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RELIABILITY - BASED DESIGN OF REINFORCED CONCRETE TWO-WAY SOLID SLABS USING EUROCODE 2

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ABSTRACT
In this work, a FORTRAN-based computer program was developed to aid the design of reinforced concrete slabs to Eurocode 2 (EC 2)[1]design requirements at constant reliability levels using First Order Reliability Method (FORM).The design variables for the design of the slab were considered random with safety indices calculated using FORM. It was shown among other findings that the FORM design procedure gave more economical designs considering the area of steel required and final depth of section, than the EC 2 design procedure when a target safety level of 3.0 was considered. Also, example of design using the program was included for various values of target safety indices at constant geometrical, loading and end condition of the slab. The developed program is therefore suitable for application.

Keywords: *Design, reliability, two-way slabs, target safety index, Eurocode 2*

1. INTRODUCTION

A structure is only as strong as its 'weakest element', and so members with good measure of reliability do contribute significantly to overall structural reliability [2].

Despite what designers often think, the parameters of loading and load-carrying capacities of structural members are not deterministic quantities, that is, quantities which are perfectly known. They are random variables, and thus absolute safety (or zero probability of failure) cannot be achieved [3].

It is in the interest of the concrete industry and the engineering community to produce concrete structures that not only have an adequate margin of safety against collapse but also provide acceptable performance in service at minimum cost [4].

The presence of uncertainty in the analysis and design of engineering structures has always been recognized. However, traditional approach simplified the problem by considering the uncertain parameters to be deterministic, and accounted for the uncertainties through the use of empirical safety factors. Safety factors were derived based on the past experience but do not absolutely guarantees safety or satisfactory performance. They do not either provide any information on the influence of different parameters of design [5].

The reliability is commonly described as the probability or likelihood of structure performing its purpose adequately for a period of time intended under the operating conditions encountered [6].

A structural element will cease to perform when the nominal value of load exceeds its nominal load-carrying capacity. The reliability basis for designing a range of structures, structural materials, safety and functional requirements, and the actions, to which these facilities are exposed, form a vital framework for a coherent suite of structural design standards [7].

The probability-based design concept is not new [8]. However, improvement on the methodology was recommended [9; 10]. The work presented in this paper therefore proposes a reliability-based design program for two-way solid slabs to EC2. The program examines the uncertainties and consequences of failure of reinforced concrete two-way solid slabs designed to EC 2 [1] requirements, as well as safety index format for the design using FORM.

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2.0 DESIGN OF TWO-WAY SOLID SLABS TO EUROCODE 2

The purpose of structural analysis is the establishment of the distribution of internal forces and moments, or stresses strains and displacements, over the whole or part of a structure. Additional local analysis shall be carried out where necessary [1].In most normal cases, analysis will be used to establish the distribution of internal forces and moments; however, for certain complex elements, the methods of analysis used (e.g finite element analysis) give stresses, strains and displacement rather than internal forces and moments. Special methods are required to use these results to obtain appropriate reinforcement areas.

2.1 Restrained Slab Spanning in Two Directions

For slab with fixity at supports, the maximum moments per unit width are calculated using:

$$M_{sx} = \beta_{sx} n l_x^2 \text{ in direction of span } l_x \quad (1)$$

$$M_{sy} = \beta_{sy} n l_x^2 \text{ in direction of span } l_y \quad (2)$$

In (1) and (2), M_{Sx} and M_{sy} are the moments at mid span on strips of unit width spans in shorter span (l_x) and longer span (l_y) respectively, n is the total ultimate load per unit area, β_{sx} and β_{sy} are the respective moment coefficients in l_x and l_y directions respectively.

The area of reinforcement A_{sx} calculated as[1]:

$$A_{sx} = \frac{M_{sx}}{0.87 f_{yk} Z} \quad (3)$$

In equation (3), M_{sx}, f_{yk} and Z are the applied bending moment, characteristic strength of steel and lever arm of the reinforced concrete section respectively.

2.2 Shear in Slabs

Shear stresses in slabs subjected to uniformly distributed loads are generally small, shear reinforcement will seldom be required and it would be usual to design the slab such that the design ultimate shear force is less than the shear strength of the unreinforced section. This can conveniently be checked on the basis of the allowable shear stress in the unreinforced slab given by

$$v_{Ed,c} = \frac{V_{Rd,c}}{bd} \quad (4)$$

The ultimate shear stress is therefore given as:

$$v_{Ed} = \frac{V_{Ed}}{bd} \le v_{Rd,c} \quad (5)$$

In equations (4) and (5), $V_{Rd,c}$ is the concrete shear resistance of the section, $v_{Rd,c}$ is the allowable shear stress in the unreinforced slab, v_{Ed} isthe applied shear stress, V_{Ed} is the applied shear force on the section, while b and d are respectively the width and effective depth of the section.

2.3 Punching Shear in Slabs

Punching shear refers to a shearing stress caused by a concentrated load around a section of the load. A new model to analyse the punching shear of concrete slabs is to assume that cracks, which are the potential failure surfaces, are already developed and the slab resistance is concentrated along these failure surfaces [12]. The maximum force that can be carried by the slab without shear reinforcement can be obtained using the values of $V_{Rd,c}$, given by the code of practice. However, checks must be undertaken to ensure that the maximum permissible shear force (values of $V_{Rd,\max}$, given by equation 6) is not exceeded at face of the loaded area.

$$V_{Rd,\max} = 0.5 v_1 f_{cd} ud = 0.5 v_1 (f_{ck}/1.5) ud \quad (6)$$

In equation (6) u is the perimeter of the loaded area, d is the depth of slab, f_{ck} is the characteristic strength of concrete and v_1, is the strength reduction factor,

2.4 Limit state of Deflection

In design, excessive deflections of slabs will cause damage to the ceiling, floor finishes or other architectural finishes. To avoid this, limits are set on the span-depth ratio. EC 2 [1] specifies equations to calculate basic span-effective depth ratios to control deflections to a maximum of span/250 and these are however given in Table 7.4N of the code.

The limiting span-to-depth ratio may be calculated using Expressions (7.16a) and (7.16b) of the EC 2 [1], written respectively as equations (7) and (8):

$$\frac{L}{d} = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_o}{\rho} + 3.2\sqrt{f_{ck}}\left(\frac{\rho_o}{\rho} - 1\right)^{3/2}\right]; \quad if \quad \rho \le \rho_o \quad (7)$$

And,

$$\frac{L}{d} = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_o}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\frac{\rho'}{\rho_o}\right]; \quad if \quad \rho \ge \rho_o \quad (8)$$

In (7) and (8),

$$\rho_o = \sqrt{f_{ck}}\, 10^{-3} \quad (9)$$

In equations (7) to (9),L/d is the limiting span/depth ratio of the slab, K is the factor to take into account the different structural systems, ρ_o is the reference reinforcement ratio, ρ is the required tension reinforcement ratio at mid-span to resist the moment due to the design loads, ρ' is the required compression reinforcement at mid-span to resist the moment due to design loads and f_{ck} is the characteristic strength of concrete.Equations (7) to (9) were derived for rectangular sections of concrete class C30/35 and for grade 500 steel.

2.5 Limit state of Cracking

Members subjected to bending exhibit flexural cracks, and even at working loads. These cracks become excessive when the width becomes greater than the permissible value given in Table 7.1 of EC 2 [1]. The maximum acceptable value suggested by EC 2 for ordinary reinforced concrete is 0.3mm for all exposure classes under the action of *quasi-permanent* load combinations [11].

For reinforced slabs in buildings subjected to bending without significant axial tension, specific measures to control cracking are not necessary where the overall depth does not exceed 200 mm and the provisions of 9.3 of EC 2 have been applied.

2.6 Minimum Area of Reinforcement

Minimum areas of steel reinforcement must be provided to control crack. The provision of minimum area ensures that the steel reinforcement does not yield when the concrete in the tension zone cracks with a sudden transfer of stress to the reinforcement.

The area of reinforcement in primary direction (for 500 grade steel) should not be less than 0.13%bh for C25/30 concrete, 0.15%bh for C30/35 concrete, 0.18%bh for C40/50 concrete and 0.21%bh for C50/60 concrete.The area of secondary reinforcement may be taken as 20% of the main reinforcement.

2.7 Concrete Cover for Durability

Table 4.4N of EC 2 gives the minimum cover to meet the durability requirements for slabs and other structural members for the various exposure classes relating to the environmental conditions given in Table 4.1 of the EC 2. Nominal cover as defined in section 4.4 of EC 2 is the design depth of concrete cover to all reinforcement including links.

3. FIRST ORDER RELIABILITY METHOD

In order to investigate the effect of the variables on the performance of a structural system, a limit state equation in terms of the basic design variable is required. This limit state equation is referred to as the performance or state function and expressed as:

$$G(X_i) = G(X_1, X_2,...., X_n) = R - S, \quad (10)$$

where X_i, for i = 1,2,..., n, represent the basic design variables, R is the strength capacity and S the loading effect(s) of a structural system which are random variables. The limit state of the system can then be expressed as

$$G(X_i) = 0. \quad (11)$$

and in terms of these reduced variates the limit state equation becomes:

$$G(\sigma_{xi}X'_1 + \mu_{xi}, \sigma_{x2} X'_2 + \mu_{x2},..., \sigma_{xn}X'_n + \mu_{xn}) = 0$$
(12)

where μ and σ are the means and standard deviations of the design variables.

The safety index β associated with the minimum distance of the failure surface to the origin can be calculated using the method of invariant solution [13]. The reliability based equation (12) using FORM is therefore given by

$$\beta = \min_{x\varepsilon F} \sqrt{\left(\left(X_1^{'}\right)^2 + \left(X_2^{'}\right)^2 + ... + \left(X_n^{'}\right)^2\right)} \quad (13)$$

where X'_1, X'_2,…………, $X'_n$are the random variables in the limit state function given by G(X)=0. The minimization of equation (13) is performed through an optimization procedure over the failure domain F corresponding to the region G(X)=0. This can be accomplished using FORM5[14], which is a subroutine that solves the FORM problem. FORM5 (written in FORTRAN) provides an approximation to

$$P_f = P(X\varepsilon F) = P(G(X)\leq 0) = \int_{G(x)\leq 0} dF_x(X) \quad (14)$$

bytransforming the non-normal variables into independent standard normal variables, by locating the most likely failure point through an optimization procedure by linearizing the limit state function in that point and by estimating the failure probability using the standard normal integral[14]. A first approximation to $P_f = P(G(X)) \leq 0$ is

$$P_f = \Phi(-\beta) \quad (15)$$

Where in equations (14) and (15), $P_f$is the probability of failure, $\Phi(.)$ is the standard normal integral, β is the safety index or reliability index[14] and other variables are as defined earlier.

3.1 Performance Functions

The calculation of the performance function is performed for discrete combination of basic variables considering the bending failure mode of the reinforced concrete solid slab in accordance with EC 2 [1] as:

$$G(X) = 0.87 f_{yx} \rho L d^2 [1 - \frac{0.97 f_{yx} \rho}{f_{cx}} - \frac{1.6 Q_k L^2 (0.875\alpha + 1)}{FN} \qquad (16)$$

Also, the performance function considering the shear failure mode for reinforced concrete slabs in accordance with EC 2 [1] is given by:

$$G(X) = \frac{0.79}{\gamma}\left[\left(\frac{100A_S}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}\right] - v_{ED} bd \qquad (17)$$

And, the performance function considering the deflection failure mode for reinforced concrete slabs considered in accordance with EC 2 [1]when $\rho \leq \rho_o$, is given by

$$G(X) = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_o}{\rho} + 3.2\sqrt{f_{ck}}\left(\frac{\rho_o}{\rho} - 1\right)^{3/2}\right] - \frac{L}{d} \qquad (18)$$

And when $\rho \geq \rho_o$, the performance function considering deflection is given as

$$G(X) = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_o}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\frac{\rho'}{\rho_o}\right] - \frac{L}{d} \qquad (19)$$

In equations (16) to (19), α (Alpha) is the ratio of dead-to-live loads and other variables are as defined earlier.

3.2 Design Procedure

As proposed in this study, a design is considered satisfactory if the following condition is satisfied[15; 16]:

$$\beta \approx \beta_T \qquad (20)$$

Where β is the calculated safety index obtained from the reliability program on the basis of the input variables and β_T is the target safety index[17;18].

4. PROGRAM DEVELOPMENT

A program was developed to aid the design of reinforced concrete slabs in accordance with EC 2 [1] design requirements as well as the requirement of FORM explained in previous section.

The program starts by requiring the user to input the values of target safety index, weight of finishes and partitions, imposed load (Q_k), diameter (Φ) of the reinforcement to be used, concrete cover (COV), characteristic strength of both the concrete (f_{ck}) and steel reinforcement (f_{yk}). The shorter and longer spans, l_x and l_y respectively, have to be fed in to the computer. The program will proceed with the design of the two-way slab based on the calculated ratio of longer to shorter span. The program then asks the user to give the proposed thickness of the slab and its end conditions.The span and support moments are calculated using the method of elastic analysis described in EC 2, which is in-built into the program. The flowchart of the program is given in Figure 1.

If the design satisfies the ultimate and serviceability limit states of EC 2, reliability analysis of the designed section will be carried out by the program. A design is finally acceptable if the calculated safety index is approximately equal to the target safety index. The design procedure is repeated by the program until this condition is satisfied.

The program terminates after a list of suitable slab section is given with the implied safety level for the design, or alternative design output given using different diameter of steel reinforcement.

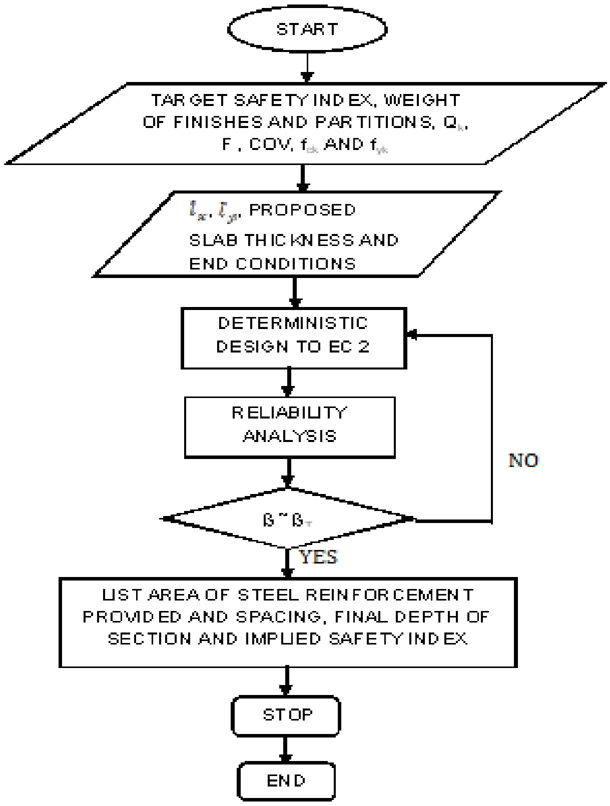

Figure 1: Program Flowchart

5. RESULTS AND DISCUSSION

5.1 Design Example of a Two-Way Solid Slab

5.1.1 Design Data

It is required to design a solid slab using the following parameters: Characteristic concrete strength, f_{ck}=30 N/mm²; characteristic steel strength, f_{yk}=500 N/mm².

The shorter and longer spans of the panel were 4m and 7m respectively, and the weight of finishes and partitions were each 1kN/m^{2}respectively. A shorter edge of the slab was assumed to be discontinuous. Also, the characteristic imposed load was 4kN/m^2, and the diameter of steel reinforcement, and concrete cover were 12mm and 20mm respectively.

5.1.2 FORM Design Procedure

The slab was designed using the developed program and assuming a target safety index, β_T of 3.0. An initial depth of 100mm was assumed and fed into the program. The stochastic models considering the failure modes of solid slabs in accordance with EC 2 [1] were prepared in accordance with FORM [14, 19], were in-built into the computer program. The developed program was used to carry out the analysis with optimum design variables automatically selected by the program for the design. The FORM design satisfied the requirements of both EC 2 and FORM [14]. Equation (20) was satisfied as a final check. The design results of the FORM design are as presented in Table 1.

5.1.3 Design using Eurocode 2 Procedure

The slab was designed using the assumed design variables stated earlier in accordance with the EC 2 design procedure. The results are also presented in Table 1.

Table 1: Results of FORM Design and Design using Eurocode 2

Design Details	Design to Eurocode2 (2004)	FORM Design
Area of Steel required (mm^2/m)	267.4	279.65
Final Depth of Slab (mm)	150	125
Implied Safety Index	4.57	3.0

From Table 1, the FORM design gave more economical design considering the final depth of section which is about 20% lower than the EC 2 [1] design method at a target safety level of 3.However, the EC 2 design gave area of steel required that is about4% lower than the FORM design. It is to be noted that the FORM design has a safety index of 3.0, which smaller than the safety index of 4.57 considering the EC 2 [1] design which therefore signifies economy of the FORM design procedure when all variables were considered in the design.

5.2 Example of Reliability-Based Design Slab at Varying Safety Indices

Reliability-based designs of the slab presented in the previous section were carried out at varying reliability levels of 3.0, 4.0 and 5.0. Each design output satisfied the EC 2 requirements and the safety index of FORM[14]. In each case, Equation (20) was satisfied as a final check. The results obtained from the program considering the three safety indices are as presented in Table 2.

Table 2: Probabilistic Design of Slab at Varying Safety Indices

Design Details	Target Reliability Level, β_T		
	β_T = 3.0	β_T = 4.0	β_T = 5.0
Area of Steel Required (mm^2/m)	279.65	325.77	336.95
Final Depth of Slab (mm)	125	135	175

From the results presented in Table 2, it was observed that:

a) As the safety index was increased, the depth of the section increased.
b) As the target safety index was increased from 3.0 to 4.0, there was an increase of 8% in the depth of the designed section. However, there was an increase of about 30% in magnitude of depth of designed section when target safety index was increased from 4.0 to 5.0.
c) Also, as the target safety index was increased from 3.0 to 4.0, the magnitudes of area of steel provided increased by about 14%. However, there was an increase of about 3% in magnitude of area of steel provided when the target safety index was increased from 4.0 to 5.0.

6. CONCLUSION

The work presented the design of reinforced concrete two way solid slabs at a uniform safety level with the aid of a computer program. The requirements of EC 2 [1] and FORM [14] were strictly adhered to. It was shown among other findings that the FORM design gave more economical designs considering the area of steel provided and final depth of section, than the EC 2 design procedure when a target safety level of 3.0 was considered. Also, example of FORM design using the program was included for various values of target safety indices at constant geometrical, loading and end condition of the slabs. It can be concluded that the developed program is suitable for application.

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