53.9
Bus 8	11	10.795	0.9814	0.829	99.7	44.3
Bus 9	11	10.790	0.9809	0.710	99.7	38.0
Bus 10	11	10.779	0.9799	0.532	99.7	28.5
Bus 11	11	10.769	0.9790	0.354	99.7	19.0
Bus 12	11	10.768	0.9789	0.177	99.7	9.5
Bus 13	11	10.875	0.9886	0.750	99.8	39.8
Bus 14	11	10.874	0.9885	0.151	99.9	8.0
Bus 15	0.415	0.405	0.9759	0.149	80.0	211.9
Bus 16	0.415	0.406	0.9783	0.041	73.2	58.0
Bus 17	0.415	0.404	0.9734	0.222	80.0	316.8
Bus 18	0.415	0.402	0.9687	0.366	80.0	524.7
Bus 19	0.415	0.402	0.9687	0.219	80.0	315.2
Bus 20	0.415	0.401	0.9663	0.146	80.0	209.9
Bus 21	0.415	0.401	0.9663	0.218	80.0	314.6
Bus 22	0.415	0.401	0.9663	0.218	80.0	314.2
Bus 23	0.415	0.400	0.9639	0.218	80.0	313.9
Bus 24	0.415	0.400	0.9639	0.218	80.0	313.9
Bus 25	0.415	0.404	0.9734	0.369	80.0	527.0
Bus 26	0.415	0.406	0.9783	0.041	73.2	58.0
Bus 27	0.415	0.404	0.9734	0.369	80.0	527.0
Bus 28	0.415	0.404	0.9734	0.148	80.0	211.4

Table 8: Cable Loading Report of the Proposed 11kV Campus Feeder

Cable ID	Ampacity (Amps)	Loading (Amps)	% Loading
Cable 1	270.43	127.11	47.00
Cable 2	270.43	120.72	44.64
Cable 3	270.43	79.29	29.32
Cable 4	270.43	69.72	25.78
Cable 5	270.43	53.85	19.91
Cable 6	270.43	44.33	16.39
Cable 7	270.43	37.99	14.05
Cable 8	270.43	28.48	10.53
Cable 9	270.43	18.98	7.02
Cable 10	270.43	9.49	3.51
Cable 11	270.43	39.84	14.73
Cable 12	270.43	7.99	2.95

6. DISCLAIMER

The views expressed in this paper are those of the authors and do not represent those of Jacobs Engineering Inc. London.

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