

# SIMULATION OF ELECTROMAGNETIC TRANSIENTS IN POWER SYSTEMS

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## ABSTRACT

*Transients in power systems are initiated by abrupt changes to otherwise steady operating conditions. These changes would be as a result of any of the following: opening or closing of circuit breakers, switching conditions, lightning or any other fault condition. For purposes of power system analysis these conditions are simulated using mathematical models. In the formulation of these mathematical models efforts are made to represent changes in transmission line characteristics, such as the variation of line parameters with frequency.*

*Electromagnetic (EM) transients produced by these mathematical models serve as a useful reference in the design of protective devices and fault locators. It is therefore pertinent to examine the effect of faithful reproduction of line characteristics in the simulated transients with respect to the performance of fault locators tested with the simulated transients. Such investigation has shown that the computational burden involved in trying to represent all line characteristics in transients calculations could be saved.*

## INTRODUCTION

Accurate fault location in power systems is very important for a speedy restoration of power supply. In modern power systems these faults are located by running programs with built-in fault location algorithms using post fault data. At the design stage, however, these faults location algorithms are usually tested with simulated fault data from electromagnetic (EM) transients calculation. The reliability of the results of such tests depends mainly on the method of calculation of the EM transient. Ideally the method should faithfully reproduce the fault being simulated.

EM transients are essentially of travelling wave form, the wave shape of the voltage and current transients being influenced by the boundary conditions imposed on individual system. EM transients program whose computational techniques are based on travelling wave principle are simple and effectively reproduce fault transients. Fault location algorithms treated by locating faults simulated with such programs produced the same order of fault location accuracy as on practical models of power transmission lines. Other methods of EM transients calculation

are based on the Fourier transform and the z-transform techniques. These can easily incorporate the frequency variance in transmission line parameters, but they involve more complicated mathematics than the travelling wave methods. The computational burden presented by Fourier and z- transforms could be saved if the incorporation of changes in line characteristics in the transients calculations does not provide any extra detail about the fault or system condition.

The computational techniques used in EM transients calculation may be broadly classified into: (a) Time-domain methods using travelling wave approach [1-4] (b) frequency-domain methods based on the Fourier transform [5-8] and (c) z-transform methods [9-11].

This paper will look at each technique and analyse the mathematical model with the view of establishing which model is the best suited for fault simulation, taking into account the computational burden vis-a-vis the details produced in the simulation.

## 2.0 The Computational Techniques

### 2.1 Times-Domain Methods

Bewley Lattice diagram [1] was the foremost graphical means of interpreting travelling wave propagation, and this presents a very good picture of Electromagnetic (EM) transients. Although basically the method is applicable to lossless lines it has been developed by various investigators [2-] to include the effects of losses and the frequency-dependence of the system parameters.

A lossless transmission line can generally be modeled by the following equations:

$$\frac{\delta V}{\delta X} = \frac{-l\delta i}{\delta t} \quad (1)$$

$$\frac{\delta i}{\delta X} = \frac{-c\delta v}{\delta t} \quad (2)$$

$v$  = voltage,  $i$  = current,  $l$  = inductance,  $c$  capacitance.

These equations have well known solutions, due to D' Alembert given as:

$$i(x, t) = F_1(x - at) + F_2(x + at) \quad (3)$$

$$v(x, t) = z[F_1(x - at) - F_2(x + at)] \quad (4)$$

where

$z = \sqrt{l/c}$  = surge impedance of the line

$a = 1/\sqrt{lc}$  = wave propagation velocity

$F_1(x-at)$  and  $F_2(x + at)$  are arbitrary functions of the variables  $(x-at)$  and  $x + at$ .  $F_1(x-at)$  is a wave travelling at velocity  $a$  in the positive (forward)  $x$  direction and  $F_2(x + at)$  is a wave travelling in the opposite direction.

In Bewley Lattice diagram lines and cables are specified by their surge impedances and travel times. The reflected and refracted waves at junctions and terminations of the lines and cables are calculated by the use of reflection and refraction coefficients  $k_1$ , and  $k_2$ , defined for a single-phase system, as

$$k_1 = \frac{Z_T - Z_0}{Z_T + Z_0} \quad (5)$$

$$K_2 = \frac{2Z_T}{Z_T + Z_0} \quad (6)$$

Where  $Z_0$  is the surge impedance of the line or cable on which the wave is travelling and  $Z_T$  is the effective surge impedance seen by the wave when it reaches the junction or termination. The travel times of lines and cables comprising a system are determined

from a knowledge of their lengths and propagation velocities.

When a single transmission line is considered the voltage (or current) at any time may be obtained by summing the waves which have arrived at that point prior to the time of interest. But for an interconnected system of lines and cables the surge impedances and propagation velocities of each circuit are in general, different. In this case the voltage (or current) is obtained by summing all refracted (or transmitted) waves prior to a particular time or alternatively summing all incident and reflected waves.

This scheme has since been developed [2,3] into systematic procedures with automatic processing facilities both for single and multinode networks of any given interconnection. Dommel's [2] contribution uses the trapezoidal rule of integration for lumped parameters. Although reference was made to the non-linear frequency-dependence of line- parameters, this is yet to be fully accounted for in solutions which are wholly in the time-domain.

### 2.2 Frequency-Domain Approach

The travelling wave method of EM transients analysis is quite simple and attractive but the necessity for accurate representation of frequency variance of line parameters over the whole frequency range involved in the computation calls for a purely frequency-domain analysis. Briefly, the frequency-domain approach involved the re-calculation of line parameters at pre-determined frequency points within the frequency range of transient evaluation.

For each single frequency, the solution to the transmission line equations can be written down, but the impedance terms in these equations must be calculated afresh for each frequency being considered. In order to determine the amount of each frequency present in the generator voltage which supplies the transmission line, the modified Fourier transform is used, and in order to determine the unknown voltages  $v(t)$  and currents  $I(t)$  in the time domain, the inverse Fourier transform must be performed on  $v(w)$  and  $I(w)$ . These transformations are infinite integrals and the inverse transform is an integral with respect to frequency,  $w$ . In

practice, these infinite integrals are evaluated numerically as finite summations [5]. Summations are performed for  $-\Omega < w < \Omega$  in steps of  $\Delta w$ .  $\Omega$  should be large and small for best accuracy. ( $\Omega$  is the maximum frequency considered). Inappropriate choice of  $\Delta w$  can cause oscillations in the results in the tone domain.

In summary therefore, a purely frequency-domain solution involves the computation of the line parameters at each frequency point; taking the modified Fourier transform of both the transmission line equations and the applicable boundary conditions; solving the problem in the frequency domain and, finally, taking the inverse Fourier transform to obtain solutions in real time. The method is applicable both to single transmission in real time. The method is applicable both to single transmission line and polyphase networks [7]. The major cause for concern with Fourier transform method is the computation time and storage requirement.

Re-evaluation of network equations at each frequency point in the analysis to take account of non-linear dependence on frequency of transmission line parameters takes a large amount of computer time. The ability to deal with these non-linearities appears to be the relative advantage of this method.

### 2.3 The z-transform method.

Following a growing development in numerical methods of Electromagnetic (EM) transient analysis, Humpage et al [9-11] explored and successfully implemented a scheme that applied the z-transform to EM transient calculation. Beginning with the basic relationships of propagation in the frequency domain, the successive steps of z-transform analysis are developed leading to recursive sequences for solution in the time domain.

Although the special role of the z - plane in transforming between the frequency domain and the time domain is central to the formulation, it does not draw and extensively on formal - transform theory. The use of the z - plane is a particular one which derives from the exponential form of transmission line impulse responses in the frequency domain. In fact the underlying proposal for the formulation emerged from a recognition of the close correspondence between the

exponential form of the z- transform and that of the principal transmission- line responses in the frequency domain.

The frequency-domain solution of the 2nd - order equations of wave propagation in a transmission line is given in equation (7) and 8

$$\begin{aligned} V_s(\omega) - z(\omega)i_s(\omega) \\ = \exp\{-\lambda(\omega)\}[v_r(\omega)i_r(\omega)] \\ V_r(\omega) - z(\omega)i_r(\omega) \\ = \exp\{-\lambda(\omega)\}[V_s(\omega)i_s(\omega)] \end{aligned}$$

subscript s and r denote sending end and receiving end of the transmission line respectively.

$$\text{let } F_x(\omega) = \exp\{-\lambda(\omega)x\} \quad (9)$$

*= forward impulse response*

(9)

$F_x(\omega)$  is a matrix in a three phase system. x is the length of the line section measured from the sending end of the line. For a transmission line of length l, the forward impulse response matrix simply becomes:

$$F(\omega) = \exp(-\lambda(\omega)l) \quad (10)$$

$z(\omega)$  is the surge impedance function. The two functions  $F(\omega)$  and  $z(\omega)$  are easily derived in the frequency domain using the + basic line parameters.

$(\omega)$  is the matrix of propagation coefficients.

The main task in the use of z-transform for the calculation of EM transients is the formation in the z-plane, of the rational fraction forms of the two principal functions, the forward impulse response and the surge impedance function. The subject of synthesising the impulse response and the surge impedance function in the z-plane has been treated [11]. Provided Hilbert transform conditions are satisfied, the minimisation procedure for finding rational function by which  $F(\omega)$  and  $z(\omega)$  can be represented are based on the magnitudes of these functions, *i. e*  $|f(\omega)|$  and  $|z(\omega)|$

Let  $F(\omega)$  and  $z(\omega)$  be represented by  $F(z)$  and  $Z(z)$  in the z-plane. Both function are derived as the following rational fractions:

$$F(Z) = A\pi \prod_{k=1}^N \frac{1+a_k z^{-1}+b^k z^{-2}}{1+a_k^{-1}+d_k z^{-2}} \quad (11)$$

and

$$Z(Z) = B\pi \prod_{k=1}^M \frac{1+e_k z^{-1}+f^k z^{-2}}{1+g_k z^{-1}+h_k z^{-2}} \quad (12)$$

The order of the function to be synthesised

basically depends on the overall accuracy required in the EM transient analysis. In general, 4th-order function can meet a wide range of requirements but 2nd-order functions, and in some cases, those of 1st-order form may be satisfactory [11]. During investigation of the method of z-transform for transient analysis, the author used the 1-st order and 2-nd order functions for the surge impedance and forward impulse response respectively. The results proved quite satisfactory [12]. The sampling interval was found such that responses were satisfactorily defined over the frequency range relevant to transient evaluations in which the responses are used in the program.

Equations (7) and (8) transform directly to the z-plane as:

$$v_s(z) - Z(z)i_s(z) = F(z)(v_r(z) + Z(z)i_r(z))$$

$$v_r(z) - Z(z)i_r(z) = F(z)(v_s(z) + Z(z)i_s(z))$$

Time domain solutions are obtained by the inverse z-transform operations.

#### Basic Transmission Data

Number of circuits	1
Number of conductors per phase	4
Number of earth wires	1
Conductor position symmetry	none
Conductor resistivity, m	$3.2 \times 10^{-8}$
Earthwire resistivity, m	$2.29 \times 10^{-8}$
Conductor strand diameter, cm	0.32
Earthwire strand diameter, cm	0.32
Geometric mean diameter for 4-conductor bundle, cm	30.94
Outer diameter of earthwire, cm	2.86
Number of effective strands in phase conductors	54
Number of effective strands in earth wire	54
Earth resistivity, m	20

#### **Case I - The Travelling Wave Method**

The calculation of transients due to a single-line-to-ground fault simulated on the 400kv system of fig. 1 was used as an example to demonstrate the travelling wave method. Basic line parameters were calculated at a frequency based on the transit time parameters being switched, using the program in ref, [12]

p. 174. The frequency in this example was 1KHz.

Fig. 2 show the voltage transient at the sending end of the line for a phase-to-ground fault 60km away. The short circuit capacities at the sending and receiving ends were 35000 MVA and 10000 MVA respectively.

The rapid fluctuation the voltage waveform is a consequence of assuming constant line parameters. Non-representation of the frequency variance of the line parameters usually results in some distortion in the transient waveform.

#### **Case II - Fourier Transform Method**

The Fourier transform method was used in calculating the electromagnetic transients in the 400kv test system, also for a single line-to-ground fault. The result is shown in fig. 3. The use of Fourier transform made it possible for the frequency variation of line parameters to be incorporated in the program. The simulation was done for a full cycle. As can be seen from fig.3 distortion of fault waveforms has been brought to the barest minimum. In both cases (travelling wave method and Fourier transform) the fault inception is at the instant corresponding to voltage maximum in the fault phase. This is the worst case from the point of view of travelling-wave distortion.

#### **Case III - z- Transform Method.**

Using the transmission line data for a typical 400kv system shown in fig. 1, the forward impulse response and the surge impedance function of the z-transform method were calculated. Fault simulation was at the instant of a voltage peak in phase a, the fault level at the switching end of the line being 20000 MVA. The result is shown in fig.4

#### **CONCLUSION**

A fault location algorithm should be tested using data that faithfully represents current and voltage waveforms experienced at a relay location and seen by a protection system in practice. it has been shown that the waveforms can be obtained by digital simulation. Three major schemes for obtaining the fault transients were discussed.

The method that faithfully reproduces the fault waveform is the frequency-domain analysis using Fourier transform. The method incorporates the frequency-dependence of transmission line parameters. In this method line parameters are re-evaluated at pre-determined frequency points. The storage requirement and execution time, however, make the method computationally expensive for practical purposes. Results from purely frequency-domain analysis are only good as a

reference for comparing other methods. Another drawback in the frequency-domain approach is the difficulty in including time related events, like non-simultaneous circuit breaker pole-closure, in the program. These are easily done on the Bewley travelling wave method, which is a purely time-domain approach. The travelling wave methods of evaluating transmission line transients are quite attractive although they lack in accurate representation of frequency variance of line parameters.

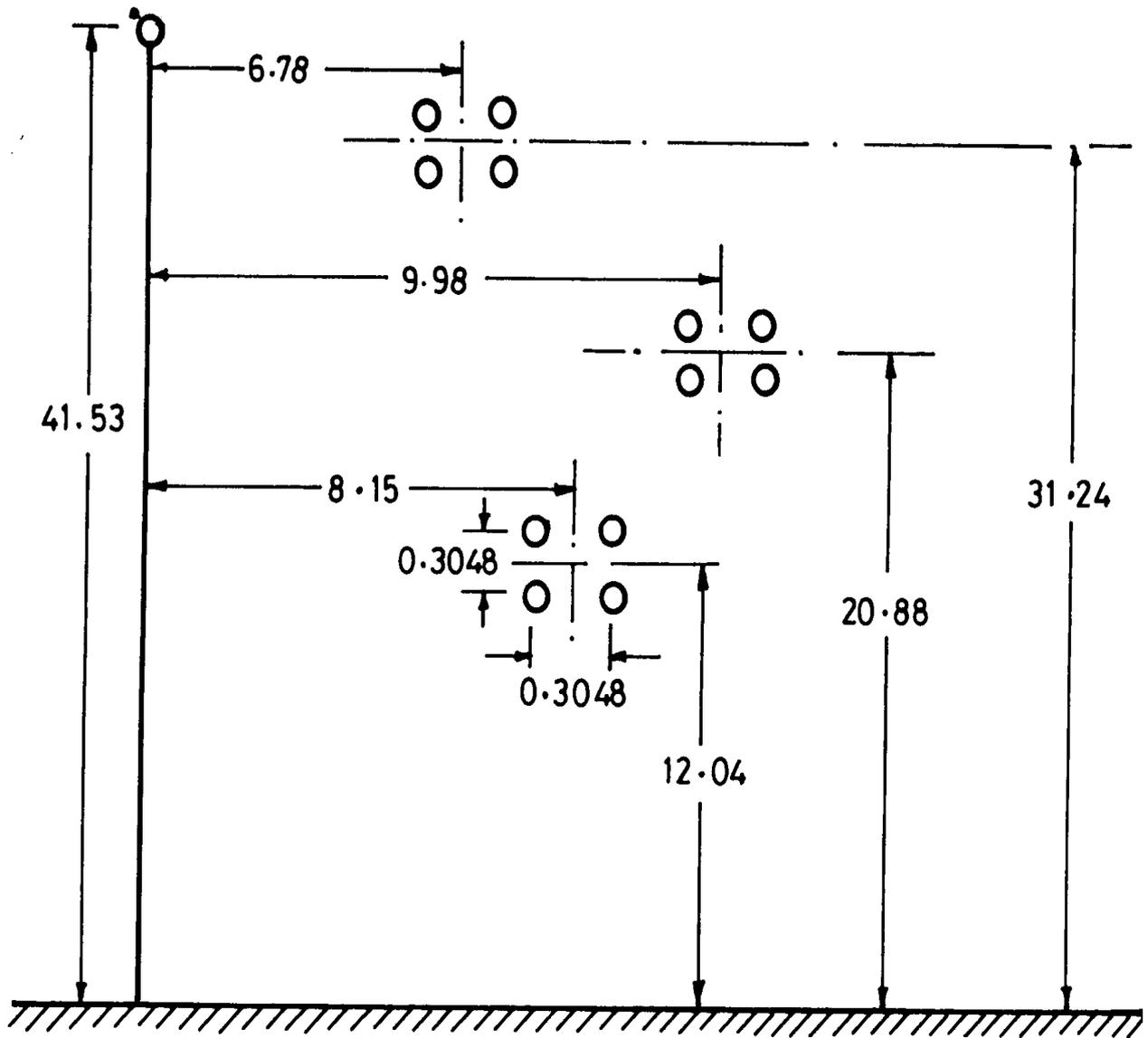


Fig. 1: 400 kV Transmission line (conductor spacing in metres)  
 Upper : Phase a  
 Middle : Phase b  
 Lower : Phase c

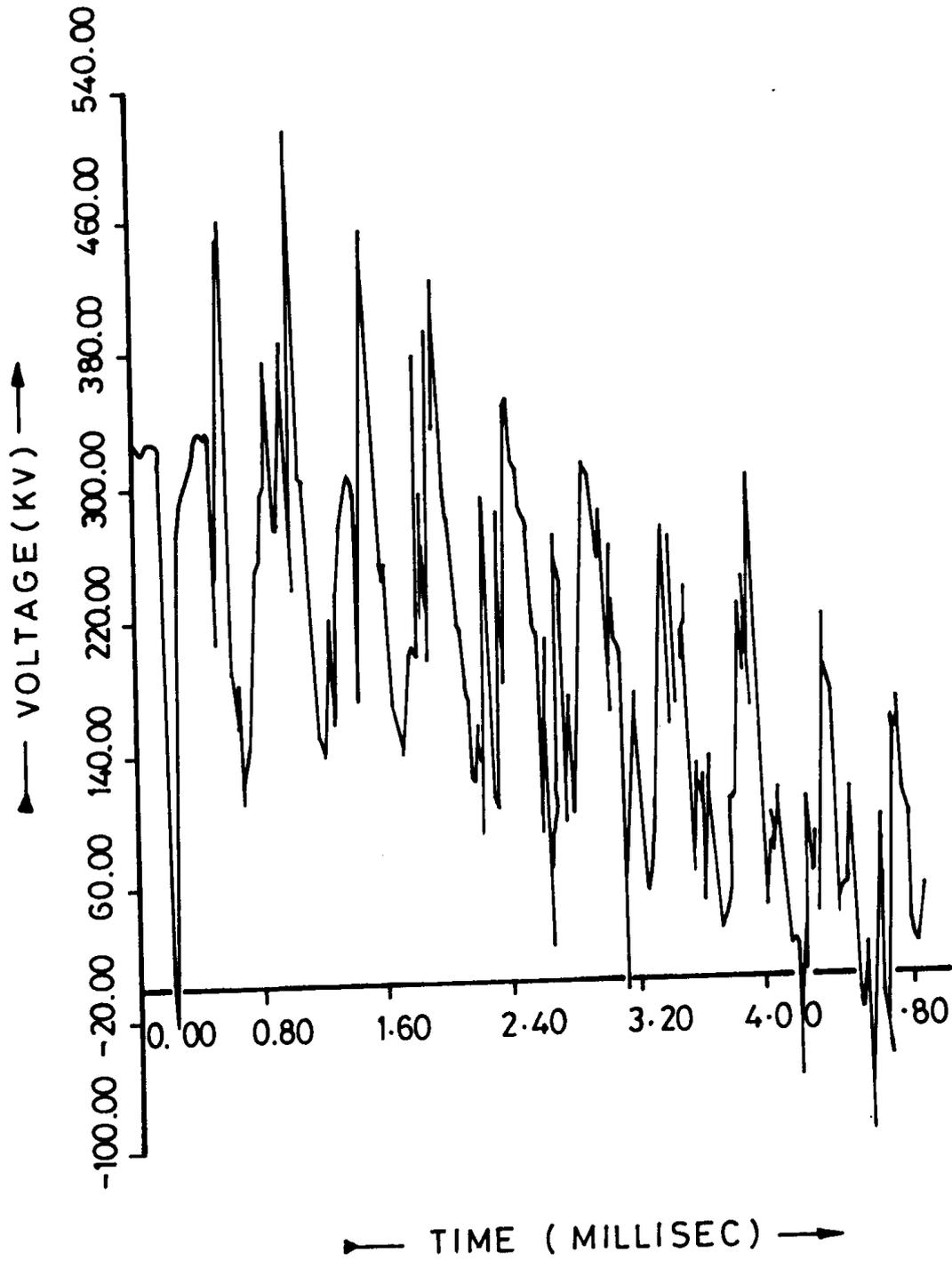


Fig. 2. Voltage Transient due to a line-to-ground fault.

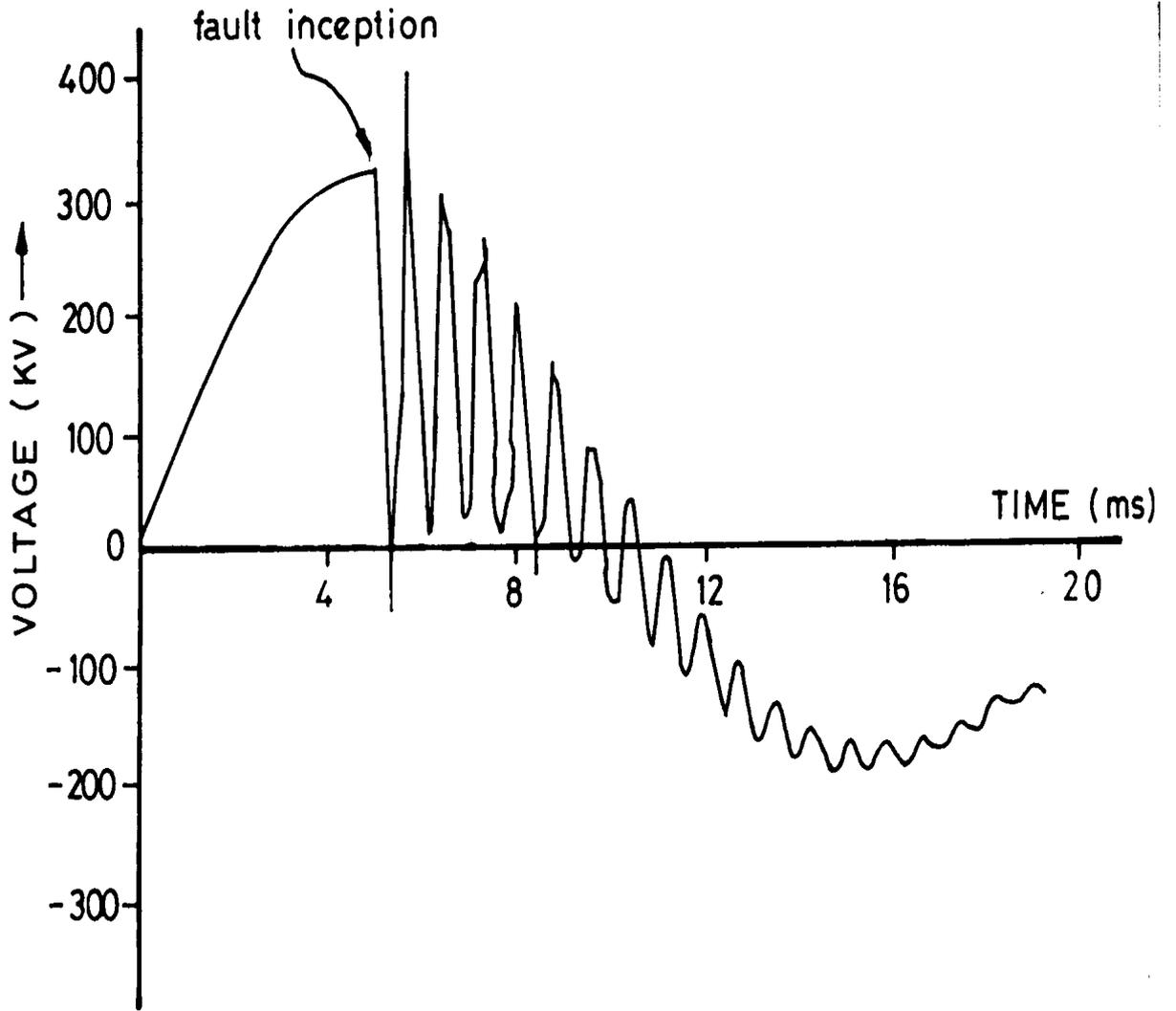


Fig. 3: Fault waveform due to a line-to-ground fault.

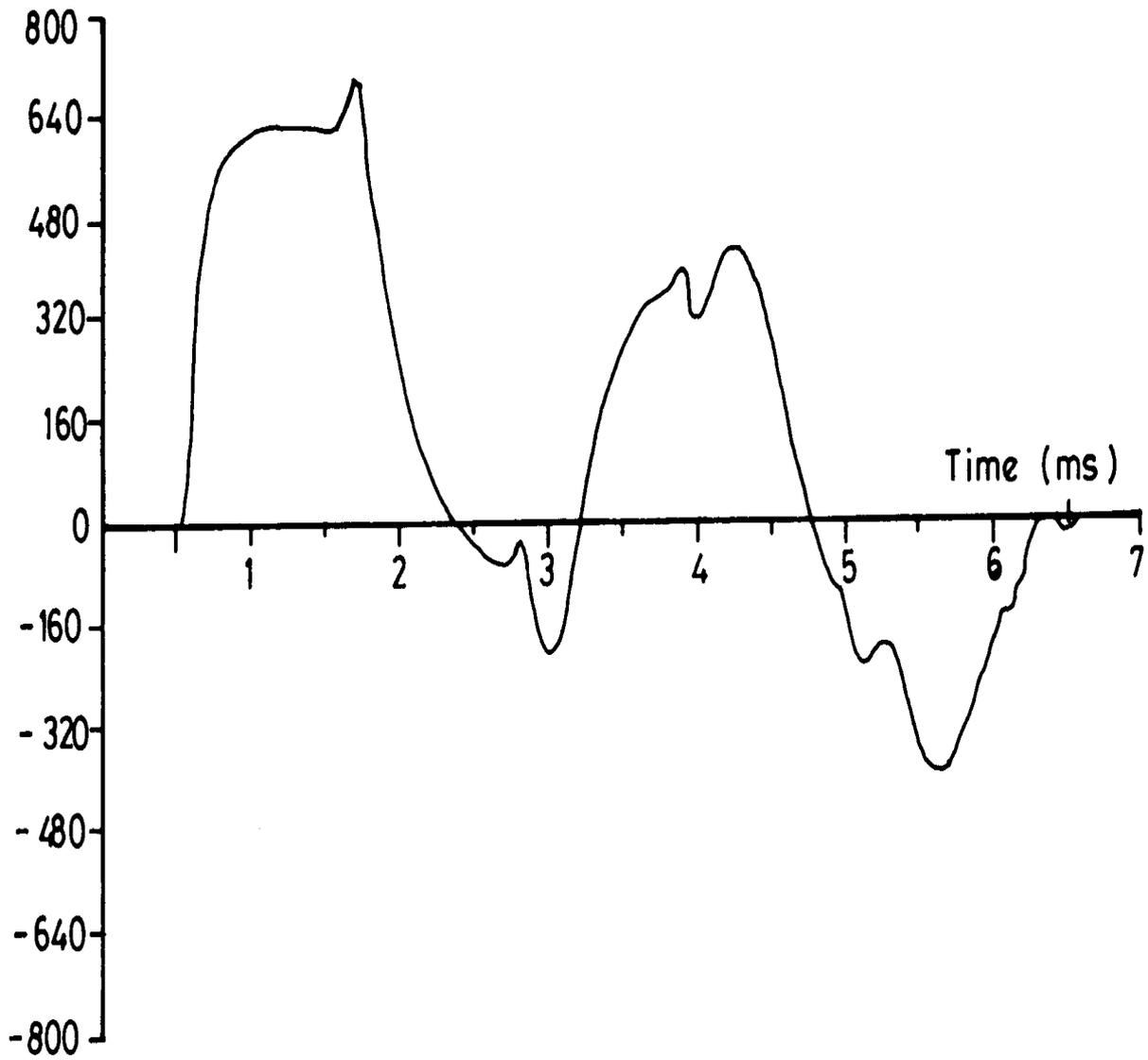


Fig. 4: Receiving end phase 'a' voltage transient in single conductor energisation.

The most recent method of transients evaluation based on the z-transform looks quite promising. Apart from the rigour of synthesising the responses and functions in the z-plane and their eventual transformation to the time domain, the actual transients evaluation takes much less time than either the time convolution or Fourier transform method. In a test case [12] also in this paper, a Fourier transform method run on CDC Cyber 72/73 required 150 seconds whereas the z-transform solution was achieved in 0.15 seconds.

In conclusion, the travelling wave methods are attractive for their simplicity. A purely time domain solution took only 1.5 seconds. The frequency dependence of line parameters does not constitute a serious problem since these parameters can be calculated at dominant frequencies expected in the transient analysis. The savings in execution simplicity of the scheme compensate for the slight loss in faithful reproduction of transients. In fact, the waveform distortion arising from non-representation of frequency variance of line parameters, will subject fault location algorithms to a better test [12]

There is, therefore, a choice of either:

- a) a simple, fast program with minimal storage requirement and which allows for the inclusion of time related events in power systems, or
- b) methods that account for frequency variance of line parameters but involve complex programs that require large storage and computation time.

Having carried out the simulation in typical 400kv systems and having tested a fault location algorithm [12] with the simulated data the first option is recommended.

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