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DIFFERENT ELEMENT METHODS IN ENGINEERING PRACTICE

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Abstract

Presented is the most common element methods used for analysis in engineering. The methods are discussed in an overall and general manner so that engineers and scientists who are increasingly, called upon to use element methods to support and check their analyses and/or designs can appreciate the essential differences and similarities in the various methods and their possible advantages and disadvantages. The aim is also to provide a background for numerical analyst who although an expert in one element method may be unfamiliar with other methods.

Keywords: element methods, discretization, analysis, approximate method, characteristic matrix

1. Introduction

Element methods are approximate procedures which tend to discretize all or part of the structure (or system) in the sense that the behavior at discrete number of points (or nodes) in the structure characterizes the overall response of the structure to loading reconstituted by assembly procedure.

A number of element methods of analysis have been developed over the years and in recent years have continued to expand and diversify into all major fields of scientific and engineering studies [1-5]. They have become popular due to rapid advancements in computer technology and its availability to engineers and scientists. Indeed, the development of faster and cheaper computers has led to greater emphasis being placed on computer-oriented methods like element methods that are used extensively in the development of computer programs or codes.

Element methods now provide a viable alternative to analytical methods that can be expensive and time consuming. The most commonly used element methods are:

1. The Line Element Method (LEM)
2. The Finite Element Method (FEM)
3. The Finite Element Difference (FED)
4. The boundary Element Method (BEM)
5. The Discrete Element Method (DEM)

Before we proceed to discuss each element method, some common fundamental concepts about the element methods should be described.

1.1. Discretization:

The key common factor of all element methods is discretization of the domain (=structure system, solution region, body, continuum etc) of analysis as emphasized by the middle letter E in their abbreviations. This common factor essentially involves dividing the domain into smaller sub-domains or parts called "elements" of various shapes held together at their boundaries called the "nodes" which define the element topology. Material and other attributes are associated with each element and may vary between elements as required. The more the number of elements (subdivisions) used to model the problem (system), the better approximation to the solution is obtained with the consequent increase in computation time and effort.

1.2. Shape function:

In element methods, assumed field variables (e.g displacements) that satisfy certain conditions (continuity and differentiability to necessary degree) are used. These variables are interpolated in terms of element coordinate systems and their values at the nodes using appropriate interpolation function called shape function associated to the model field variables or degrees of freedom.

1.3. Characteristic matrices:

All quantities and equations in element methods are formulated ab initio as matrices and manipulated using associated algebra; thereby making all element methods computer-oriented techniques. The characteristics matrices and characteristic vectors (also called Element equation) is written as:

$$F^e = K^e U^e \tag{1}$$

where K^e is characteristic matrix and is given by:

$$K^e = \int_{vol} B^T H B dvol \tag{2}$$

F is the characteristic vector of load and U is the characteristic vector of unknowns. This element equation can be derived by using any of the following approaches:

a. Direct approach - in this method, direct physical reasoning is used to establish the element properties (Characteristics matrices and vectors) in terms of pertinent variables. Although the applicability of this method is limited to simple type of element, a study of this method enhances our understanding of the physical interpretation of the element methods.

b. Variational approach - In this method, the element analysis is interpreted as an approximate means for solving variational problems. Since most physical and engineering problems can be formulated in variational form, the element method can be readily applied for finding their approximate solutions. The variational approach has been most widely used in the literature in formulating element equations. A major limitation of the method is that it requires the physical or engineering problem to be stated in variational form which may not be possible in all cases.

c. Weighted residual approach - In this method, the element equations are derived directly from the governing differential equations of the problem without reliance on the variational statement of the problems. This method offers the most general procedure for deriving element equations and can be applied to almost all practical problem of science and engineering [6-7]. Again, within the weighted residual approach, different procedures like Galerkin method and Least squares method can be used in deriving the element equation.

Whichever formulation approach is used, the resulting element equation are solved using appropriate matrix solvers like Gauss elimination, Frontal approach, Sky-line approach, Cholesky factorization etc. Displacement formulation is implicitly assumed throughout the discussion.

1.4. Assembly process:

Since the structure is composed of several elements, the individual characteristic matrices and (load) vectors are assembled in a suitable manner and the overall equation formed as $F_g^s = K_g^s U_g^s$ and solve for the unknowns.

2. The Line Element Method (LEM)

The first process in LEM is discretization where the structure is subdivided into smaller straight line segments (elements) and equilibrium equation written for each element such that the response of the entire structure is reconstituted by assembly procedure.

The LEM is virtually indistinguishable from classical matrix stiffness method of structural analysis except that the element stiffness matrices are derived from assumed displacements instead of exact displacements; hence making LEM an approximate procedure. The early line element (LE) work continued the framework analogy with the structure being envisaged as "separated" into members (elements) [as a sort of discretization] defined by constructional features.

As energy basis was recognized, more mathematical approaches were developed and the element stiffness matrix is derived from calculus of variations. These developments make LEM which was mainly applied to skeletal structures like (beams, trusses, frames, arches, cables, etc) to be extended to the analysis of simple continuum structures like axisymmetric shells (cylindrical and spherical shells of revolution) [8-9].

LEM is mainly used for calculating deflections and resultant forces in skeletal structures. The major drawback of the method is its inability to analyze point variables like stresses as can easily be done with the methods described later.

Coupled LEM and FEM has been applied successfully in the aircraft industry, Rao [8] where the discretization of an aircraft wing was done using LE for flange areas and FE for coverplates, spar and rib regions. LEM may be regarded as an elementary version of the finite element method.

3. The Finite Element Method (FEM)

This is the most popular element method in science and engineering. The important influence of minimum weight structures in aircraft industry led to the development of FEM. It was Turner, Clough, Martin and Topp [2] who combined the idea of discrete element with "stiffness" approach to matrix structural analysis to produce a systematic procedure which later became known as the FEM. Now any physical phenomenon governed by differential equations can be modeled by the FEM formulated via principles of variational calculus. The basis of formulation gives variants of FE, namely Weighted Residual FE, Least Square FE, Virtual work FE, Variational FE etc.

The method essentially involves dividing the body in smaller "elements" of various shapes (triangles and rectangles in 2D cases and Tetra hedrons and "bricks" in 3D cases) held together at the "nodes" which are corners of elements. Thus, a standard FEM uses two or three dimensional element shape obtained by discretization of continua while LEM uses one dimensional element shape obtained by line segment idealization of structures.

FEM has been applied to a large number of problems in widely different fields Zienkiewicz [7]. Its popularity, particularly for bad-deformation problems, largely depends on the fact that it is very appealing to engineers and scientists. They are able to relate it to a large extent to the background of structural mechanics as the physical meaning of the steps of calculations are relatively apparent. A large part of the FE programs can remain as a "black box" to the user and even a beginner can obtain interesting results with minimal effort. This does not mean that the method is easy and no experience is required in solving engineering problems of practical importance. On the contrary, to make use of the full potentials of the method and interpret the results of calculations, considerable expertise is required.

Advantages of FEM can be summarized as follows:

- Intricate boundaries can be described with a fine mesh grid.
- Ability to use elements of various sizes, types and shapes to model system.
- It can accommodate arbitrary support conditions. Both force and displacement boundary conditions are treated expeditiously.
- Ability to deal with complex material property laws.

Disadvantages of FEM are:

- It is being limited by computer capacity and sound knowledge of computer programming and mathematics.
- It cannot yield accurate results in the area of high stress concentrations where interpolation procedures must be followed for estimations. (eg contacts and interfaces).
- A general closed-form solution which would permit one to examine response changes in various parameters is not produced, that is aspecific numerical solution is obtained for specific problem.
- The major disadvantage of the FEM is that considerable effort is required in preparing data for a problem. This is particularly crucial in 3D (three dimensional) problems and has led to "mesh generation" programs. These programs produce (to large extent) the input data required for the FEM. Still considerable effort is required in "starting up" the problem. The method is also expensive in computer time as a large set of simultaneous equations (several hundreds of thousands) have to be solved (often iteratively) to obtain solution. The computer time goes up further if the problem is non-linear and/ or three-dimensional.

Inspite of these disadvantages, FEM remains the dominant form of engineering analysis tool. Its strength lies in its generality (applicable to all problems) and flexibility (ability to be coupled to other element methods) to handle all types of loads, sequence of construction, installation of supports, etc.

4. The Finite Element Difference (FED)

This method is popularly known as finite difference method but designated here as finite element difference to emphasize discretization. Strictly speaking, FED is not an element method but a pure numerical method like "weighted residual" technique. This is because they discretize the differential equation governing the system and not the system itself.

The idea behind FED is to replace the governing differential equations and the equations defining the boundary conditions by the corresponding finite-difference equations. This process reduces the problem to set of simultaneous algebraic equations which can be solved without much mathematical difficulty.

Thus before applying the method, the governing differential equations must be available unlike true element method in which the governing differential equation is consequence of formulation procedure. Another major disadvantage of the FED involves the difficulties in dealing with complex boundary conditions and irregular geometries which introduce irregular meshes. In the recent version of the method, both difficulties can be lessened by adopting an energy formulation, Mohr[10]. Not only does this approach lead to a symmetric stiffness matrix, but also the non-essential boundary conditions do not have to be considered explicitly but are instead satisfied in a weighted average sense as a result of the stationary process.

FED has been applied to many engineering problems in the areas of solid, structural and fluid mechanic especially in non linear analyses.

5. The Boundary Element Method (BEM)

This method is becoming increasingly popular. It uses approximate functions that satisfy the governing equations in the domain but not on the boundary.

In BEM, also called boundary integral method, only the surface of the body to be analyzed needs to be discretized. Thus, for two dimensional situations line elements at the boundary (contour) represent the problem while surface elements represent three dimensional problems.

Consequently, there are generally less unknowns than will be generated by FEM and FED. Other advantages include the relative ease, with which singularities and boundaries at infinity may be treated. Also the data preparation is relatively simple but the computer program is not so transparent. BEM appears to be a very efficient method for homogeneous, linear elastic problems and less suitable for elasto-plastic problems since it is more difficult to find approximate functions that satisfy the governing equations in the domain.

In comparison with the FEM, the major disadvantage of BEM would appear to relate to the added mathematical complexity, that is, BEM requires a higher level of understanding of mathematical complexities. The implementation of the BEM is more complicated than that of the FEM because the integration of the singular fundamental solution (called kernels) require special programming techniques apart from this, the implementation of BEM follows a similar process to the FEM, that is, element contributions are computed and then assembled into global matrix. In contrast to FEM, however, the resulting global matrix is fully populated and not symmetric resulting in a greater effort for the solution. This has in the past been claimed as a major drawback of the method. With the advancement in computer hardware in the last decade, however, this does no longer apply. For example, vector-processing super computer is more efficient in solving a fully populated matrix because the vector length can be made large and the book-keeping is dispensed with (eg. CDC Cyber 205 computer) [11,12].

Recognizing the advantages and disadvantages of the two methods (FEM and BEM), many researchers have combined the two methods, that is, coupled FEM/BEM method in which for a certain region (usually close to an opening or some other features of interest) FE discretization is used, while for other far regions BE discretization is adopted. For example, in tunneling and mining, the stress state near the tunnel face is of importance and are modeled by FE while the elastic far region can be modeled by using infinite domain BE [13].

6. The Discrete Element Methods (DEM)

The element methods described so far (LEM, FEM, FED, BEM) are founded on the assumption of continuity as the basis for discetization of many systems. Thus in conventional analysis of a continuum body using these methods, a mesh of elements is constructed which are interconnected at the nodes and maintain displacement compatibility along the inter-element boundaries. The system of equations is written for the entire assemblage of elements, including the constraints on the system. These have proved extremely successful analysis tools for engineers and scientists.

However, there are many situations where continuity cannot be assumed and other solution techniques must be utilized. For example, it has been shown that in situations where the effects of joints, contacts and/or fissures, etc are significant the response cannot be represented accurately by continuum techniques like FEM and BEM. This is the basis for the development of discrete element method (DEM) to handle such cases. While this term (DEM) was originally applied in the 1960s to the FEM, it is used here to describe those techniques whose basic assumption is that of discontinuity between bodies and whose emphases is on the solution of contact and impact between multiple bodies. DEM has evolved from various disciplines such as physics of particles, geomechanics, rock mechanics, ice and river mechanics etc. The theoretical background for the method is derived from the fields of aeroelasticity but other fields such as molecular dynamics, multibody dynamics and computer animation use methods which share much in common, with DEM.

Early applications of DE work continued these dynamic applications where the method solves the dynamic equilibrium equation for each body subject to body and boundary interaction forces. Like FEM, DEM is completely general in its ability to handle a wide range of material behavior (complex constitutive laws), interaction laws and arbitrary geometries. Highly dynamic effects are simulated including stress wave propagation, vibration and damping.

The benefits of the technique make it particularly useful for analyzing discontinua such as rocks where many variations of DEM exist like rigid block method, distinct element method, rigid block spring method and discontinuous deformation analysis model. Investigation of rock fragmentation and cratering using DEM codes like BLOCK, BUMP, CAROM, or DMC has been demonstrated [13]. DEM is ideally suited to analyzing rock joints behavior since complex joint models are readily accommodated. DEM is also used in ice mechanics especially in the behavior of ice in the Arctic region for shove oil exploration and with respect to submarine and missile penetration. DEM is applied to problems in tunneling, mining, construction of dams especially mechanical behavior of jointed rock masses. Although these analyses are possible by FEM using interface finite elements and by BEM using infinite domain BE, solutions are easier with DEM.

DEM is now being applied to such diverse areas as automobile impact, analysis of composites, buckling, machine vibration, weapon effects and animation, numerical analysis of deformable bodies. The method has also been applied to fluid flow through fractures and heat conduction through jointed medium.

Since in dynamic application the equation of dynamic equilibrium for each body are repeatably solved till the laws of contact and boundary conditions are satisfied, computation time required to solve even simple problems can be excessive.

While DEM is presented as being distinct from FEM, it has been shown that the underlying basis can be cast into familiar FE form. Indeed, a standard FEM code can be embedded in a DEM package so that continuum deformations are handled by FEM, while the contact and body interactions are handled using DE techniques. This is called coupled FE/DE methods. Also the graphic capabilities of DE and FE codes can often provide a useful tool for generating a detailed visual representation of the solid mass.

7. Conclusion and Future Outlook

Five element methods have been presented of which FEM have acquired prominent position in the analysis of engineering structures. With computers becoming cheaper and faster, direct application of element methods in industry will also increase.

Recognizing the advantages and disadvantages of each element method, it is best to combine FEM and one other method in a "marriage a la mode" where the advantages of each coupled methods are retained – assuring a mutually beneficial techniques.

The future trend of element techniques in engineering analysis may be summarized as follows-

- More emphasis on three dimensional (3D) analysis.
- Less man-hour spent in inputting data for analysis because the CAD sytems and preprocessors which are now available take pain out of specifying the mesh and drastically reduce the time spent in defining and inputting the mesh.
- Greater flexibility in presenting results of analyses because with user friendly post-processors and the display of results in colours, the element modeling has become a routine tool for engineers and scientists.

Infact using these pre-and post-processors the likelihood of errors is greatly reduced and the analyses are becoming increasingly error-free at first time they are submitted, thus making element modeling to be a fun!. However, the most important danger in these developments involves the educational gap between the researchers who develop the element techniques and computer codes and the engineers who will use them.

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