FLEXURAL CRACKS DEVELOPMENT IN REINFORCED CONCRETE BEAMS UNDER PULSATING LOADS

BY

V. V. LEVCHICH
DEPARTMENT OF CIVIL ENGINEERING
FEDERAL POLYTECHNIC
IDAH
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ABSTRACT

Cyclic loadings have an exceptionally adverse influence on structures' behaviour as compared with static loading, and in order to meet limit states requirements, we have to analyses particular conditions and consider additional precautions. This work attempts to describe the stress-strain state of beams which is gradually changing with the number of load cycles applied and, especially, to analyses formation and development of cracks which greatly affect the whole behaviour of the beams. The method of assessment of maximum cracks' width giving good agreement with experimental data was found. The width of cracks is treated as closely related not only with appearance and corrosion of reinforcement but also with deflection and ultimate limit state of beams.

Due to the complex nature of this phenomenon the existing test results are very frequently contradicting each other, and up till now there are very few general recommendations on how to deal with this problem.

1. INTRODUCTION

The experiments reported here were undertaken in order to investigate behaviour of the flexural reinforced concrete member under cyclic loads of different parameters. Beams having an overall cross-section of 100 x 220mm and an effective span of 1800mm were made of concrete with characteristic strength of fcu = 44 N/mm^2 and were reinforced by two ribbed bars of diameter 16mm, giving a percentage of reinforcement = 2.0. (Fig.1).

Characteristic strength of steel f_y = 470N/mm^2, its modulus of elasticity Es = 1.98 x 10^{5}N/mm^2. Shear reinforcement provided was adequate to prevent shear failure (Fig. Ib) Beams were subjected to sinusoidally varying loading at 420 cycles per minute (Fig. 2), applied at the third points of the beam span as concentrated loads (Fig. 1a}, The maximum cycles load level (γ = maximum cycle moment divided by ultimate moment of resistance = Mmax/Mu) varied from 0.3 to 0.8 of the short term static ultimate moment of resistance. Maximum and minimum cycles loads and ultimate moment of resistance are shown in Fig. 2. The load asymmetry coefficient is the ration of the minimum cycle's moment (Fig. 2) to the maximum value of cycle's moment (ζ= Mmin/ Mmax) and was within the range 0.0 - 0.6

2. CRACKS DEVELOPMENT UNDER STATIC LOADS:

2.1 Cracks formation and their heights

This knowledge forms a basis on which cyclic load investigations may proceed. Before applying pulsating loads, all beams were loaded by static loads up to the level of the maximum load cycle (Mmax). Besides, to get basic data about beam behaviour under static load, four of them were investigated until rupture due to static loads

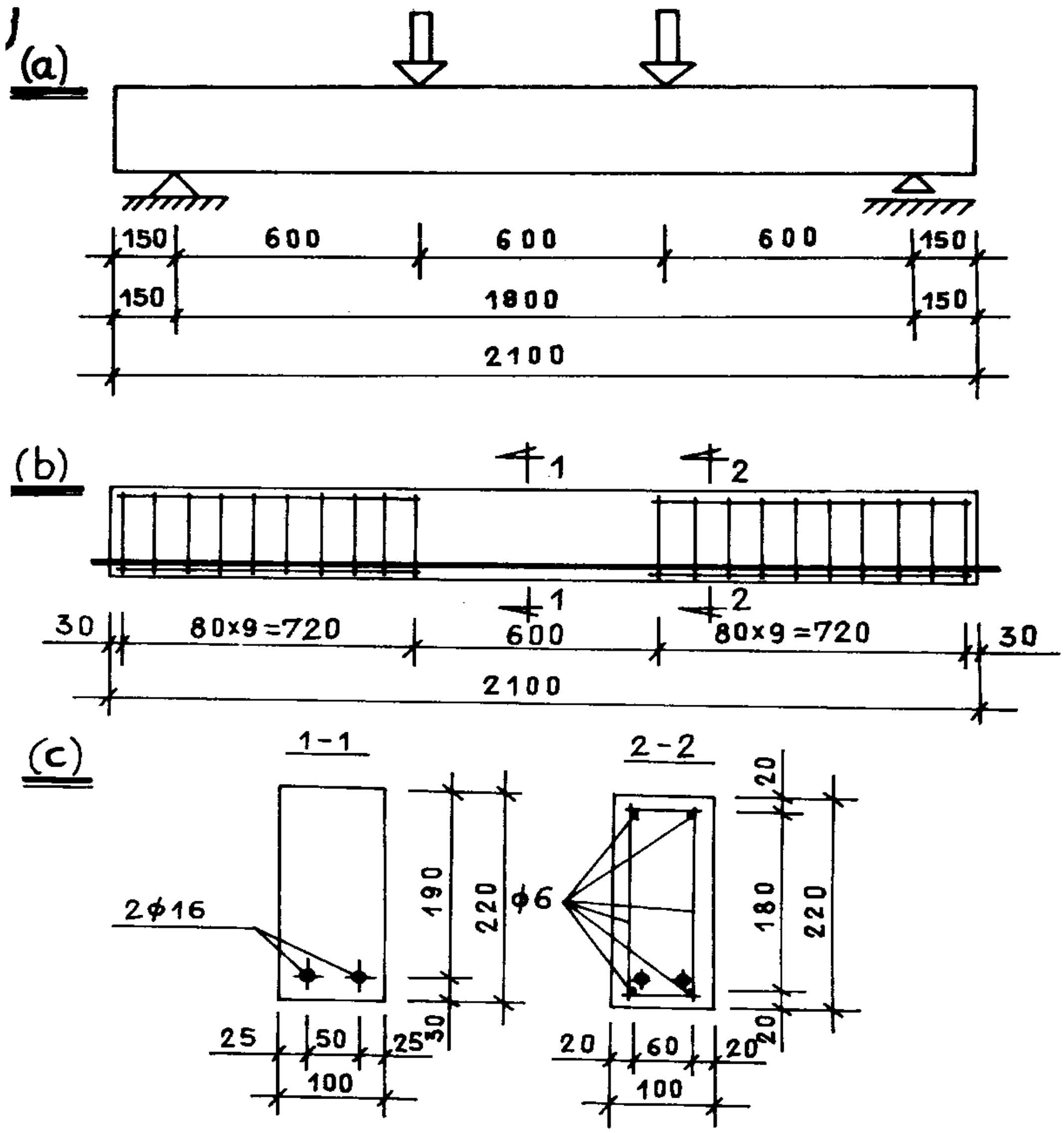

FiG.1. BEAMS INVESTIGATED. (a) STRUCTURAL SCHEME; (b) LONGITUDINAL SECTION; (c) LATERAL SECTIONS.

The first vertical cracks appeared under static bending moment of approximately 0.14 of the ultimate moment. With the increment of bending moment, the number of cracks and consequently the spacing between them reached nearly constant value and further load increment did not change them considerably. Formation of new cracks within zero shear zone practically stopped at the load level γ = 0.5 and crack spacing remained basically constant (Table 1). Up to the load level of γ = 0.5 crack heights were increasing rapidly, but at higher levels development was not so fast, and at γ= 0.8 practically there was no increment of crack height. Table 1, gives some information about cracks under static loads.

Table 1: Crack spacing and height under static loads. (Where 1, 2, 3 . . . crack numeration, **Mmax** - maximum level of cycle load, Mu - ultimate bending moment corresponding to rupture).

No	LOAD LEVEL $\frac{M_{max}}{Mu} = \gamma$	SPACING BETWEEN CRACKS (mm) REIGHT OF CRACKS (mm)													
		1-2	2-3	3-4	4-5	5-6	6-7	AVER	1	2	3	4	5	6	AVER
1	STAT	116.0	101.0	76.0	101.0	76.0	88.6	93.0							
2	STAT	139.0	57.0	69.5	63.0	38.0	76.0	73.8							
3	STAT	126.5	115.0	139.0	1330	126.5	-	128.2							
4	STAT	82.3	57.0	44.3	69.3	120.0	-	74.6							
5	0.50	98.0	175.0	91.0	121.0	-	-	122.0	55.0	90.0	95.0	82.0	76.0	—	79.6
6	0.50	149.0	53.0	1950	66.0	-	-	119.0	79.0	76.5	54.0	86.0	-	—	73.9
7	0.50	130.0	133.0	175.0	39.0	-	-	119.0	62.0	91.0	82.0	75.0	-		77.5
8	0.50	118.0	112.0	93.0	113.0	64.0	-	127.0	74.2	66.0	60.8	22.0	23.0	—	50.8
9	0.50	157.0	95.0	131.0	133.0	-	-	129.0	90.0	100.0	86.0	87.0	84-0	57.0	87.5
10	0.50	126.0	132.0	152.0	134.0		-	136.0	45.0	70.0	82.0	90.0	79.5	—	73.3
11	0.65	110.0	100.5	174.0	110.0	119.0	-	123.0	85.0	75.0	90.0	83.0	85.0	—	85.5
12	0.65	128.0	155.0	146.0	-	-	-	143.0	43.0	96.0	40.0	80.8	-	94.0	66.9
13	0.80	74.0	77.0	77.0	77.0	1180	140.0	96.0	76.5	94.0	78.5	90.0	104.0	—	87.5
14	0.80	118.0	132.0	130.0	97.0	-	-	119.0	87.0	113.0	68.0	76.0	75.0	80.0	83.8
15	0.80	165.0	92.0	134.0	96.0	99.0	-	118.0	82.0	95.0	97.5	98.0	67.5	—	89.0
16	0.80	18.0	84.0	189.0	176.0		-	132.0	-	82.5	104.5	103.0	108.0	86.0	99.5
17	0.80	142.0	90.0	123.0	-	-	-	118.0	96.0	94..5	85.7	105.0	—	—	95.5
18	0.80	112.0	160.0	129.0	113.0	-	-	128.0	59.0	98.5	112.5	98.0	83.5	—	97.5
19	0.80	96.0	183.0	80.0	70.0	145.0	-	117.0	82.0	90.0	94.5	70.0	85.0	—	85.0
20	0.80	104.0	112.0	106.0	88.0	120.0	-	106.0	99.0	95.0	102.0	94.0	93.0	—	96.6

NOTE: The investigations for the first four beams were purely static until collapse occurred. The width of the cracks were therefore not measured at all levels. For the other beams, results were taken at the maximum load levels of the cycles loaded statically up to the maximum levels of the cycles initially before cyclic loadings.

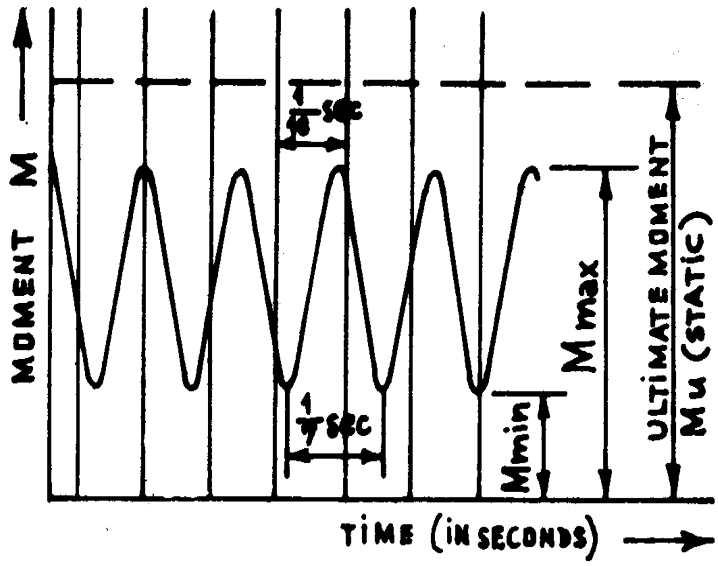

FIG.2. OSCILLOGRAPHIC DIAGRAM OF CYCLIC LOADINGS.

As it is seen from Table 1, the crack depths differ greatly even for one beam. When Level of loading is increasing the difference between crack depths tend to decrease, as cracks initially having smaller depth are developing faster. For example, for beams loaded up to $\gamma = 0.8$ the ratio between biggest and smallest cracks was 112.5:59.0 = 1.9. For $\gamma = 0.65$ it was 96.0:40.0 = 2.4, and for $\gamma = 0.5$ the ratio was 100.5:22.0 = 4.6.

Just the same phenomenon is typical for the spacing between adjacent cracks. The number of vertical cracks was increasing only up to the load level of $\gamma = 0.5$ and after that this process practically stopped. The crack spacing varied widely and the ratio between maximum and minimum spacings may be as big as 5.0.

2.2 Crack width

Experimental data are presented in table 2. As CP110:1972 method [1] is oriented to average crack width, the actually measured steel deformations at the crack sections were used to calculate maximum width of cracks. Crack widths may differ greatly and the maximum crack width can be several times bigger than the average value, so the limit state in terms of crack width should be related with the maximum crack width, which will be at the weakest or critical section. It should be noted that both methods employed (USSR CODES AND CP110:1972) underestimate the crack width at $\gamma = 0.8$, (Table 2) and additional modification factors are to be used. It should be noted that average deformations of steel reinforcement calculated by CP110:1972 method and those measured in experiments were fairly corresponding to each other. Hence, average crack width will be in a good agreement with experimental data. A comparison of these strains is given in table 3. But depending upon the speed of crack development these maximum crack heights were reached after different numbers of load cycles. For example beams loaded with $\gamma = 0.8$ sustained practically the same number of load cycles before collapse (for $\zeta = 0.0$, N= 413250 and for ζ = 0.6, N = 419387) because the rate of crack development did

Table 2: Experimental and analytical maximum crack widths under static loading

LOAD LEVEL	CRACK WDITH (mm)			
BENDING MOMENT (KNm)	$\gamma = \frac{mmax}{mu}$	EXPERIMENTAL	ANALYTICALLY CALCULATED BY:	
			USSR CODE OF PRACTICE	CP110:1972 USING EXPERIMENTAL STRATINS
9.9	0.30	0.040	0.055	0.043
16.5	0.50	0.081	0.084	0.082
21.5	0.65	0.115	0.126	0.106
24.6	0.80	0.171	0.160	0.135

TABLE 3: Experimental and analyica1 strains of reinforcement under static loads

BENDING MOMENT (KNm)	AVERAGE STRAIN OF TENSIL REREINFORCEMENT ($X10^{-6}$)		DIFFERENCE %
	EXPERIMENTAL ALUES	ANALYTICAL VALUES BY CPllO :1972	
9.9	520.0	536.07	+2.5
16.5	934.0	957.12	+2.5
21. 5	1242.4	1276.55	+2.7
26.4	1566.0	1588.00	+1.4

3. CYCLIC LOADING

3.1 Crack spacing and height of penetration

As a rule, cyclic loadings did not load to the creation of new vertical cracks within the zero shear zero of beams. At the same time many vertical and inclined cracks appeared outside this zone.

Cyclic loads of all parameters considered caused the increment of height and width of all vertical cracks. The rate of development depends upon the level of loading (γ) and the coefficient of load asymmetry (ζ). Under γ = 8.5(s = 0.0; 0.18; 0.30; 0.60) height increment of cracks was growing with decrement of asymmetry coefficient "ζ;", but under γ = 0.8 it was practically independent upon "ζ".

It is of great interest that the maximum (limiting) height of cracks observed in beams loaded under γ=0.50; 0.65; and 0.80, which collapsed after number of load cycles less than two million was approximately the same, irrespective of values of "γ" and "ζ". not depend on asymmetry coefficient of loadings. Consequently, in these cases the maximum crack heights were reached irrespective of value of "ζ" and practically simultaneously. Beams loaded with γ = 0.5 had bigger rate of crack height development under lower asymmetry coefficients. Hence, beams with lower "ζ" collapsed after smaller number of load cycles. While increasing the asymmetry coefficient the number of load cycles before collapse is also increasing (for ζ= 0.0, N=135285 and for ζ = 0.3 N = 1850000), When beams were able to sustain more than two million cycles, the maximum value of crack height was not reached.

In spite of the fact that crack heights were increasing considerably the neutral axis depth in the same sections were changing less visibly, although under all load parameters the compressed concrete zone depth slightly decreased with load cycles increment. It was found that compression zone depth of concrete was decreasing mainly due to diminishing of the tensile concrete between the neutral axis and crack's peak. This phenomenon may explain the higher rate of crack development under lower levels of "γ" when the tensile concrete zone ever the crack peak is bigger than under the higher levels of "γ". Generally, crack heights were developing until the tensile concrete over the crack is not switched off, i.e. cracked.

Just before collapse the relative compressed concrete zone depth in beams with γ = 0.5 and γ = 0.8 for all levels of ζ were practically the same. The nature of crack height development indicates a possibility to assume that the limit state of flexural members within the ranges of "γ" and "ζ" causing collapse before 2 men load cycles is predetermined by the limiting compressed concrete zone depth, that before collapse will be of the same value irrespective of levels of "γ" and "ζ".

A typical example of crack height and compressed zone depth developments is given in table 4.

During the initial period of loading (up to 200 000 cycles) crack development was very fast but after that this process was considerably slower. Cracks which were less developed during first static loading were subject to more substantial development due to cyclic loading.

3.2 Crack width

Crack height development was accompanied by increment of their width, which equally is dependent on cyclic loading parameters. Some beams with γ = 0.3 and 0.5 had 200 - 250% increment of crack width. Depending upon the number of cyclic loadings the crack width development is presented in Table 5.

Table 4: Development of crack height, depth of tensile concrete over crack peak and compression concrete depth for the beam with γ = 0.5 and ζ = 0.0

Note: hcr. l. ht. l, Xl are initial crack height, tensile concrete depth over crack and compression concrete depth respectively, and hcr^*, ht^*, X^* - the same after certain number of load cycles.

NUMBER OF LOAD CYCLES (N)	$\overset{*}{hcr}$ (mm)	$\frac{hcr^{*}}{hcr.1}$	DEPTH OF TEMSILE CONCRETE OVER CRACK PEAK (mm)		COMPRESSION CONCRETE DEPTH - "X"(mm)	$X^{*}/x1$
			ht	$ht^{*}/ht.1$		
N=1 (Initial)	hcr.1 = 76.4	1.000	35.2	1.000	78.4	1.000
$0.02x10^{6}$	98.0	1.283	17.1	0.485	75.2	0.959
$0.02x10^{6}$	107.6	1.408.	10.4	0.295	71.2	0.908
$0.4x10^{6}$	109.0	1.426	8.6	0.244	70.4	0.897
$0.7x10^{6}$	112.0	1.466	7.2	0.205	70.3	0.896
$1.0x10^{6}$	114.0	1.492	5.6	0.159	69.8	0.890

Table 5: relative increment of crack width with load cycles numbers

Number of load cycles (N)	γ = 0.3 ζ=0.3	γ = 0.5 ζ=0.0	γ = 0.5 ζ=0.3	γ = 0.5 ζ =0.6	γ = 0.65 ζ=0.23	γ = 0.8 ζ=0.0	γ =0.8 ζ=0.18	γ = 0.8 ζ=0.03	γ=0.8 ζ =0.6
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20 000	1.50	1.30	1.17	1.05	1.27	1.28	1.17	1.11	1.27
50 000	1.75	1.47	1.33	1.14	1.36	1.39	1.33	1.22	1.33
100 00	1.88	1.98	1.55	1.29	1.46	1.50	1.50	1.45	1.33
200 000	1.88	2.16	1.78	1.43	1.55	1.55	1.66	1.55	1.40
400 000	2.03	2.33	1.80	1.52	1.55	-	-	-	-
700 000	2.15	2.42	1.87	1.57	1.63	-	-	-	-
$1X10^6$	2.25	2.50	1.91	1,64	-	-	-	-	-
$2X10^6$	2.25	-	1.94	1.67	-	-	-	-	-
$2x10^6$	2.25	-	-	1.72	-	-	-	-	-

Under constant load asymmetry coefficients with increment of the maximum load cycle level (γ), the rate of crack width development was diminishing. Under constant levels of load (γ) more rapid development of cracks was observed with lower load asymmetry coefficients (ζ). This is in good agreement with the development of crack height and indicates that these two processes have a common nature. The level of initial crack width due to the first static loading affects the level of their increment due to cyclic loadings.

3.3 Crack width calculations

As fatigue limits of steel and concrete and corresponding deformations and also the nature of their development depend on the load asymmetry coefficient, it is reasonable to guess that the crack width is dependent on "ζ". These experiments have proved it. In beams with γ= 0.5 after 1 x 10^6 cycles of loading under ζ = 0.6 the crack width increment was 64%, while under = 0.0 it was found to be 150%. It is important that degree of "ζ" influence depends on the load level "γ".

Generally, with γ increasing gradually, the influence of "ζ" is gradually dimishing and under γ = 0.8 the effect of "ζ" is very small. None of efficially recognized methods of crack width calculations takes into account the influence of load asymmetry coefficient "ζ". As in these experiments the relationship between width and ζ was clearly established it resulted in an attempt to evaluate this influence analytically for crack width calculations. It rurned out that basically the most suitable method for crack width computations is one recommended by USSR code of practice [2], but modified as for its content. The original equation recommended by USSR code of practice is:

$$acr = KCg\eta 20\frac{\sigma s}{E_s}\ (3.5 - 100\ \mu)^3\sqrt{\emptyset} \quad (1)$$

Where K = coefficient, taken for flexural members as 1.0,

Cg = Coefficient taking care of load

nature), which for cyclic loads is 1.5 irrespective with load asymmetry coefficients),

η = Coefficient dependent on type of reinforcement used. For separate deformed bars it is 1.0. σs and Es = stress and modulus of elasticity of steel respectively.

$\mu = \frac{AS}{bd}$ =longitudinal reinforcement coefficient (As = reinforcement sectional area; b = breadth;

d = effective depth of the beam).

ϕ = reinforcement bar diameter in mm

Using an experimental data it was found possible to evaluate the rate of influence of "ζ" by the following expression:

$$Cg = 2 - 2\zeta + \zeta^2 \quad (2)$$

So, instead of constant value of "Cg" recommended by codes *[2]* the variable value of this parameter depending on load asymmetry coefficient calculated by equation (2) was found to be more reasonable.

The second amendment, to the code's expression is that the stress in tensile reinforcement should be calculated after 2 million loadings or just before collapse in cases when beams did not sustain 2 million cycles. While calculating stress in reinforcement after cyclic load applications the level of loading and pulsating nature of this loading are taken into account. The method of reinforcement stress calculations, which proved to be acceptable was suggested by the author and described in *[3]*, but any reliable method for these calculations may be employed. It should be worth repeating that the most crucial point in calculating is an evaluation of stress in reinforcement after cyclic loadings. Using the approach described above the maximum widths of cracks for all beams investigated were calculated. The comparison between experimental and analytical data is presented in Table 6. Bearing in mind that not so long ago the possibility of crack width prediction was not conclusive, the difference between experimental and analytical values found in this investigations should be treated as satisfactory. May be, if experimental data are more representative in cases γ= 0.3; ζ = 0.3 and γ = 0.8; ζ = 0.3 the differences could be even less.

4. CONCLUSION

Development of normal cracks in flexural members under cyclic loading is a continuing process depending upon the load parameters and number of cycles. Stage of crack development affects the behaviour of the structure as a whole. Crack height may be used for fatigue assessment of beams. For serviceability limit state the maximum width of vertical cracks is regarded to be more reasonable than their average values. The width of the cracks after cyclic loadings can be calculated with a satisfactory accuracy by equations given in this work.

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Table 6: Comparison of Experimental and analytical values of crack width due to cyclic loads.

LOAD-PARAMETERS γ	ζ	CRACK WIDTH (mm) EXPERIMENTAL INDIVIDUAL	AVERAGE UNDER ζ= CONST	ANALYTICAL	DIFFERENCE %
0.3	0.3	0.090	0.090	0.131	+45.5
0.5	0.0	0.270	0.210	0.250	+19.0
0.5	0.0	0.150			
0.5	0.18	0.170	0.170	0.188	+10.6
0.5	0.3	0.115	0.140	0.158	+12.8
0.5	0.3	0.165			
0.5	0.6	0.120	0.120	0.128	+6.7
0.65	0.23	0.195	0.188	0.224	+19.1
0.65	0.23	0.180			
0.80	0 .0	0.330	0.305	0.347	+13.8
0.80	0.0	0.280			
0.80	0.18	0.260	0.230	0.258	+12.2
0.80	0.18	0.200			
0.80	0.3	0.280	0.280	0.220	-21.4
0.80	0.3	0.280			
0.80	0.6	0.210	0.205	0.195	-4.9
0.80	0.6	0.200			

REFERENCES

1. CP110: part 1:1972. Code of practice for the structural use of concrete.

2. USSR's code of practice for Reinforced Concrete Structures, Moscow, 1976.

3. Kuryll0, A.S., Kwasha, V.G., Levchich, V.V., Fatigue strength of Reinforced Concrete Flexural Members. NIJOTECH, VOL. 4., NUMBER 1, MARCH 1980, pp. 65 - 69.