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COMPARISON OF EULER-LAGRANGIAN AND FISCHER'S METHODS OF PREDICTING DISPERSION COEFFICIENT

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Abstract

This paper is aimed at comparing the prediction of dispersion coefficient of a natural stream using a new Euler-Lagrangian model, Fischer's models and Levenspiel and smith equation. In order to achieve this, stream data were extracted from "American Stream Tracer Analysis on Humboldt River" and applied to the models. Results showed that slightly higher values of dispersion coefficients were obtained using the Euler-Lagrangian model than Fischer's. The percentage difference ranges from 4.26% to 16.59%. The Levenspiel and smith approach predicted values 1 to 3 times larger than the new method. In addition to the available data from literature, limited field data collected from River Ebere in Enugu State, Nigeria were used in the analysis. The new approach is less labourious, more time saving and more economical.

Keywords: dispersion coefficient, euler-lagrangian method, analysis, comparison

1. Introduction

Mixing and dispersion phenomenon that occur as natural processes in river system help to reduce the local pollution level considerably by distributing the dissolved substances gradually with time [1, 2]. The dispersion process is important in water quality management and pollution control and determines the capacity of a stream to assimilate contaminants [3, 4]. If the capacity of a stream to assimilate contaminants is over-estimated, serious pollution can occur. Underestimating can lead to under-utilization of the stream. This would involve more expenditures in treatment facilities. Adequate prediction of waste concentration downstream from a waste discharge position enables the engineer to design more rationally the outflow [1]. Dispersion studies are also very relevant in the determination of re-aeration capacity of streams. The extent of dispersion is quantified by the dispersion coefficient (D) or its dimensionless number, dispersion number which is the inverse of the pellet number which has been used widely in chemical reactor engineering [5].

Several dispersion models have been proposed. The earlier ones include those proposed by Taylor [6], Elder [7], Fischer [8], McQuivey and Keefer [9], Ojiako [3], and Agunwamba [10]. An improved expression for the transverse mixing coefficient equation and the direct integration of Fischer's triple integral are employed to determine a new theoretical equation for the longitudinal dispersion coefficient. By comparing with 73 sets of field data and the equations proposed by other investigators, it was shown that the derived equation containing the improved transverse mixing coefficient predicts the longitudinal dispersion coefficient of natural rivers more accurately [11].

Deng et al. [12] proposed a method using 70 sets of field data collected from 30 streams in the United States ranging from straight man-made canals to sinuous natural rivers. The new method predicts longitudinal dispersion coefficient. More than 90% calculated values range from 0.5 to 2 times the observed values. Won and Cheong [13] used empirical equations to compute the dispersion coefficient analytically in order to predict dispersion characteristics in natural streams. They reported a comparative analysis of previous theoretical and empirical equations.

Dispersion number is usually determined experimentally by tracer studies [14, 15]. This involves tracer injection by using such tracer materials as chemical salts at the upstream and then sampling at the downstream until the concentration reduces to an insignificant level. This method is based on the one-dimensional equation of diffusion and on variable-time, constant-distance method of sampling. There is usually marked disparity between measured and predicted values of δ [17, 18]. None of the new models has proved very useful in bridging this disparity between experimental and predicted values of δ [15]. A new Euler-Lagrangian approach was proposed by Agunwamba [10]. Unlike the existing method which is based on fixed point continuous monitoring until the tracer concentration has been reduced to negligible concentration, the new approach assumes that sampling takes place at various equidistant point along the pond or channel simultaneously or at variable times. The new approach was illustrated with laboratory data and gave results 0.9 to 3.3 higher than those obtained with the conventional Levenspiel and Smith fixed point continuous monitoring system. The advantage over the existing method include savings in time, economy, convenience, flexibility and reduction in sampling times [10]. However, this approach has not been applied in the study of dispersion in river. Therefore, the research objective is to use the Euler-Lagrangian approach [19] of sampling at constant time-variable distance to compare Fischers approach of evaluating dispersion coefficient (D) and dispersion number (δ). The comparison will be based on data collected from literature as well as from field experiments conducted on Ebere River in Nigeria. Comparism could not be extended to other models because of lack of access to the relevant publications.

2. Methodology

2.1. Study area/source of data

River Ebere is located in Nrobu community in Uzo-Uwani Local Government Area, Enugu State. The river originated from the hilly boundary terrain between Nrobu and Edem-Ani in Nsukka and flows very fast from the valley of this hill through the forest savanna towards Nrobu. The river meanders through the community and has different tributaries which flow across Nrobu to a distant town called Abi which is about 2.5 km from Nrobu. Finally, the river joins with other rivers at Adani, further from Abi. The river water is used for drinking, cooking and washing.

The population of the area is about 7000 inhabitants and since the river flows through accessible areas in the community, it is used for drinking, cooking and washing.

2.2. Experimental plan

The experiment was carried out along the river channels at safe locations. Ten stations spaced approximately 200m apart were established at different points along the river.

2.3. Field measurements and sampling

The dimensions of the river were obtained with a calibrated rod and a tape. The stopwatch and a surface float (cork) were used to determine the flow velocity. The bank sample was collected before the injection of 50kg of NaCl tracer at a point upstream the river. The method of injection was based on dissolving the salt first in a bucket and then pouring it inside the river at station 1. The First sampling was conducted for three days. The second sample scheme lasted for four days at the same stations and time interval.

2.4. Laboratory analysis and computation

The samples were taken to the laboratory and 50ml each of the sample was titrated with standardized silver nitrate solution according to the procedures specified in the standard methods [20]. The methods used for the computation of the dispersion number (δ) were the Levenspiel and Smith [5] approach and the one proposed by Agunwamba [10]. According to Levenspiel and Smith (1957).

$$\delta = \frac{1}{8}\left[\sqrt{8\sigma^2+1}-1\right] \tag{1}$$

Where σ^2 = normalized variance given by

$$\sigma^2 = \frac{1}{\overline{\theta}^2}\left[\frac{\sum^k c_i t_i^2}{\sum c_i} - \overline{\theta}^2\right] \tag{2}$$

t_i = time after injection of tracer (seconds); c = tracer response concentration at the exit stream (g/L); k = Number of samples; and $\overline{\theta}$ = The average flow time (Marecos do Monet and Mara, 1987) given by

$$\overline{\theta} = \left(\frac{\sum^k c_i t_i^2}{\sum c_i}\right) \tag{3}$$

In addition, data for the analysis of dispersion coefficient were extracted from stream data (Mckechnie, 1988) and include:

i. Date of injection ranges from 1st June–13th June 1987.
ii. Date of collection 1st June–16th June 1987
iii. Sampling Depth (0m–4m)
iv. Depths varies from 3m–7.5m)
v. River reach (7,300m)
vi. Channel widths varies from (106m–12.16m)
vii. Average velocity (0.078m/s and 0.29m/s)
viii. Average velocities of slices varies (0.008m/s–0.132m/s)
ix. Concentration of tracer materials
x. Energy slopes (1:3,000, 1:1,500, 1:1,200)

From the above data the discharge, transverse mixing coefficients, shear velocity and area of channel were determined. Other parameters calculated include the dispersion number and dispersion coefficient.

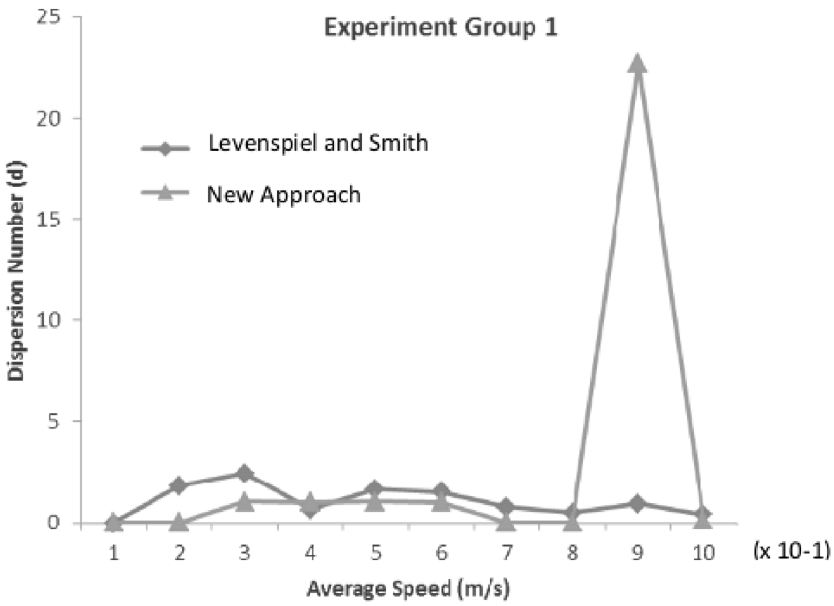

Figure 1: Variation of Dispersion Number with Average Speed for Experiment 1.

3. Data Analysis

The dispersion analysis is based on the Euler-Lagrangian and Fischer's models. Based on the moment method and Euler-Langrangian approach, Agunwamba [10] obtained the variance of C-x-t curve as:

$$\sigma^2 = \int_0^1 \left(\frac{2\delta}{1-\xi} + \frac{12\delta^2}{??} \right) dE - 4\delta^2 \left(\int_0^1 \frac{d\xi}{1-\xi} \right)^2 \tag{4}$$

where, δ = dispersion number; ξ = non-dimensional length ratio (Z/L); Z = relative distance between racer injection and collection; L = length of river reach. Equation (4) was further simplified by Agunwamba (1994) into

$$\sigma^2 = \left[\frac{\sum c\lambda^2}{\sum c} - \left(\frac{\sum c\lambda}{\sum c} \right)^2 \right] \left[\frac{\sum c}{\sum c^{-1}\lambda} \right]^2 \tag{5}$$

Where σ^2 = variance, λ = time-length constant and c = concentration of tracer. The variance of the data is computed in relation to dispersion number (δ) a dimensionless equivalent of dispersion coefficient (D).

$$\sigma^2 = \frac{\sum_{i=1}^{n} \left(\frac{\tau}{-\xi} \right)^2 c}{\sum_{i=1}^{n} c} - \frac{\left(\sum_{i=1}^{n} \frac{\tau}{1-\xi} c \right)^2}{\sum_{i=1}^{n} c} \tag{6}$$

$$\delta = \frac{1}{29} \left(\sqrt{1 + 15\sigma} - 1 \right) \tag{7}$$

where, $D = u\delta L$, D = Dispersion Coefficient; λ = time-length constant, L = length of river reach and u = mean velocity. Considering variation in the transverse direction, Fischer (1968) obtained

$$D = \frac{1}{A} \int_0^b q'(z)dz \int_0^z \frac{1}{Ezd(z)} \int_0^z q'(z)dz \tag{8}$$

In which

$$q'(z) = \int_o^{d(z)} u'(y,z)dz \tag{9}$$

where A = cross-sectional area, $u'(y,z)$ = velocity at any point on the cross-section relative to the mean velocity i.e $u'(y,z) = u(y,z) - u$, u = mean velocity.

$$E_z = 0.23 d_i u* \tag{10}$$

where, d = channel depth at the beginning of slice, $u*$ = shear velocity. Then

$$u* = \sqrt{gRS} \tag{11}$$

g = acceleration due to gravity, R = hydraulic radius, S = energy slope. For calculation, integrals are replaced by a summation

$$D = \frac{1}{A} \sum_{k=2}^{n} q'k\Delta z \left[\sum_{j=2}^{k} \frac{\Delta z}{E_{zj} d_j} \left(\sum_{i=1}^{j=1} q'_j \Delta z \right) \right] \tag{12}$$

where

$$q'_1 = \frac{1}{2} (d_i + d_{iH}) U'_1 \tag{13}$$

but $U'_1 = u_1 - u$ where, u_i = mean velocity in the ith vertical slice, u = mean velocity of flow with the x-section, Δz = width of a vertical slice.

4. Results and Discussion

4.1. Presentation of results

The results for the analysis of dispersion coefficients of natural streams using the new approach and Fischer's approaches are shown in Figures 1-4. The Levenspiel and Smith method gave results that have relatively higher dispersion number values than the new approach, except only in one station.

The reason for these differences in the two approaches cannot be explained for the time being. It may require further experimental work and studies to clearly determine and explain the order of the differences. However, the variation in dispersion number was more pronounced in the Levenspiel and Smith method.

$\sigma^2 = 0.358$, $\delta = 0.076$ and from equation (7b), $D = 0.078 \times 0.076 \times 7300 = 43.27$ m^2/s. The normalized dispersion coefficient $(D) = 0.078$m/s $\times$ 0.076 $\times$ (19485 - 17483)/15 = 0.792m^2/s.

$$\begin{aligned} D &= \frac{\text{Column (5)} \times \text{Column (9)}}{\text{Total area}} \\ &= \frac{4.0245\text{m}^3 \times 219,9519\text{m}}{8\text{m} \times 142\text{m}} \\ &= 0.7792\text{m}^2/\text{s} \end{aligned}$$

The discharge of a natural stream greatly influences Fischer's approach. Slice depths and slice mean velocities determine the discharge. The discussion for the analysis is based on the following parameters:

4.2. Retention time

Retention time (θ) is the ratio of channel length to the mean velocity. Retention times used are 25.997 hrs, 281.29 hrs and 69.92 hrs. Higher retention time results in lower dispersion number. There is direct use of retention time in Fischer's approach.

4.3. Geometry

Channel depth is an important parameter in dispersion studies. Velocities vary at different depths of a channel section. Generally, velocities are lower at the channel bed and the banks, thus at deeper channel depth the lower the dispersion coefficients for the two approaches. Bigger channel width results in higher dispersion coefficients. This is in accordance with several research findings in literature where dispersion decreases with increase in the length to width ratio (L/W) ratio. As L/W increases, plug flow condition is attained [19].

5. Implication of Results

In all cases, Agunwamba's model predicted slightly higher values of δ than Fischer's model. Higher values will result in higher dispersion of pollutants in streams. As the pollutants are dispersed, the stream has greater dilution effect and consequently, reduces its pollution effect on the aquatic living organisms. Hence, prediction of values δ higher than the actual values will result in ascribing higher assimilatory capacity to the stream than is actually the case. If the difference between actual and the predicted is high, this implies discharging more waste into the stream than it can assimilate which may jeopardize aquatic life. On the other hand, under estimation will result in under utilization of the assimilatory capacity of the stream. This will subsequently result additional expenditure due to higher treatment.

6. Conclusion

Slightly higher values of dispersion coefficient was evident in Agunwamba's approach than in Fischer's. This might be as a result of the introduction of a transverse mixing coefficient, which reflects shear velocity. Quantities like tracer concentration, travel time, channel reach were prominently applied in Agunwamba's model. Shear velocity which reflects gravitational pull, transverse mixing coefficients and channel slope were used in Fischer's model. Combinations of more similar quantities were evident in the two equations. However, Fischer's approach is less time consuming in calculation than Agunwamba's while the later requires less sampling time, labour and cost.

Finally, both models can be applied to predict the dispersion coefficient of a natural stream. While none of the two models appears superior in terms of accuracy, it may be envisaged that a more accurate approach will result when factors omitted in either are included.

Further research work is needed in using the two approaches to predict concentration–time curves so as to ascertain the predictive capacity of the two. This aspect could not be handled in the project because of incomplete data.

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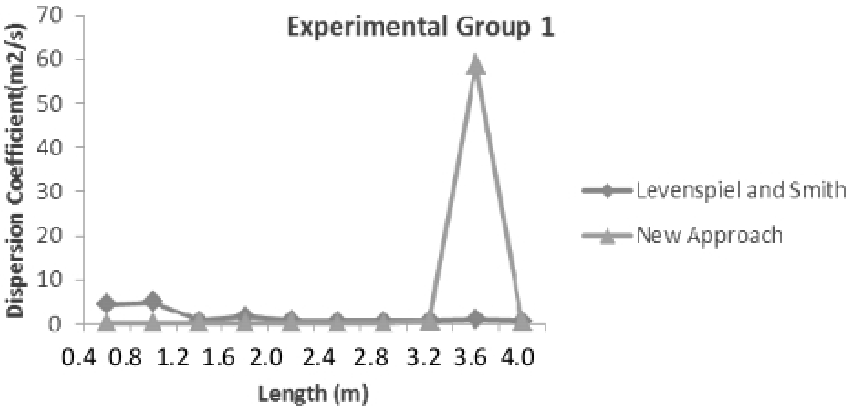

Figure 2: Variation of Dispersion Coefficient with Sampling Length for Experiment 1.

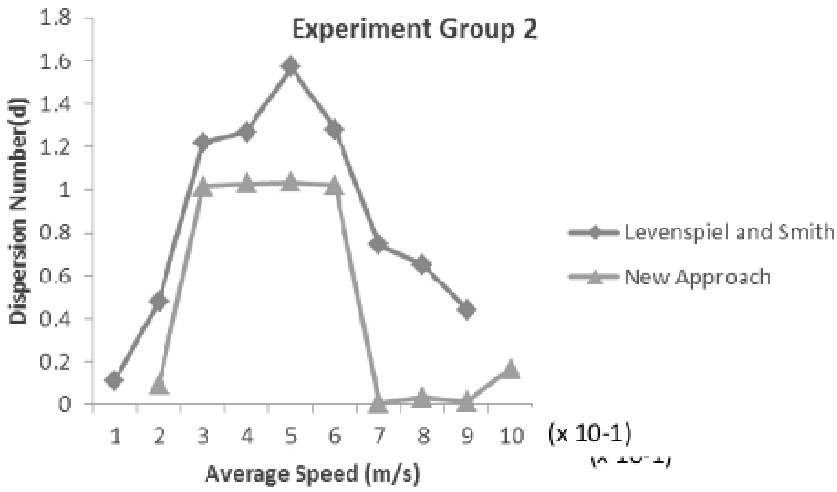

Figure 3: Variation of Dispersion Number with Average Speed for Experiment 2.

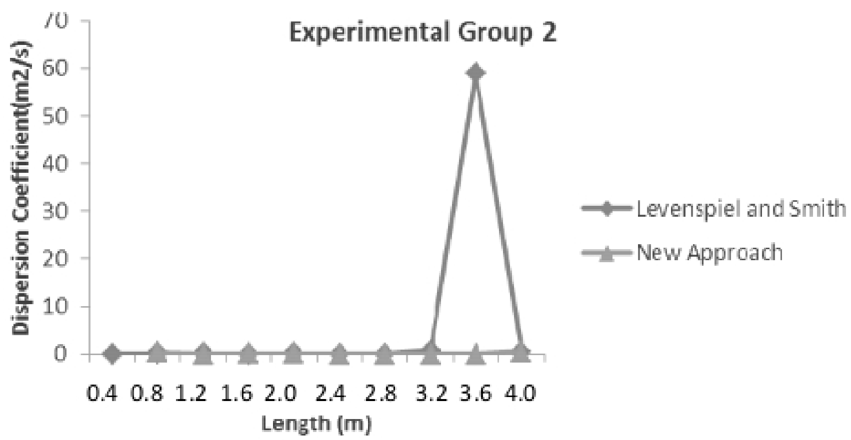

Figure 4: Variation of Dispersion Coefficient with Sampling Length for Experiment 2.

Table 1: Dispersion coefficient result for tracer injected on 1/6/87 and Collected on 1/6/87 at a sampling depth of 1m (θ = 25.997hrs), Reach = 7300m.

C	**t** (hrs)	**Z** (m)	**1 - E**	τ	$\lambda = \tau/(1\text{-E})$	λ = ???? (mg/1)	Cλ (mg/l)	Cλ^2 (mg/1)
0.742	5.78	2285	0.3130	0.222	0.709	0.526	0.503	0.373
0.494	5.67	2129	0.2916	0.218	0.748	0.370	0.560	0.277
0.510	5.50	1973	0.2703	0.213	0.788	0.402	0.621	0.317
0.64	5.37	1827	0.2503	0.207	0.827	0.496	0.683	0.430σ
1.107	5.22	1780	0.2438	0.201	0.824	0.912	0.679	0.752
0.911	5.08	1554	0.2129	0.195	0.916	0.834	0.839	0.764
1.005	4.98	1422	0.1948	0.192	0.986	0.991	0.972	0.977
0.916	4.83	1291	0.1768	0.186	1.052	0.964	1.107	1.014
0.122	4.65	1148	0.1573	0.179	1.138	0.139	1.293	0.158
1.906	4.40	1006	0.1378	0.169	1.226	2.337	1.503	2.865
1.190	4.20	875	0.1199	0.162	1.351	1.608	1.825	2.172
1.384	3.02	745	0.1018	0.155	1.523	2.108	2.320	3.211
0.524	3.87	590	0.0808	0.149	1.844	0.966	3.4	1.782
1.473	3.75	426	0.0584	0.144	2.466	3.63	6.08	8.956
1.532	3.58	283	0.0388	0.138	3.557	5.450	12.65	19.38
14.446						21 .733	43.43	

Table 2: Determination of dispersion coefficient using Fishers method mean velocity = 0.078m/s, channel width = 142m, channel slope = 1:1,200, width of slice (Δz) = 7.47m.

Slice No	Distance from left bank to start of slice (m)	Slice depth (m)	Mean velocity in each slice (m/s)	Discharge through slice w.r.t mean velocity $q_1\Delta z$	Cumulative relative discharge $\sum q_1\Delta z$ (m^3/s)	(7) $\frac{\Delta z}{\sum z_j d_j}$ (s/m^2)	(8) $\sum_{i-1}^{j-1} q_1\Delta z$ (m^3/s)	(7) x (8) (m)
1	0	3.78	0.022	-1.7318	-1.73184	8.8883	0	-10.8614
2	7.47	4.5	0.044	-1.3333	-3.06523	6.2715	-1.731	-10.8135
3	14.94	6	0.044	-1.6191	-4.68436	3.5277	-3.065	-13.0571
4	22.41	6.75	0.066	-0.6386	-5.32304	2.7873	-4.684	-12.0182
5	29.88	7.5	0.066	-0.6723	-5.99534	2.2577	-5.323	-13.5361
6	37.35	7.5	0.088	0.5229	-5.47244	2.2577	-5.995	-16.4497
7	44.82	6.5	0.088	0.4855	-4.98689	3.00591	-5.472	-14.9902
8	52.29	6.5	0.11	1.5537	-3.43313	3.00591	-4.986	-10.3197
9	59.76	6.5	0.132	2.3597	-1.07336	3.00591	-3.433	-5.04131
10	67.23	5.2	0.132	2.0975	1.02421	4.69673	-1.073	4.810453
11	74.7	5.2	0.132	2.0975	3.12178	4.69673	1.024	14.66222
12	82.17	5.2	0.132	2.1076	5.22944	4.69673	3.121	24.09574
13	89.64	5.25	0.11	1.2549	6.48440	4.60770	5.229	29.18789
14	97.11	5.25	0.088	0.4295	6.91393	4.60770	6.484	
15	104.58	6.25	0.088	0.4668	7.38080	3.25119	6.913	
16	112.05	6.25	0.066	-0.560	6.82055	3.25119	7.380	23.9964
17	119.52	6.25	0.066	-0. 4414	6.37908	3.25119	6.820	22.17496
18	126.99	3.6	0.044	-0.9016	5.47745	9.79936	6.379	62.51095
19	134.46	3.5	0.044	-0.8254	4.65201	10.3673	5.477	56.78655
20	141.93	3	0.022	-0.6274	4.02453	14.1110	4.652	65.64502
				4.0245				219.9519

Table 3: Summary of the predicted dispersion coefficient of the models.

Date of Injection/ sampling	θ (hrs)	Depth of sampling (m)	U (m/s)	δ	Agunwamba		Fischer D (m^2/s)
					unormalized D (m^2/s)	normalized D (m^2/s)	
1/6/87 1/6/87	25.997	1	0.078	0.076	43.27	0.792	0.7792
1/6/87 3/6/87	25.997	3	0.078	0.328	186.76	3.62	3.42
7/7/87 7/7/87	25.997	1	0.078	0.081	46.12	0.467	0.43
7/7/87 8/7/87	25.997	3	0.078	0.116	66.05	1.29	1.076
6/5/87 7/6/87	281.92	1	0.078	0.158	89.97	5.43	5.061
6/5/87 8/6/87	281.92	1	0.078	0.13	74.022	3.11	2.927
13/7/87 13/7/87	69.92	0	0.029	0.029	6.14	0.13	0.07556
13/7/87 14/7/87	69.92	1	0.029	0.469	99.19	6.09	5.7
13/7/87 15/7/87	69.92	1	0.029	0.15	31.76	1.23	1.178
13/7/87 16/7/87	69.92	3	0.029	0.96	203.23	8.27	7.89282

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