RIGID AND ELASTIC SUB-BASE EFFECTS ON TALL STEEL FRAMES

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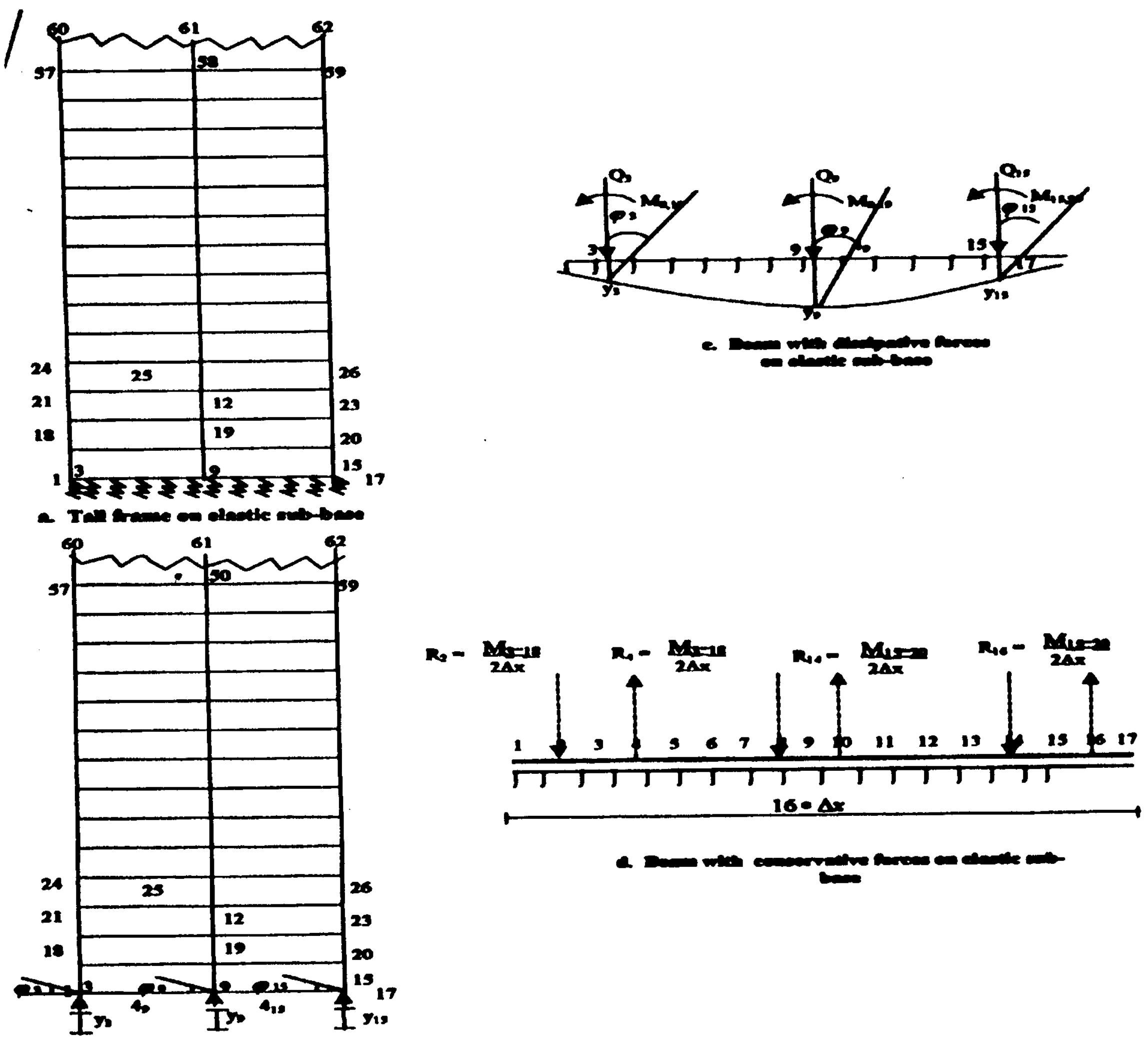
ABSTRACT

The function of the frame system in a structure is to carry loads (dead, imposed and wind) acting on the structure. The frames strengthen the high-rise steel building and guarantee the stability of the whole system. Additionally, they transfer these load unto the foundation. However, the main structure's subsequent behaviour on the sub-base (soil) which is elastic in nature (5) has more to be desired: The paper investigates the working conditions of tall steel frames (twenty storeys) under dead, imposed and lateral toads. The whole frame-system are designed resting on rigid strip support ($k=\infty$) and on elastic sub-base ($k=1500T/m^3$). The vertical loads (dead and imposed) and the horizontal loads were treated separately for the frames. One of the oldest models of elastic sub-base proposed by E. Winkler (1867) was employed. It is believed that this gives a realistic behaviour of the frame during construction. The result confirmed that some assumptions in existence both in, soil mechanics, as well as, in structural analysis can be improved upon with the new information technology. The result achieved, is remarkable.

INTRODUCTION:

Investigation into the working condition of structures and the settlement of the elastic sub-base (1,3) under the influence of load on structures connotes one of the basic problems of soil mechanics today. Additionally the structural analysis has its own shortcomings too. The bone of contention here is the joint action between the frames, the foundation and the elastic sub-base. Even the approach of structural analysts to tall frames supports needs more investigation. It is also vital to have in mind that structures normally are on elastic sub-base (Fig. l) - a situation that is not always considered in everyday structural analysis. Tomlinson M. J., Bowles J. E. et al, (1,11), do accept that determination of buildings settlements is very complex, ambiguous and difficult to properly define. This is believed to be inherent in the properties of soils, embedded in soil mechanics. The assumption also in structural analysis that the supports are vertically nondeformable is also faulty (Fig, 02). This working relationship has never been well defined. The sub-base is elastic in nature, and usually deflects downwards during construction. Also the pressure distribution at the immediate level of the foundation in relation to the magnitude of eccentricity of the resultant loads vis-a-vis the neutral axis is normally assumed. This is not fair to the structure. It is often necessary to arrive at foundation settlement accurately or, at least have indication of the probable magnitude for such structural displacement.

The main objective of this paper, therefore, is to compare steel tall frames twenty storeys on rigid strip supports ($k=\infty$) with those on strip foundation testing on elastic sub-base ($k = 1500T/m^3$) The former is believed to reflect the status quo and the latter is believed depicts the real activities on the construction site where all designed inner forces (bending moments, shearing forces etc) are usually redistributed. Cracks that often appear within the walls of tall buildings during constructions point to this phenomenon. It has also been recognized that foundation engineering is complicated (1). Also settlement has been accepted as stress induced and time dependent quantity. The approach to this work is to introduce frame system, which will approximately and convincingly represent the activities within the cross-section of the structure during construction on site. The Winkler model (1867) has been employed to achieve this. This represents the strip on elastics base (k=l500T/m^3) - Fig. 1, which gives way for the vertical displacement. Additionally the result here can be compared with those on rigid strip (k= ∞) supports Fig 2.



b. tall frame and displacements.

Fig 1. Principles of frame as elastic sub-base

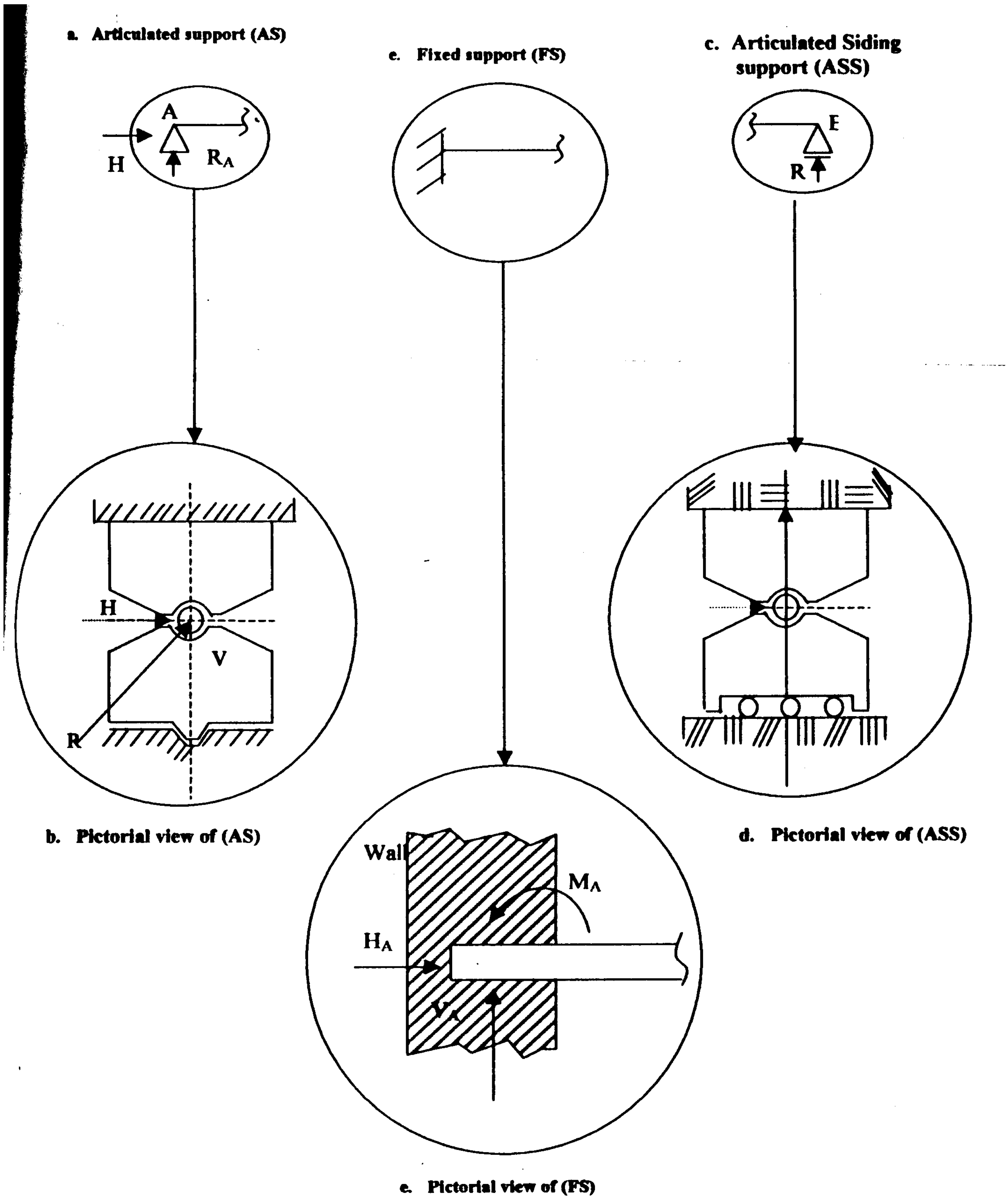


Fig.2. non –deformable rigid supports

2. METHODOLOGY

2.1 SUB- BASE

Gorbanow –posadow and other researchers (1,2,6,7,) described the coefficient (k) for the elastic sub-based as;

$$K = \frac{P}{A} * \frac{1}{\delta} \qquad (1)$$

Where,
p = load acting on a rigid slab, pressed into the soil,
A = area of pressed slab,
δ = vertical settlements of sub-base under the slab.
The equation denotes the basis for this research and the approach to the description of the value of the coefficient (k).

2.2 BEAMS ON ELASTIC SUB-BASE

The basic differential equation used evolved from;

$$EI\frac{dy^4}{d\times^4} = q(\times) \qquad (2)$$

Where,
E = modulus of elasticity (KN/mm^2),
I = moment of inertia (mm^4),
In the case of the sub-base pressure,

$$q(\times) = P(\times) - g(\times) \qquad (3)$$

Where,
P(X) = external pressure due to uniformly distributed loads
Assuming a one parameter sub-base (Windler model) g(X) =k.y (4)
.

g(x) = intensity of the sub-base pressures and K=k_o b (5)
Where,
K_o (T/m^3) = the coefficient of sub-base settlement, b (cm) = width of the beam

Thus, taking the above consideration, the equation (4) now takes the form;

$$EI\frac{dy^4}{dx^4} + ky = P(\times) \qquad (6)$$

The principle of beam on one parameter elastic sub-base is presented on fig.010. fig. 01 also displays the approximate pattern that the structural analysis follows. These are the analysis of;

i. Tall steel frames on rigid strip support (K=∞)
ii. Tall steel frames on strip foundation resting on elastic sub-base (K=1500T/m^3)
iii. Strip foundation (beam) on elastic sub-base; (K=1500T/m^3)
iv.

2.3 TALL STEEL FRAMES

The analysis of tall steel frame on rigid strip foundation (K=∞) and those on the elastic sub-based (K=1500T/m^3) is a complex assignment. The analysis involves calculation of indeterminate systems elastic support into the whole frame analysis. It is better to have the full understanding of the statically supports, their corresponding reactions and elastic sub-base. (figs. 01 &02). The load acting, on the frame system includes, dead, imposed and wind loads. Methods adopted for the structural analysis included, the Guldan and slope deflection methods

2.1.1 SLOPE DEFLECTION METHOD

The slope deflection equation is in matrix from as below;

$$\begin{bmatrix} a_{11} & a_{12} — -a_{1n} & a_{1n} \\ a_{21} & a_{22} — -a_{2n} & a_{2n} \\ - & - \; - \; - & - \\ - & - \; - \; - & - \\ a_{I1} & a_{I2} — -a_n & a_m \\ a_{m1} & a_{m2} — -a_m & a_{mn} \end{bmatrix} \times \begin{bmatrix} X_1 \\ X_2 \\ - \\ - \\ X_1 \\ X_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ — \\ y_1 \\ y_n \end{bmatrix} \qquad (7)$$

This stiffness matrix can briefly be presented as follows;

$[A]_{m,n} * [X] = [y]$ (8)

Where,

[A] = square matrix termed stiffness matrix,

[X] = Vector column matrix,

[y] = displacement column matrix,

;

A computer program was developed also, to solve the numerous differential equations form the method.

Changing to the specific matrix notation, the reaction components and displacements' components can further be written into the following from

i. For member fixed at both ends,

$$\begin{bmatrix} m_{p1} \\ m_q \\ — \\ - \\ m_{ri} \\ m_v \end{bmatrix} = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & - & - & - & \frac{6EI}{L^2} & \frac{6EI}{L^2} \\ \frac{2EI}{L} & \frac{4EI}{L} & - & - & - & \frac{6EI}{L^2} & \frac{6EI}{L^2} \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ \frac{6EI}{L^2} & \frac{6EI}{L^2} & - & - & - & \frac{12EI}{L^3} & \frac{12EI}{L^3} \\ \frac{6EI}{L^2} & \frac{6EI}{L^2} & - & - & - & \frac{12EI}{L^3} & \frac{12EI}{L^3} \end{bmatrix} \begin{bmatrix} p \\ q \\ - \\ - \\ r \\ s \end{bmatrix} + \begin{bmatrix} m_{p1}^0 \\ m_q^0 \\ - \\ - \\ m_n^0 \\ m_v^0 \end{bmatrix} \qquad (9)$$

This can briefly be presented as

$[m_1] = [K_1] * [\delta_1] + [m_1^0]$ (10)

Where

$[m_1]$ = vector column of general forces acting at members ends

$[K_1]$ = stiffness matrix of member

$[\delta_1]$ = vector column of displacements' components of members;

$[m^0{}_1]$ = vector column of the free expression showing values of reaction of member joints by assuming full fixed-supports of members loaded with external loads.

$$[K]^P = \begin{array}{c} P \\ q \\ r \\ s \end{array} \begin{bmatrix} \frac{3EI}{L} & 0 & \frac{-3EI}{L^2} & \frac{3EI}{L^2} \\ 0 & 0 & 0 & 0 \\ \frac{-3EI}{L^2} & 0 & \frac{3EI}{L^3} & \frac{-3EI}{L^3} \\ \frac{3EI}{L} & 0 & \frac{-3EI}{L^3} & \frac{3EI}{L^3} \end{bmatrix}$$

(11)

And with left side having articulated support, the stiffness matrix takes the notation as

ii. The stiffness matrix for the case of member with one –end fixed and other right-side with articulated support takes the form;

$$[K] = \begin{array}{c} P \\ q \\ r \\ s \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{3EI}{L} & \frac{-3EI}{L^2} & \frac{3EI}{L^2} \\ 0 & \frac{-3EI}{L^2} & \frac{3EI}{L^3} & \frac{3EI}{L^3} \\ 0 & \frac{3EI}{L^2} & \frac{-3EI}{L^3} & \frac{3EI}{L^3} \end{bmatrix}$$

(12)

These equations were employed throughout the analysis.

3. OBSERVATION

Tall steel frames (twenty storeys) loaded both with vertical (dead and imposed) loads and lateral (wind) loads were considered in this paper. The frames are analysed on rigid strip supports (K=∞) as well as resolved as resting on elastic sub-based ($K=1500T/m^3$). The results achieved are very interesting and be presented as follows;

3.1 There is no vertical deflection under the tall steel frames on rigid supports ($K=\infty$). See (fig.3a) supports-(y5 =0; y9=0; y15 =0).

3.2 However, there are vertical displacement under the tall steel frames on strip foundation resting on elastic sub-base, ($K=1500T/m^3$). Fig. 4

3.3 Dead and imposed loads are the major influential contributors to the vertical displacement, under the strip foundation resting on the elastic sub-base.

3.4 The lateral load is however the main cause of the existing horizontal displacement; (fig.5)

3.5 The vertical settlements form the lateral loads (wind) are minimal (1.7mm) (table 1).

3.6 Horizontal displacement of the frames resting on elastic sub-base is 30% larger than the frame on non-deformable supports, (fig 5 & table 1).

4. CONCLUSION

The research examined the implications of the working conditions of orthogonal 20-storeys frame systems when supported on the elastic sub-base. The summarized results are as follows.

I. There are redistribution of inner forces in frames resting on elastic sub-base ($K=1500T/m^3$).

II. The winkler model used, proved that vertical settlements always occur on construction sites, under structural tall steel frames.

III. The rigid support ($K=\infty$) without the consideration of the elastic support modeling does not represent well the true activities of that foundation, on site.

IV. The lateral displacement of the frame resting on elastic sub-base is well above those frames on rigid foundation (> 30%).

V. Vertical displacement caused by dead and imposed loading y_3=9.7mm; y_9=12.7mm; y_{15} =9.7mm; are well above those caused by wind loads (y_3=1.7mm; y_9=0.00; y_{15} =1.7mm).

VI. The wind load has more influence on the horizontal deflections than the vertical deflections.

VII. The lateral load effect on the vertical displacement is minimal (8=1.7mm).

The values of the displacements are as in table 1 below.

Table 1: VALUES OF DISPLACMENTS (mm)

Description	Displacement (mm)	
	$K=\infty$	$K=1500T/m^3$
Support settlements for 20 storeys loaded vertically	y_3 =0.0 y_9-0.0 y_{13} = 0.0	y_3 = 9.7 y_9 = 12.7 y_{13} = 9.7
Support settlements for 20 storeys loaded horizontally	y_3 = 0.0 y_9 = 0.0 y_{15} = 0.0	y_3 = 1.7 y_9 = 0.0 y_{15} =1.7
Maximum lateral deflection of the 20 storey loaded Horizontally	δ = 60	δ =84

Note: y=(mm) δ = (mm)

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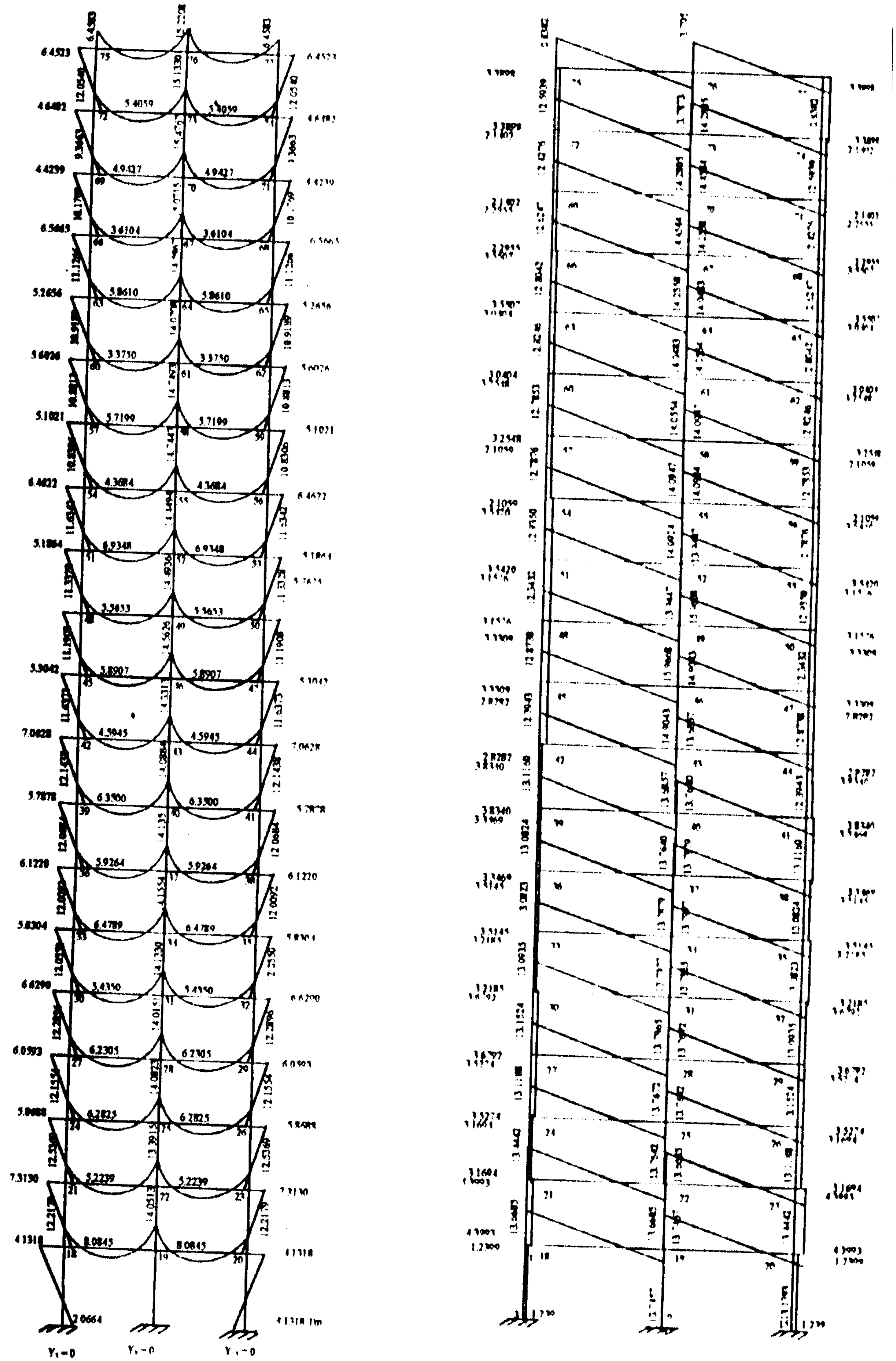
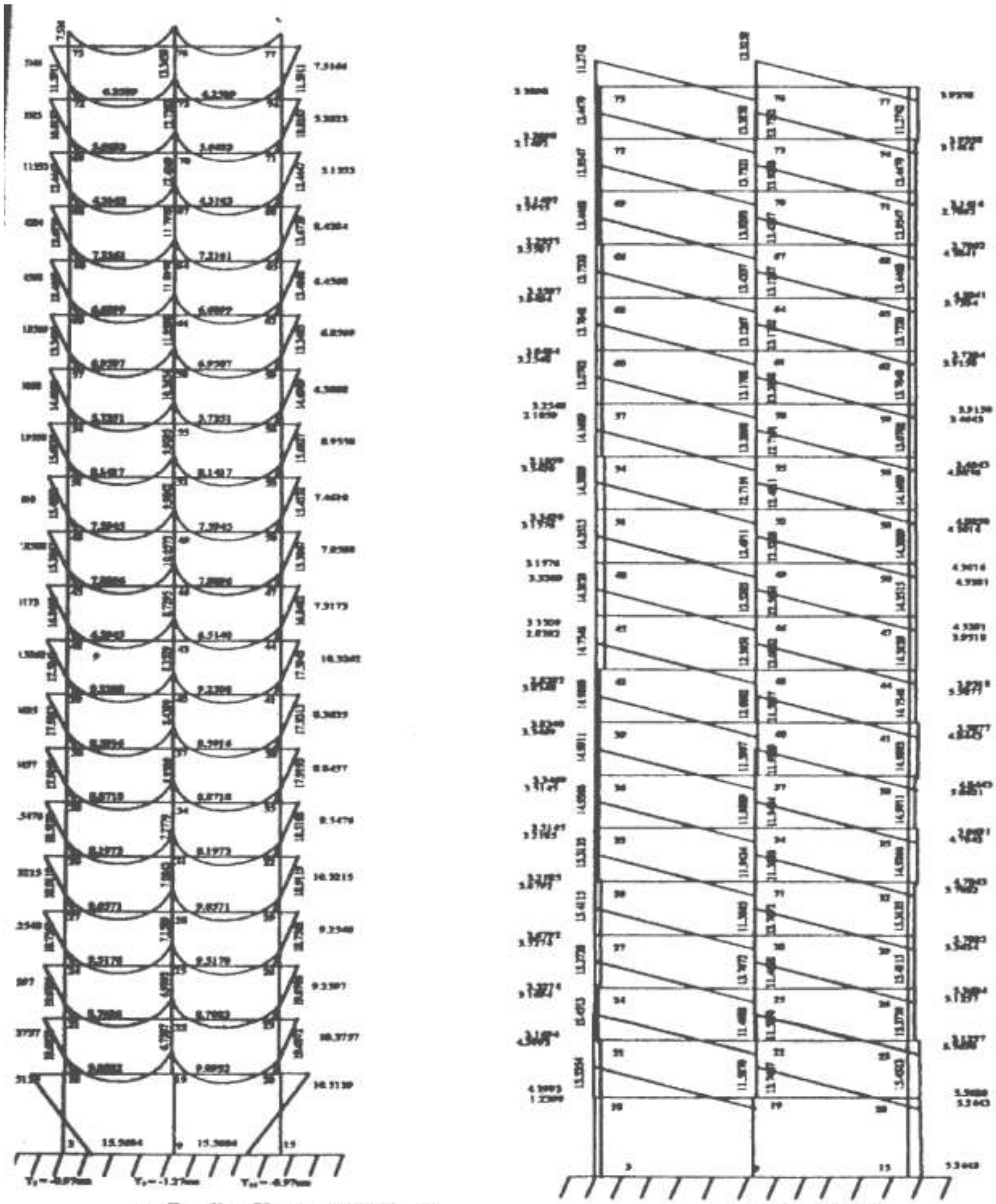


a. Bending Moments (Tm)

Fig. 5: Frames rigid foundation (k = ∞)

b. Shear (T)



a. Bending Moments (M_x[Tm]) b. Shear Q_x[T]

Fig. 4: Frames with foundation on elastic sub-base (k = 1500 T/m³)

20 storeys

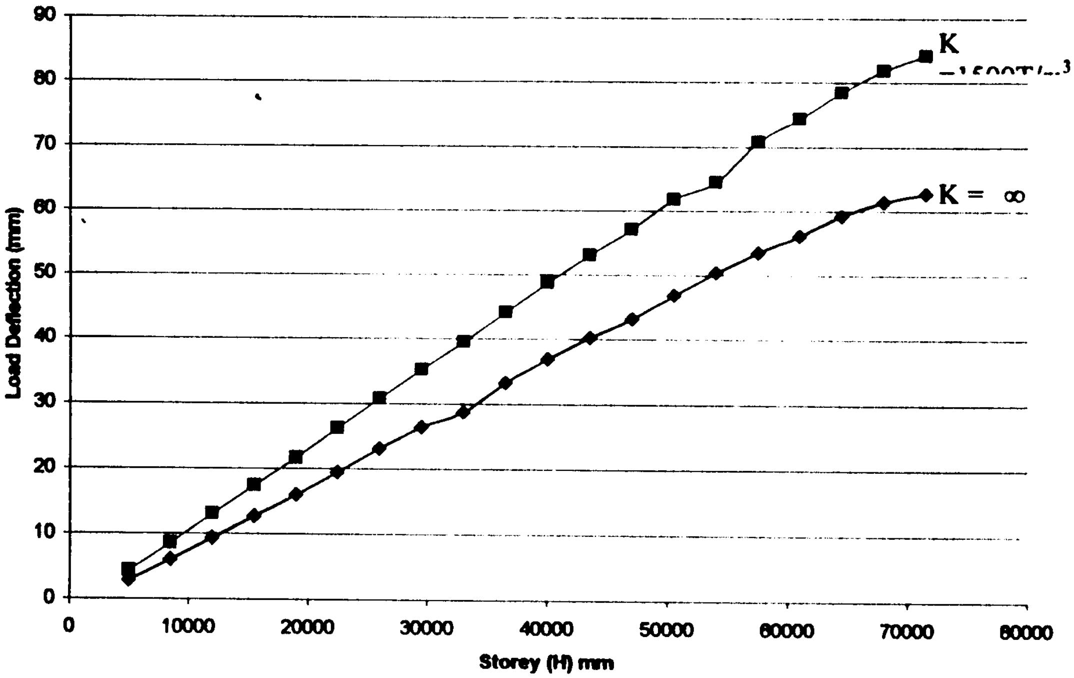


FIG 5: Storey / displacement curves