

**FATIGUE STRENGTH OF REINFORCED CONCRETE FLEXURAL MEMBERS**

by

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**ABSTRACT**

It is well known that reinforced concrete flexural members subjected to cyclic loads behave differently compared with static bending and can collapse due to the fatigue of concrete, reinforcement or both when maximum fatigue stresses of concrete and steel are well below the corresponding static strengths. But up till now there are only very few methods of fatigue and serviceability limit calculations. Besides, even existing methods in many particular cases cannot be accepted because of poor agreements with experimental observation. Meanwhile with the rapid increasing interest in the use of reinforced concrete in various structural elements subjected to severe fatigue loadings the possibility to predict safe behaviour of these structures is becoming more and more vital.

**1. INTRODUCTION**

To investigate behaviour of the flexural members under cyclic loads and to elaborate a method of analytical evaluations of fatigue strength singly reinforced concrete beams, reinforced concrete members under axial tension and axially compressed concrete prisms were subjected to sinusoidally varying loadings at 420 and 550 cycles per minute. Beams were loaded by two concentrated loads applied at third points of the beam span. Longitudinal reinforcement of the beam consisted of 2 ribbed bars of diameter 16mm (percentage of reinforcement = 2) having yield stress 470 N/mm, and modulus of elasticity  $1.98 \times 10^5 \text{ N/mm}^2$ . Shear reinforcement provided excluded the possibility of the collapse of the beam due to shear. Actual characteristic strength of concrete at the beginning of testing (8 months after casting) was  $44/\text{mm}^2$ . Initial stresses in reinforcement of beams and tensile members were kept the same.

Maximum stresses of concrete in compressed zone of beams and axially compressed concrete prisms were also maintained at equal levels. The maximum cycles load varied from 30 to 85% of the short term static crushing load, The initial asymmetry coefficient ( $\zeta = f_{\min}/f_{\max}$ ) varied from 0.00 to 0.700.

Deformations of reinforcement and concrete and deflections of beams were measured directly during pulsating loadings and periodically during static loadings after certain numbers of load cycles

**2. BRIEF SUMMARY OF TEST RESULTS AND COMMENT**

In beam the increment of deformations of compressed concrete due to cyclic creep of concrete after  $2 \times 10^6$  cycles of loading or before the collapse (in the cases when failure occurs) was within the range of 15-47 per cent deformation in tensile reinforcement increase with number of load cycles and before the beam collapses (or after  $2 \times 10^6$  cycles) they were 18-49 per cent higher than the initial deformations at the first static cycle of loading. Deflections were 1.3-2.2 times more than those at the first cycle of loading. The increment of vertical cracks widths was between 33 and 185 per cent. The heights of the vertical cracks in comparison with the initial heights were increased by 25-65 per cent. At the same time, reductions in the depth of the compressed zone of concrete were between 4 and 11.7 per cent only. It means that the positions of the neutral axes were practically not changed and

the increments in the crack height were mostly because of the reduction in height of tensile concrete between the neutral axis and the peak of the vertical Crack. Cyclic loadings did not cause new vertical cracks within the beam length of pure bending, but new inclined and vertical cracks appeared outside the pure bending zone, between supports and points of load application. The major parts of increment of deformations, deflections and rack width took place during the first 50,000-500,000 cycles of loading and after that the rate of increment was very small. Because of bigger stress amplitude; cyclic loadings with lower asymmetry coefficients create more severe conditions and increments-in deformations were more rapid. The changes in the beam conditions due to cyclic loading were more considerable under low and medium load levels, than under high load levels.

It was found that the stresses asymmetry coefficient does not correspond to the load asymmetry coefficient (which was maintained constant) because of stress changes while cyclic loadings. Due to the effects of cyclic creep of concrete in compressed zone and to some extent due to the bond deterioration between steel and concrete and racks development, the absolute values of maximum and minimum cycle's stresses of reinforcement increase and this leads to the increment of the stress asymmetry coefficient ( $\zeta$ ). Generally with regard to  $\zeta$  it creates more preferable conditions for steel, but at the same time the absolute value of the cycle's stress is growing up and the difference between this gradually increasing level and the fatigue limit is reducing and subsequently failure occurs. For this reason the fatigue limit of the-axially tensed members (where  $\zeta$  and level of maximum stress were constant during loading) was higher than that of the flexural members. On the contrary the fatigue limits of the compressed zone of concrete in the beams were higher than that of the axially loaded concrete. This is because of unequal stress distribution, within the depth of the concrete compressed zone. Actually the concrete compressed zone

is loaded eccentrically and due to this its fatigue limit is higher than that of axially loaded prism.

Beams with percentages of longitudinal reinforcement usual for static loads including balanced designed sections collapsed due to reinforcement collapse. It means that percentage of reinforcement for beams subjected to cyclic loads should be higher than for beams resisting static loads.

A substantial increment in deflections and crack width has been found. This increment was due to the progressively accumulated cyclic creep of compressed concrete, gradual deterioration of bond between reinforcement and concrete and due to the steel relaxation. Using the experimental values of maximum and minimum stresses in steel reinforcement, parameters of external loading, elastic and plastic deformations of concrete compressed zone width, deflections, crack and number of cycles, attempts were made to calculate thereal stresses of the steel reinforcement after cyclic loading and stresses in the compressed concrete. More detailed test results are presented in [1,2]. Analysis of practical observations and experimental data lead to the analytical method of calculations elaborated

**3. FATIGUE STRENGTH CALCULATIONS**

It was found that for calculations instead of the constant value of  $f_{pr}$  ( $f_{pr}$  is the concrete prism strength;  $p$  which are usually taken as  $0.7f_{cu}$ ) it is more reasonable to use variable value of the compressed concrete zone strength for flexural members,  $f_b$ , which depends upon the percentage of longitudinal reinforcement and depth of the concrete compressed zone.

Limit strength of concrete in the compressed zone should be determined from balanced section design when actual stress in tensile reinforcement  $f_r$  is equal to the steel yield stress  $f_y$  as

$$f_b = \frac{A_s^{max} \times f_b}{X \times b} \tag{1}$$

Where  $A_s^{max}$  is the cross - sectional

area of longitudinal reinforcement required for the balanced design section, X is the depth of the concrete compressed zone and b is the width of the beam.

In fact the relationship between the level of loading

$$\gamma = M_{max} / M_u$$

( $M_{max}$  is the maximum bending moment of cyclic load and  $M_u$  is the ultimate bending moment corresponding to the static collapse of the beam) and stresses in compressed concrete is a curvilinear one, but for practical purposes it can be represented by two linear functions. These functions will meet at the load level of  $\gamma_0$ .

$$\gamma_0 = M_o / M_u \quad (2)$$

$$\text{Where } M_o = \frac{1.75 f_{cr}^0 \times I}{S_c} \quad (3)$$

$f_{cr}^0$  is the bottom conventional level of the concrete micro cracking which can be calculated from the equation elaborated by Berg [3]

$$f_{cr}^0 = f_{pr} (0.351g f_{pr} - 0.5) \quad (4)$$

and I is the moment of inertia of the beam cross-section with concrete in the tension zone neglected; and X is the concrete compressed zone depth, which can be determined from the equation  $S_c = n S_r$  ( $s_c$  is a statically moment of the concrete compressed zone,  $n = E_r / E_c$  is the ratio between modulus of elasticity of reinforcement and concrete, and  $S_r$  is the static moment of the tensile reinforcement).

$$\text{If } \gamma = \frac{M_{max}}{M_u} \text{ is less than } \gamma_0 = \frac{M_o}{M_u}$$

the calculation stress of the concrete compressed zone can be determined from the equation:

$$f_c = \frac{\gamma}{\gamma_0} 1.75 f_{cr}^0 \quad (5)$$

If  $\gamma > \gamma_0$

$$f_c = 1.75 f_{cr}^0 + (\gamma - \gamma_0) \frac{f_b - 1.75 f_{cr}^0}{1 - \gamma_0} \quad (6)$$

Due to repeating load the shape of the

concrete compressed zone stress diagram can be represented as a trapezium with maximum stress of f and different parts of the depth of the compressed concrete Zone  $R^*$  having maximum stress  $f_c$ .  $R^*$  is increasing with number of cycles and can be calculated from the following equation:

$$R^* = m\gamma \left(1 + \frac{1-\gamma}{\xi + \mu/\mu}\right) \times (7)$$

Where m corresponds to the static collapse of the beam with the given  $\mu$  and is equal to

$$\frac{2 f_y A_s}{f_b \times b \times X} = 1 \quad m \quad (8)$$

$\mu$  and  $\mu_{max}$  is the real and balanced static design section percentages of the longitudinal reinforcement of the beam; and  $\xi$  is the asymmetry load coefficient. Experimental observations show that the depth of the concrete compressed zone has a tendency to decrease while cyclic loading but it can be taken as constant and equal to its value during the first static loading.

Having  $f_c$ , x, and  $R^*$  maximum stresses in the tensile reinforcement  $f_r^*$  before fatigue collapse of after  $2 \times 10^6$  cycles of loading can be calculated

$$f_r^* = \frac{f_c \cdot X \cdot b \cdot X (X \cdot R^*)}{2 A_s}$$

Comparing calculated  $f_r^*$  and  $f_c$  with corresponding fatigue limits of axially tensed reinforcement and axially compressed concrete the possibility of the beam resistance to the given cyclic load can be determined. The experimental data [4, 5, 6, 7] were used to check the validity of this method. Comparison between the experimental data of different researchers and analytical results obtained by using the method of calculations suggested by authors is given in table 1. So, as we can see maximum deviations between experimental and analytically calculated stresses in reinforcement were +15.7% and - 14.6% which can be treated as satisfactory.

Table 1. Comparison of Stress in the Tensile Reinforcement

Beam overall depth (mm)	Beam Breadth (mm)	% of Longitudinal of reinforcement	F <sub>cu</sub> (N/mm <sup>2</sup> )	F <sub>y</sub> (N/mm <sup>2</sup> )	γ	ζ	Tensile Reinforcement Stress – f*r (N/mm <sup>2</sup> )		Differences (%)
							Experimental	Analytical	
MIKHAILOV AND SELUKOV'S INVESTIGATIONS [4]									
300	150	1.385	39.0	400.0	0.528	0.25	212.0	227.0	+7.08
300	150	1.380	39.0	400.0	0.472	0.25	195.0	201.0	+3.08
300	150	1.406	39.0	400.0	0.464	0.25	194.0	196.0	+1.03
SAMBOR'S INVESTIGATIONS [5]									
300	150	1.88	56.2	432.0	0.433	0.22	216.8	250.0	+15.13
300	150	1.88	56.2	432.2	0.763	0.52	385.4	380.0	-1.14
300	150	1.88	56.2	432.2	0.743	0.52	363.3	375.0	+3.22
KARPUKHIN'S INVESTIGATIONS									
180	120	1.17	20.0	370.0	0.700	0.33	295.0	267.5	-9.32
180	120	1.17	20.0	370.0	0.833	0.50	331.0	312.0	-5.74
180	120	1.17	20.0	370.0	0.850	0.70	370.0	316.0	-14.59
TEREKHOVA'S INVESTIGATIONS									
300	150	1.09	41.0	560.0	0.510	0.30	332.0	317.0	-4.52
300	150	1.09	60.7	560.0	0.466	0.30	267.0	292.0	+9.36
MUSTOV'S INVESTIGATIONS									
300	150	1.09	30.0	625.0	0.510	0.33	306.0	354.0	+15.68
AUTHORS' INVESTIGATIONS									
220	100	2.00	44.0	470.0	0.518	0.00	330.6	327.0	-1.09
220	100	2.00	44.0	470.0	0.548	0.18	344.0	315.0	-8.43
220	100	2.00	44.0	470.0	0.548	0.30	321.3	311.0	-3.21
220	100	2.00	44.0	470.0	0.548	0.60	291.0	290.0	
220	100	2.00	44.0	470.0	0.876	0.18	440.0	434.0	-1.36
220	100	2.00	44.0	470.0	0.876	0.13	454.5	435.0	-4.29
220	100	2.00	44.0	470.0	0.876	0.60	441.0	426.0	-3.40

**4. CONCLUSION**

The method of reinforced concrete flexural members fatigue limit calculations presented above is based on completely different concept from the one existing now. It can be regarded as more realistic and promoting because of consideration of the reinforcing steel stress increment with number of cyclic loadings due to plastic

deformations of compressed concrete, crack development and bond deterioration between steel and concrete. Also the important advantage of this method is that the results of fatigue limit calculations can be used for further deflections and crack width Calculations. This means that ultimate and serviceability limit

states can be considered on the same basis.

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