IMPEDANCE CALCULATIONS OF INDUCTION MACHINE ROTOR CONDUCTORS.

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ABSTRACT

The exact calculation of the impedance of induction machine rotor conductors at several operating frequencies are necessary if the dynamic behaviour of the machine is to give a good correlation between the simulated starting torque and current and the experimental results. This paper describes a method of' calculating the impedance of Rectangular and Trapezoidal rotor bars. An R-L parallel network is used to model each of the Rotor bars. A computer optimisation Algorithm is developed and from which the Rotor circuit parameters at several frequencies are estimated. The model solutions may then be used to find any desired steady-state or transient performances of induction machines.

INTRODUCTION

For the past 30 years, the dynamic behaviour of induction machines has received a considerable attention in most researched works [1.2, 3.4.5.6]. However, the analysis is based on the simple idealised machine model without deep-bar effect and with the assumption that the rotor resistance remains constant at the d.c. value. Consequently, this method of analysis usually leads to a very large error in the computation of the starting torque and current at several frequencies for squirrel-cage machines with deep rotor bars.

Pertinently, the need for the development of accurate models for induction machines becomes highly necessary. This is necessitated by the fact that in an induction machine, the lower portions of the bars of the rotor cage are linked by more slot leakage flux than the upper portions of the bar. Under dynamic conditions, the lower section of the rotor bar experiences a higher inductance than the upper section of the rotor bar due to non-uniform flux distribution, thereby causing the current to flow primarily in the upper portion of the bar [7] Also, the re-distribution of the current flowing in the rotor bar effectively increases the resistance of the bar. This phenomenon of decrease in inductance and increase in resistance of rotor conductors is known as the deep-bar effect or the skin -effect [7.8.9,]. This effect is predominant in machines operated from non-sinusoidal power sources and large induction machines with rotor bars that have large bar depth to bar width ratio The paper describes a method of calculating the impedance of rectangular and trapezoidal rotor conductors. An R-L parallel network used to represent the Rotor bar types is also discussed as well as the optimisation technique applied.

2 The Rectangular Rotor-Bar

Figure 1 shows n sections of rectangular rotor bar. In order to derive the impedance equation for the rectangular rotor bar, the rotor bar conductor is divided into n-equal number of sections [7]. By so doing, the current distribution within each section can be considered uniform.

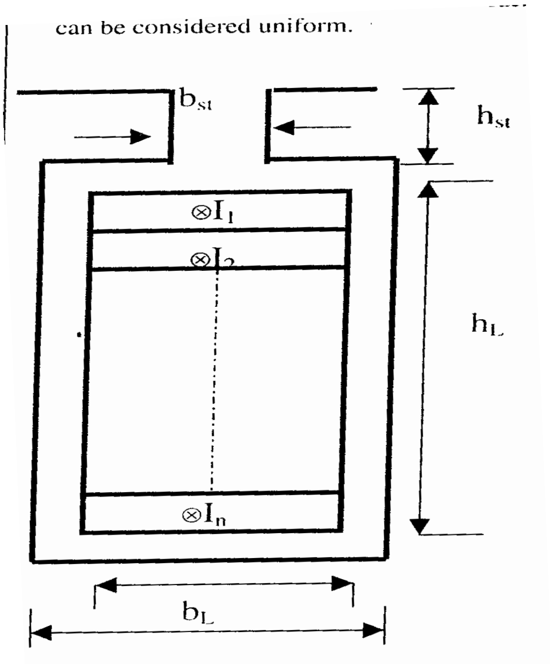

Figure 1: n sections of Rectangular Bar

Total resistance of the rectangular bar is,

$$R_T = \frac{L_s}{(\chi_{cu} h_L b_L)} \quad (1)$$

The resistance of each section is therefore given by

$R_L = (L_s)/(\chi_{cu} h_{sec} b_L)$ (2)

Where

$h_{sec} = \frac{h_L}{N}$ (3)

The magnetic flux density in section one of the bar is

$\underline{B}_1 = \mu O \underline{I}_1 / b_{Nut}$ (4)

Therefore, the magnetic flux between the first and ficitions section becomes,

$\underline{\phi}_1 = B_1 L_s h_{sec} = \underline{B}_1 L_s h_{sec}$
$= \mu o\, h_L\, L_s\, \underline{I}_1 / N b_{Nut}$ (5)

In analogy, the magnetic flux between sections 2 and section 3, becomes

$$\underline{\phi}_2 = \left(\mu_o L_S h_L / N b_{Nut} \right) * (\underline{I}_1 + \underline{I}_2) \quad (6)$$

In general, the magnetic flux between the section n and (n+1) of the rotor bar is,

$$\underline{\phi}_n = \left(\mu_o L_S h_L / N b_{Nut} \right) * \sum_{v=1-v}^{n} I \quad (7)$$

It can be seen from figure 1 that a loop results between the nth and (n+1)th section. Therefore kirchhoff's voltage equation could be applied.

$U = 0 = iR + \frac{d\Phi}{dt}$ (8)

Between the nth and (n+1)th section, we have

$0 = \underline{I}_n R_L = \underline{I}_{n+1} R_L + j\omega \underline{\phi}_n$ (9)

Combining equations (7)and (9), and with little manipulation, the total bar current is

$$\underline{I}_{n+1} = \underline{I}_n + j \left(\omega \mu_o h_L^2 b_L \chi_{cu} / N^2 b_{Nut} \right) * \sum_{v=1-v}^{n} I \quad (10)$$

The total voltage is

$\underline{U} = R_W \underline{I}_n + j\omega L_{wi}\, \underline{I}_n$ (11)

The total bar impedance of the rotor is calculated by adding the bar outside inductance as shown in figure 1,

$$L_a = \left(\mu_o h_{st} L_S / B_{st} \right) \quad (12)$$

$\underline{Z}_{total} = R_{Wi} + j\omega(L_a + L_{wi})$ (13)

3. The Trapezoidal Rotor Bar

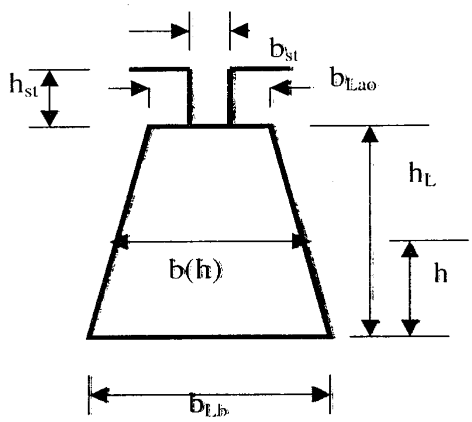

Figure 2: Trapezoidal Rotor – Bar

Here also, the trapezoidal rotor-bar of figure 2 is divided into n-equal sections. Each section width being a function of h and is given by [9]:

$$b(h) = b_{1b} \left(1 - h_{(n)} / h_1 \right) + \frac{h(n)}{h_1} b_{La} \quad (14)$$

And

$h(n) = \frac{h_{sec}}{2} + h_{ec}(n-1)$ (15)

$h_{sec} = \frac{h_1}{N}$ (16)

The resistance of each section is,

$R_L = \frac{(N L_S)}{\chi_{cu} h(n) b(n)}$ (17)

$\underline{U} = R_L\, \underline{I}_n$ (18)

The current is as given in equation (10). With equations (12), (10),and (18), the total impedance of the trapezoidal rotor conductor can be computed.

4 R-L Rotor Model

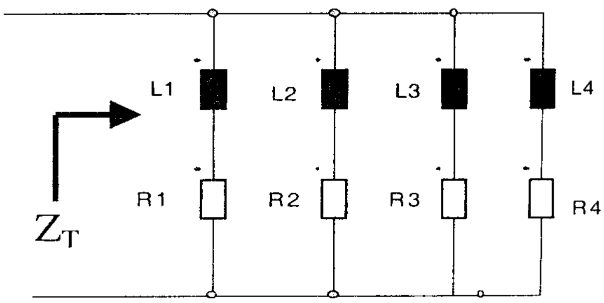

Figure 3: Parallel R-L network

The parallel R-L network shown in figure 3 is used in the modeling of the rotor bars. The network total impedance is given by,

$$Z_T = \frac{(Z_1 Z_2 Z_3 Z_4)}{(Z_2 Z_3 Z_4 + Z_1 Z_3 Z_4 + Z_1 Z_2 Z_4 + Z_1 Z_2 Z_3)} \quad (19)$$

Where,

$Z_1 = R_1 + j\omega L_1$
$Z_2 = R_2 + j\omega L_2$
$Z_3 = R_3 + j\omega L_3$
$Z_4 = R_4 + j\omega L_4$

5. simulation Results

MATLAB m-file for the calculation of the total impedance of the rectangular and trapezoidal rotor bars is developed [10]. The parameters of the bars used in the simulation are as shown in Appendix

A. the frequency- impedance plots are presented in figures 4 and 5 for rectangular and trapezoidal rotor bars respectively.

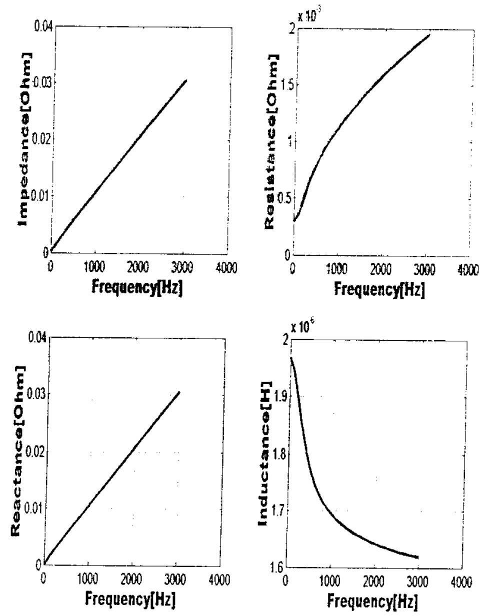

Figure 4: Rectangular Rotor-bar plot

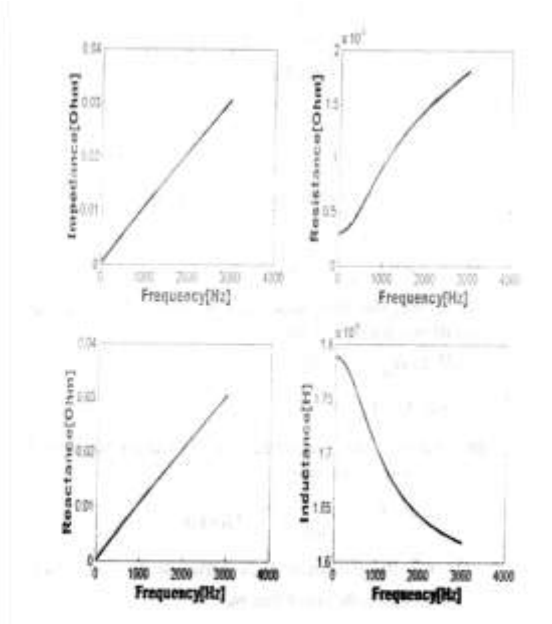

Figure 5: Trapezoidal Rotor-bar plot

To obtain the parameter of the rotor circuit model of figure 3, a simple constrained optimisation algorithm that attempts to give the best agreement between the actual impedance of the rotor bar and the model impedance is developed. In the optimisation algorithm, the cost function, F being minimized is defined by,

$$F = \sum_{i=1}^{N} \left[\left(\left(real(Z_{opt}) - real(Z_T) \right)^2 \right) + imag(Z_{opt}) - imag(Z_T) \right)^2 \right] \quad (20)$$

Z_T give the actual rotor bar impedance at frequency, f and the optimization technique provides the values for the rotor circuit, Z_1, Z_2, Z_3, and Z_4, such that the effective rotor impedance in model is Z_{opt}. the frequency-impedance optimisation plots are shown in figure 6and Figure 7 while the results that give the estimated rotor circuit parameter are shown in Appendix B

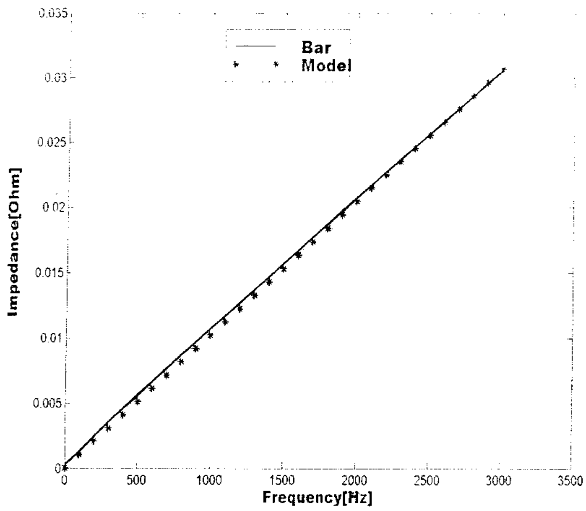

Figure 6: Rectangular Rotor-bar optimisation plot

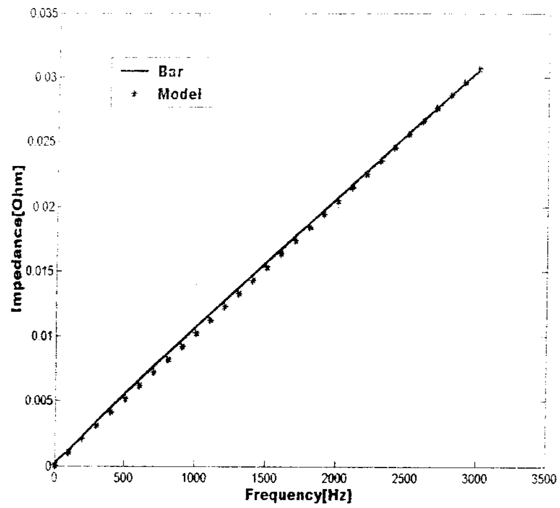

Figure.7: Trapezoidal Rotor-bar optimisation plot

6 CONCLUSIONS

A simple method for calculating the impedance of rectangular and trapezoidal rotor conductors has been presented in this paper. The optimised solution of the R-L-net work rotor model as shown in Figures 6 and 7 are in good agreement with the actual rotor bar characteristics of the induction machines. The optimised solutions may then be used to find any desired steady-state or transient performances of induction machines operating over a wide range of frequencies where the deep-bar effects are noticeable.

7 Nomenclature

L_s = length of the rotor bar
χ_{cu} = conductivity of copper conductor
h_L = height of rotor conductor
b_L = width or rotor conductor for rectangular bar
b_{Lao} = Upper width or the trapezoidal bar
b_{Lbo} = lower width or the trapezoidal bar
b_{st} = width of the end ring
h_{st} = height of the end ring
N = total number of sections
μ_o = permeability of free space
La = Bar outside inductance
R_{wi} = the Bar resistance
b_{Nut} = Width of the rotor bar
ω = angular frequency
L_1, L_2, L_3, L_4 = inductance of the R-L Rotor model
R_1, R_2, R_3, R_4, = resistance of the R-L Rotor model
R_L = resistance of each section of the bar
Z_T = total mode I impedance
j = complex operator
F = Cost function
f = frequency

8 REFERENCES

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Appendix A: Machine Data

Description	Rectangular bar	trapezoidal
Type of rotor cage	Steel	Steel
Conductivity of rotors bars	$56 Sm/mm^2$	$56 Sm/mm^2$
Number of pole pairs	3	3
Number of rotor slots	24	24
Number of stator slots	36	36
Stator bore diameter	200mm	200mm
Width of end ring	2mm	2mm
Height of end ring	4mm	4mm
Insulation thickness	0.5mm	0.5mm
Bar length	0.6m	0.6m
Stator outside diameter	195mm	195mm

Stator inside diameter	185mm	185mm
Thickness of the circular element	10mm	10mm
Permeability of free space	$4\pi * 10^{-7}$ H/m	$4\pi * 10^{-7}$ H/m
Number of rotor bar divisions	100	100
Height of rotor bar	10mm	10mm
Width of rotor bar	5mm	-
Bottom width of bar	-	8mm
Upper width of bar	-	5mm

Appendix B: estimated rotor circuit parameters

Bar types	L_1	L_2	L_3	L_4	R_1	R_2	R_3	R_4
rectangular	4.12μH	8.2μH	4.0μH	2.5mH	110.250μΩ	50μΩ	23μΩ	5.2mΩ
Trapezoidal	2.1μH	15.0μH	41.0μH	21.0μH	111.300μΩ	0.10 mΩ	5.2μΩ	20μΩ

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