DYNAMIC MODELING OF NATURAL CONVECTION SOLAR ENERGY COLLECTORS FOR AGRICULTURAL DRIERS I: THEORY AND COMPLETE RUN OF THE SOLUTIONS TO THE DYNAMIC MODEL EQUATIONS OF SSSCA FLAT PLATE COLLECTOR

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ABSTRACT

An analysis is *made of the dynamic model of a flat plate solar energy collector to be employed in natural convection solar energy driers used for drying agricultural products like meat, fish, grains and vegetables and bring them to acceptable equilibrium moisture content to avoid spoilage when stored. Analytical solutions of the derived model equations enabled expressions for the mean glazing, absorber and outlet fluid temperatures to be obtained. Expressions were also obtained for the collector instantaneous energy delivery rate, efficiency, heat removal factor, and combined plates coefficient of performance (formally called plates efficiency). A plot of the collector output parameters against time from* Os *(at 6.00a/m.) showed that each possessed a maximum, which occurred between 1800s and 3600s after solar noon where global radiation had its maximum. The shift is attributable to the heat capacities of the materials of the collector Separation of the transient terms from the steady- state terms for the plates' temperatures showed the transient contribution to be very small and practically die out between 14400s and* 16200s *from start, showing that the transient terms can be neglected. This was confirmed when the transient terms were completely deleted as the maximum values of the output parameters and their times of occurrence remained the same in both cases. The developed output expressions (in closed form) for the dynamic model of flat plate solar energy air heating collectors can easily be used for optimization studies and design of better air heating solar energy collectors.*

KEYWORDS AND EXPRESSIONS*: Solar collector, Natural Convection, Dynamic and Transient Analyses, Dynamic model closed form output expressions.*

1. INTRODUCTION

Without the radiant energy from the Sun in the forms of heat and light (two forms of electromagnetic radiations), life processes on earth will be impossible. However, the efficiency of natural conversion of solar energy is so low (about 1 %) with attendant wastages that the use of man-made equipment for its collection and utilization becomes inevitable. Solar energy collectors in current use can be broadly classified into concentrating collectors, which employ reflecting mirrors (e.g. parabolic and spherical dishes) and refracting lens in solar energy collection and, non-concentrating collectors like the flat plate collector. Solar energy collectors act as secondary thermal energy sources for agricultural driers, water heaters, thermal ovens, and low temperature and industrial process heat generation systems.

Though the radiant energy from the sun is constant, the fraction that actually reaches the earth's surface and is available for use is variable in time and space due to the sphericity of the earth's surface (zonal variation), the rotation of the earth about its axis (diurnal variation) and its revolution around the sun (sideral variation). The above effects are taken care of by the latitude L, the hour angle! (=ωt), and the declination, δ, in the expression for the solar energy falling on it. Hence, a dynamic model incorporating these variations becomes inevitable. The solar energy available for collection and use is also dependent on climatological variables like cloud cover, wind, fog, precipitation, dust and haze. The presence of any of these reduces the amount of collectable solar energy by a solar collector.

Though many papers and books have appeared for the solution of the solar energy heat transfer problem in a forced convection environment (Bhargave et al, 1982; Bala and Woods, 1994, Grainger, et. al.. 1983; Tyer, 1985; Ocsthuizien, 1987; Ready, 1987; Sukhatme, 1984), such solutions under a purely and exclusively natural convection environment remains unresolved as some of the results obtained under the accompanying assumptions are conflicting. Moreover, the transient solutions given so far employ numerical methods and in some cases coupled with analytical methods of which are more cumbersome than a direct analytical solution. The avoidance of direct analytical solution of the derived model equations for a solar collector by past investigators has made it impossible for them to obtain relations for the temperatures of the cover and absorber plates and the outlet fluid stream and this paper is an improvement on the, whether in a stand- alone configuration (Ghargava, et. al, 1982; Grainger, et. al. 1983), or in conjunction with a crop drier (Bala and Woods, 1994; Iyer, 1985; Oosthuizien, 1987).

Though climatological variables like cloud cover, fog, mist, haze, precipitation, dust and wind affect the operation of solar collectors, their direct incorporation will overcomplicate the model equations and hence, their effects where possible, will be reflected through the heat transfer coefficients. The temperature dependence of properties (density, thermal conductivity, viscosity, and heat capacity) of the materials of the collector and the fluid stream will be assumed constant during the analysis because of their small variation within the temperature range of the collector operation in order to make the model equations solvable but will be incorporated where the relations are available in the output calculations. The seasonal nature of cropping and the desire to dry the products in a few days to avoid spoilage and consequent wastages when stored makes the dynamic analysis given below imperative. Moreover, solar driers in current use are designed to operate under constant inlet fluid temperatures. Such conditions do not obtain in practice as the inlet fluid (ambient air) temperature always changes with time of day. The temperature range of crop driers coupled with the high diffuse component of solar radiation in tropical regions and the ability of flat plate collectors to capture this component and hence operate under cloud cover conditions necessitated its choice in the given analysis. The experimental validation of the solutions to the derived dynamic model equations are given in (Onuoha, a) in this volume. Nomenclature and references are also given there.

2. THE DYNAMIC MODEL OF A FLAT PLATE COLLECTOR

2.1 Dynamic Model Equations

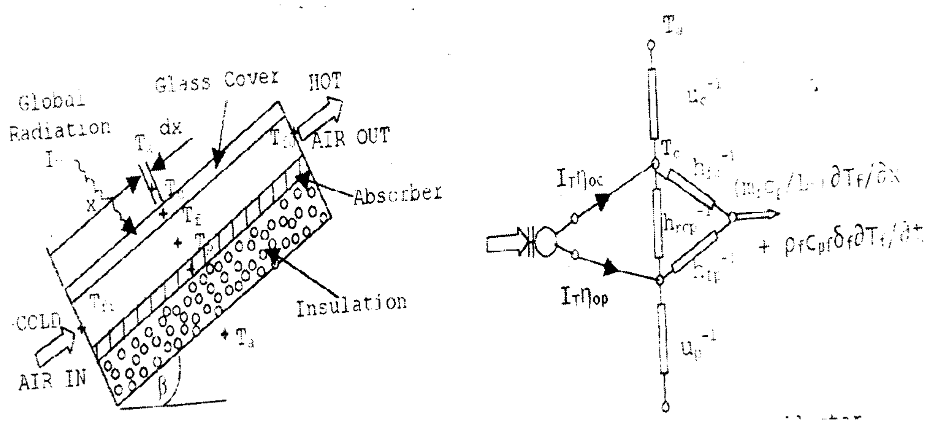

Fig. 1: Cross Section of SSSCA Flat Plate Solar Energy Thermal Collector showing the temperatures at position x and at inlet and outlet sections, indicated by +, and its termal Energy Circuit.

NOTE: SSSCA means single glazing, single-pass, single-flow, and air heating flat plate solar energy collector with flow between the cover and the absorber plates.

β = collector tilt.

Fig. 1 shows the cross-section of the collector under consideration together with its thermal energy circuit from which can be written the model equations derived from the nodal equations for an element of the collector dx, at position x from the inlet fluid boundary and at time t s from start:

COVER: $(\rho c_p\delta)_c \partial T_c/\partial t = I_T\eta_{oc} - u_c(T_c - T_a) - h_{rcp}(T_c - T_p)\text{-}h_{fc}(T_c - T_f)$ **(1)**

FLUID: $(\rho c_p\delta)_f \partial T_f/\partial t + (^{mf}C_{pf}/L_1 \partial T_f/\partial x) = -h_{fc}(T_f - T_c) - h_{fp}(T_f - T_p)$ **(2)**

ABSORBER: $(\rho c_p\delta)_p \partial T_p/\partial t = I_T\eta_{op} - u_p(T_p - T_a) - h_{rcp}(T_p - T_c) - h_{fp}(T_p - T_f)$ **(3)**

2.2 Initial Condition

At $t = 0$, $\partial T_c/\partial t = \partial T_p/\partial t = \partial T_f/\partial t = 0$ (4)

$T_c(0) = T_{c0}),\quad T_p(0) = T_{p0}, T_{fi}(0) =, T_{fio}, T_{f0}(0) = T_{foo}$ (5)

Also for operation from cold at t= $0s, T_{c0} = T_{p0} = T_{fi0}, = T_{foo} = T_{a0}$ (6)

where T_{a0} is the ambient air temperature at time, t = 0s.

2.3 Boundary Conditions

At x = 0, $T_f = T_{fi} = T_a$' $t \geq 0$ (7)

At x = L_2, $T_f = T_{fo}$ $t \geq 0$ (8)

2.4 Simplifying Assumptions

(i) The temperatures of the cover and absorber plates are uniform and can be replaced by their mean values necessitating the replacement of their time rates of change by total differentials.

(ii) Since the heat capacity perunit area of the heat transfer fluid (air) is much smaller than those of the cover and absorber plates, the time rate of change of the fluid temperature will be much smaller than its position rate of change (especially, with good or perfect insulation) and can thus be neglected since the heat capacity per unit area of the heat transfer fluid (air) is much smaller than those of the cover and absorber plates, the time rate of change of the fluid temperature will be much smaller than its position rate of change (especially, with good or perfect insulation) and can thus be neglected. (Representative values: $(\rho c_p\delta)_c =$

2720.0 x 840.0 x 0.004 = 9139.2; $(\rho c_p\delta)_p$ 7820.0 x 473.3 x 0.0035 = 12954.221; and $(\rho c_p\delta)_{air}$ = 1.204 x 1004.547 x 0.02 = 24.189,. all in $JK^{-1}m^{-2}$. $(\rho c_p\delta)_{air}/(\rho c_p\delta)_c$ = 0.002648 or 0.2648%, $(\rho c_p\delta)_{air}/(\rho c_p\delta)_p$ = 0.001867or 0.1867%]. Hence, the position rate of change of the fluid temperature can be replaced with a total differential. This means that the temperature gradient of the fluid along the plate in the flow direction is the same at all times. Hence, the' position rate of change of the fluid temperature can be replaced with a total differential. This means that the temperature gradient of the fluid along the plate in the flow direction is the same at all times. Similarly, assumption (i) implies that the time rate of change of the plates temperatures is the same at all positions for each plate.

2.5 Solutions of the Dynamic Model Equations

With the above simplifications, we can write

$$T_f = T_f(x), T_c = T_c(t), \text{ and } T_p = T_p(t), \tag{9}$$

With the initial condition, eqn. (4) replaced by

$$dT_c/dt = dT_p/dt = 0 \ at \ t = 0 \tag{10}$$

and all other conditions remaining as before. Eqn. (1) - (3) now become,

$$(\rho c_p\delta)_c dT_c/dt = I_T\eta_{oc} - u_c(T_c - T_a) - h_{rcp}(T_c - T_p) - h_{fc}(T_c - T_f) \tag{11}$$

$$(\dot{m}_f c_{pf}/L_1)dT_f/dx = h_{fc}(T_f - T_p) \tag{12}$$

and

$$(\rho c_p\delta)_p dT_p/dt = I_T\eta_{op} - u_p(T_p - T_a) - h_{rcp}(T_p - T_c) - h_{fp}(T_p - T_f) \tag{13}$$

Since $T_f = T_f(x)$ $\forall t$, integration of eqn. (41) yields

$$T_f = \psi T_{fi} + (1 - \psi)(h_{fc}T_c + h_{fp}T_p)/(h_{fc} + h_{fp}) \tag{14}$$

where the flow parameter,

$$\psi = \exp[-(h_{fc} + h_{fp})L_1x/\dot{m}_f c_{pf}] \tag{15}$$

Because of assumption of constant properties, substitution of eqn. (14) into eqns. (11) and (13) and soling simultaneously yields:

$$[e_cD + u_{cp})(e_pD + u_{pp}) - h_r^2]T_c = (e_pD + u_{pp})(I_T\eta_{oc} + u_cT_a + \psi h_{fc}T_{fi}) + h_r(I_T\eta_{op} + u_pT_a + \psi h_{fp}T_{fi}) \tag{16}$$

and

$$[e_cD + u_{cp})(e_pD + u_{pp}) - h_r^2]T_p = (e_cD + u_{cp})(I_T\eta_{op} + u_pT_a + \psi h_{fp}T_{fi}) + h_r(I_T\eta_{oc} + u_cT_a + \psi h_{fc}T_{fi}) \tag{17}$$

where $D = d/dt$.

$$e_c = (\rho c_p\delta)_c \text{ and } e_p = (\rho c_p\delta)_p \tag{18}$$

$$u_{cp} = u_c + h_r + \psi h_{fc}, \text{ and } u_{pp} = u_p + h_r + \psi h_{fp}, \tag{19}$$

and

$$h_r = (1 - \psi)h_{fc}h_{fp}/(h_{fc} + h_{fp}) + h_{rcp} \tag{20}$$

The solution of eqns. (16) and (17) can be obtained by first substituting for the value of I_T which is given by Liu and Jordan (1960)) as

$$I_T = I_b\cos\theta + 0.51_d(1+\cos\beta) + 0.5\rho I(1-\cos\beta) \quad (21)$$
$$= (I - I_d)r_{b,T} + 0.5I_d(1+\cos\beta) + 0.5\rho I(1-\cos\beta) \quad (22)$$

and by (Onuoha, 1997) as

$$I_T = A\cos2\omega t + B\sin\omega t + C \quad (23)$$

where
$$A = 0.5b\hat{H}\zeta \quad (24)$$
$$B = \hat{H}(a\zeta - b\xi) - \hat{H}_d\sigma_1 \quad (25)$$
$$C = \hat{H}(0.5b\zeta - a\xi) + \hat{H}_d\Omega \quad (26)$$
$$\zeta = R_s[\cos(L-\beta)/\cos L + 0.5\rho(1-\cos\beta)] \quad (27)$$
$$\xi = R_s[\cos(L-\beta)/\sin\omega t_{rr}/\cos L + 0.5\rho(1-\cos\beta)\sin\omega t_r] \quad (28)$$
$$\sigma_1 = R_s[\cos(L-\beta)/\cos L + 0.5(1+\cos\beta)] \quad (29)$$
$$\Omega = R_s[\cos(L-\beta)/\sin\omega t_{rr}/\cos L - 0.5(1+\cos\beta)\sin\omega t_r] \quad (30)$$

and $R_s = 0.5\pi/[\cos\omega t_r - \pi/2 - \omega t_r)\sin\omega t_r]$ (31)

For $\hat{H}$ and $\hat{H}_d$ in MJm^{-2}, $R_s = 625\pi/54[\cos\omega t_r - (\pi/2 - \omega t_r)\sin\omega t_r]$ (32)

By substituting for I_T from eqn. (23) into eqns. (16) and (17) and solving, we obtain:

$$Tc = c_{t1}\exp-(m_1t) + c_{t2}\exp-(m_2t) + c_{AC}\cos2\omega t + c_{AS}\sin2\omega t + C_{BC}\cos\omega t + C_{BS}\sin\omega t + \lambda_c/e_m \quad (33)$$

and
$$T_p = p_{t1}\exp-(m_1t) + p_{t2}\exp-(m_2t) + P_{AC}\cos2\omega t + P_{AS}\sin2\omega t + P_{BC}\cos\omega t + P_{BS}\sin\omega t + \lambda_p/e_m \quad (34)$$

where
$$C_{AC} = (A_{cc}A_r - A_{cs}A_i)/A_{ri} \text{ and } C_{AS} = (A_{cs}A_r + A_{cc}A_i)/A_{ri} \quad (35)$$
$$C_{BC} = (B_{cc}B_r - B_{cs}B_i)/B_{ri} \text{ and } C_{BS} = (B_{cs}B_r + B_{cc}B_i)/B_{ri} \quad (36)$$
$$P_{AC} = (A_{pc}A_r - A_{ps}A_i)/A_{ri} \text{ and } P_{AS} = (A_{ps}A_r + A_{pc}A_i)/A_{ri} \quad (37)$$
$$P_{BC} = (B_{pc}B_r - B_{ps}B_i)/B_{ri} \text{ and } P_{BS} = (B_{ps}B_r + B_{pc}B_i)/B_{ri} \quad (38)$$
$$A_r = u_2 - 4\omega^2, \quad A_i = 2\omega u_1, \text{ and } A_{ri} = e_ce_p(A_r^2 + A_i^2) \quad (39)$$
$$B_r = u_2 - \omega^2, \quad B_i = \omega u_1, \text{ and } B_{ri} = e_ce_p(B_r^2 + B_i^2) \quad (40)$$
$$e_m = e_ce_pm_1m_2 = e_ce_pu_2 = u_{cp}u_{pp} - h_r^2 \quad (41)$$
$$A_{cc} = A(u_{pp}\eta_{oc} + h_r\eta_{op}), \quad A_{cs} = -2A\omega e_p\eta_{oc} \quad (42)$$
$$B_{cc} = B\omega e_p\eta_{oc}, \quad B_{cs} = B(u_{pp}\eta_{oc} + h_r\eta_{op}) \quad (43)$$
$$\lambda_c \; C(u_{pp}\eta_{oc} + h_r\eta_{op}) + (u_{pp}u_c + h_ru_p)T_a + \psi((u_{pp}h_{fc} + h_rh_{fp})T_{fi} \quad (44)$$
$$A_{pc} = A(u_{cp}\eta_{op} + h_r\eta_{oc}), \text{ and } A_{ps} = 2A\omega e_c\eta_{op} \quad (45)$$
$$B_{pc} = B\omega e_c\eta_{op}, \text{ and } B_{ps} = B(u_{cp}\eta_{op} + h_r\eta_{oc}) \quad (46)$$
$$\lambda_c = C(u_{cp}\eta_{op} + h_r\eta_{oc}) + (u_{cp}u_p + h_ru_c)T_a + \psi((u_{cp}h_{fp} + h_rh_{fc})T_{fi} \quad (47)$$

$-m_1$ and $-m_2$ are the roots of the equation in D.

$$(e_cD + u_{cp})(e_pD + u_{pp}) - h_r^2 = e_ce_p(D^2 + u_1D + u_2) = 0 \quad (48)$$

where $u_1 = (u_{cp}/e_c + u_{pp}/e_p)$, and $u_2 = (u_{cp}u_{pp} - h_r^2)/e_ce_p$ (49)

$m_1 = ½(u_1 + m_0)$, $m_2 = ½(u_1 - m_0)$, and $m_0 = (u_1^2 - 4u_2)^{½}$ (50)

The coefficients of the exponential terms are obtained by applying the initial conditions. The

Outlet fluid temperature, T_{fc} is obtained by substituting for T_c and T_p from eqns. (33) and (34) into equn. (14) and x = L2 in egun. (15); i.e.

into equn. (14) and $x = L_2$ in equn. (15); i.e.

$$T_{fo} = \psi T_{fi} + (1 - \psi)\ (h_{fc}T_c + h_{fp}T_p)/h_{fc} + h_{fp}) \quad (51)$$

$$= T_{fi} + [(1 - \psi) F_p/(h_{fc} + h_{fp})]\ \{[f(t)/F_p\eta_{op} + C]\eta_{op} - U_L(T_{fi} - T_a)\} \quad (52)$$

where the flow parameter, $\psi = \exp[-(h_{fc} + h_{fp})A_c/\dot{m}_{fc}C_{pf}]$ (53)

and the collector area exposed to solar radiation, $A_c = L_1L_2$ (54)

$$F_p = [u_{cp}h_{fp} + h_rh_{fc} + (u_{pp}h_{fc} + h_rh_{fp})\eta_{oc}/\eta_{op}]e_m \quad (55)$$

is the combined plates coefficient of performance,

$$U_L = [h_{fc}\ (u_{pp}u_c + h_ru_p) + h_{fp}\ (u_{cp}u_p + h_ru_c)]/F_pe_m \quad (56)$$

is the collector overall heat loss coefficient, and the dynamic time function, f(t) is given by

$$f(t) = h_{fc}\ (T_c - \lambda_c/e_m) + h_{fp}\ (T_p - \lambda_p/e_m) \quad (57)$$

The energy delivery rate of the collector is given by

$$Q_{uD} = \dot{m}_fC_{pf}\ (T_{fo} - T_{fi}) = F_RA_c\ \{[f(t)/F_p\eta_{op} + C]\eta_{op} - U_L\ (T_{fi} - T_a)\} \quad (58)$$

where $F_R = (\dot{m}_f\ C_{pf}/U_TA_c)\ \{1 - \exp[-F_pU_TA_c/\dot{m}_f\ C_{pf}]\}$ (59)

is the collector heat removal factor and $U_T = (h_{fc} = h_{fp})/F_p$ (60)

is the collector overall heat transfer coefficient. The instantaneous collector efficiency, η_{ci}, is given by the ratio of the collector energy delivery rate to the rate of radiant energy incident on it plane, i.e, $\eta_{ci} = Q_{uD}/I_TA_c = (F_RI_T)\ \{f(t)/F_p\eta_{op} + C]\eta_{op} - U_L(T_{fi} - T_a)\}$ (61)

The collector overall efficiency, η_{co}, is given by

$$\eta_{co} = (\int Q_{uD}dt)/(\int A_cI_Tdt) \approx (\Sigma Q_{uD}\Delta t)\ /(\Sigma A_cI_T\Delta t) = Q_{uD.T}\Delta t/A_cH_T\Delta t = Q_{uD.T}/A_cH_T \quad (62)$$

3.0 PERFORMANCE PARAMETERS OF DYNAMIC FLAT PLATE COLLECTOR MODEL

3.1 Fluid Velocity and Mass Flow Rate

The velocity and mass flow rate of the heat transfer fluid generated in a natural conection air heating flat plate solar collectors are given by Onuoha (c, in press) respectively as

$$V_f = (32\mu_bL_2/\rho_fD^2)\ \{[1 + (\rho_a - \rho_f)\rho_fD^4 g\sin\beta/512\mu_b{}^2L_2]^{1/2} - 1\} \quad (63)$$

and $\dot{m}_f = \rho_fA_fV_f = (32\mu_b\delta_fA_c/D^2)\ \{[1 + (\rho_a - \rho_f)\rho_fD^4 g\sin\beta/512\mu_b{}^2L_2]^{1/2} - 1\}$ (64)

where $D = 2L1\delta_f/(L1 + \delta_f)$ (65)

is the hydraulic diameter of the fluid flow channel cross-section of width L_1, and depth δ_f, and μ_b is the fluid bulk dynamic viscosity.

3.2 Optical Efficiencies of the Cover Plate, η_{oc}, and Absorber Plate, η_{op}.

These are given by Onuoha (a, in press) respectively as

$$\eta_{oc} = 1 - \rho - \tau + \tau\ (1 - \alpha_{rp})(1 - \rho_d - \tau d)/\{1 - \rho_d(1 - \alpha_{rp})\} \quad (66)$$

and $\eta_{op} = \tau\alpha_{rp}/\{1 - \rho_d(\{1 - \alpha_{rp})\}$ (67)

and computed as average of different computations for the polarization of the reflectivity of light, i.e. $\eta_{oc} = 0.5\ (\eta_{oc}{}^1 + \eta_{oc}\mathrm{I})$ and $\eta_{op} = 0.5(\eta_{oc}{}^1 + \eta_{oc}\mathrm{I})$ (68)

3.3 Heat Transfer Coefficients

Cover-to-Ambient Heat Loss Coefficient, $u_c = h_w + h_{rcs}$ (69)

where h_w is the wind speed in ms^{-1}, and h_{rcs} is the cover-to-pay radiation heat loss coefficient, given by McAdams correlations as

$h_w = 5.7 + 3.8V_w$ (70)

where V_w is the wind speed in ms^{-1}, and h_{rcs} is the cover-to-pay radiation heat loss coefficient, given by $h_{rcs} = \sigma\varepsilon_c (T_c^4 - T_s^4)/(T_c - T_s)$ for $T_c \neq T_s$ (71)

and $h_{rcs} = \sigma\varepsilon_c (T_c - T_s)(/T_c^2 - T_s^2)$ for $T_c = T_s$ (72)

where σ is the Stefan-Boltzmann constant, ε_c is the emissivity of the cover plate and T_s is the effective sky temperature given by Klein's correlation as $T_s = T_a - 6$ (73)

Fluid-to-Cover and Absorber Heat Transfer Coefficients, h_{fc}, and h_{fp}, are given by

$$h_{fc} = h_{fp} = h_f = Nuk_f/D \quad (74)$$

The Nusselt Number, Nu, to be used is given by the brown and Gauvin (1965) correlation as recommended by Holman (1976) and Kreider and Kreith (1981), i.e.,

$$Nu = 1.75 (\mu_b/\mu_w)^{.14} \{Gz + 0.012 (GzGr^{1/3})^{4/3}\}^{1/3} \quad (75)$$

Gz is the Graetz Number given by $Gz = RePrD/L, 10^{-2} < PrD/L < 1$ (76)

Pr is the Prandtl Number, given by $Pr = \mu_b C_p/k_f$ (77)

Re is the Reynolds Number, given by $Re = \rho_f V_f D/\mu_f$ (78)

Gr is the Grashof Number, given by $Gr = g\beta_T (T_w - T_b)D^3\rho_f^2/\mu_f^2$ (79)

β_T is coefficient of cubical expansion, given by $\beta_T = 1/T_f$ (80)

T_f is the film temperature, given by $T_f = \frac{1}{4}(T_c + T_p + T_{fi} + T_{fc})$ (81)

T_w is fluid wall temperature, given by, $T_w = \frac{1}{2}(T_c + T_p)$ (82)

T_b is fluid bulk temperature, given by, $T_b = \frac{1}{2}(T_{fi} + T_{fc})$ (83)

All physical properties are evaluated at the fluid film temperature unless as specified.

Cover Plate-to-Absorber Plate Heat Transfer Coefficient, h_{rcp} is given by

$$h_{rcp} = \sigma(T_c + T_p)(/T_c^2 + T_p^2)/(\varepsilon_c^{-1} + \varepsilon_p^{-1} - 1) \quad (84)$$

Absorber-to-Ambient Heat Loss Coefficient u_p is given by

$$u_p = u_s + u_b = L_3 (L_1^{-1} + L_2^{-1}) k_i/\delta_{is} + (\delta_{ib}/k_i + 1/h_{ia})^{-1} \quad (85)$$

Where us and u_b are the side-, and bottom-to-ambient heat loss coefficients, L_3 is the side heat transfer depth, L_1 is the collector width and L_2 is its length, k_1 is the insulation thermal conductivity, δ_{is} is the side insulation depth, δ_{ib} is the bottom insulation depth, and h_{ia} is the insulation to ambient heat loss coefficient, taken as infinite.

4. OUTPUT CALCULATIONS, RESULTS AND DISCUSSION

4.1 Input Parameters

The transport properties of the heat transfer fluid were obtained from a least-square analysis of tables of properties of air at low atmospheric pressure from Mayhew and Rogers (1977), Holman (1976), and Welty, et al (1976), severally compared in the temperature range: $250 \leq T \leq 400$ K, for the dynamic viscosity μ, specific heat capacity, c_p, and thermal conductivity, k.

μ = 3.5555E-14T^3 - 7.009E-llT^2 + 7.9882E-08T - 1.557E-07 Hsm^2 (86)
C_p = 5.3333E-07T^3 - 5.257E-08T^2 + 6.3333E-03T + 1003.65714 Jkg^{-1}K^{-1} (87)
k = 1.7777E-11T^3 - 5.257E-08T^2 + 1.043E-04T - 7.978E-04 Wm^{-1}K^{-1} (88)
The density of air, ρ = 353.0001988T^{-1} (89)
is obtained from the perfect gas equation, $P/\rho = R_oT/M$ (90)
with R_o = 8.3143 J mol^{-1}K^{-1}, from Zemansky (1968), P = 101325 Nm-^{2}, and M = 0.0289657kg mol^{-1}, calculated using the table of composition of air by Agrawal and Rao (1974).
The ambient air temperature, T_a is givern by the expression (Onuoha, a)

$$T_a = T_{a0} + T_{a1}t + T_{a2}t^3 + T_{a4}t^4, for\ 0 \leq, \leq 43200\ s \quad (91)$$

where $T_{a0}, T_{a1}, T_{a2}, T_{a3}\ and\ T_{a4}$ are constant

4.2 Other Parameters Used in the Calculations

α = 0.95, β = 0.174533 rad., C_{cp} = 840.0 Jkg^{-l}k^{-1}, C_{pp} = 473.3 Jkg^{-l}k^{-1}, δ_o = 0.004m, δ_f = 0.02m, δ_p = 0.0035m, δ_{ib} = 0.1925m, δ_{is} = 0.07m, ε_c = 0.94, ε_p = 0.97,g = 9.81 ms^{-1} , H = 12.6MJm^{-2} k_{ec} = 30 m^{-1} , k_{ib} = k_{is} = 0.059 Wm^{-1}K^{-1} , L = 0.0119555 rad (for Nsukka, Nigeria), L_1 = 0.94m, L_2 = 1.225m, L_3 = 0.23m, n = 82, n_{rc} = 1.526, ρ_c = 2720.0 kgm^{-3}, ρ_p = 7820.0kgm^{-3}, ρ_g = 0.2, σ = 5.6697E-08Wm^{-2} K^{-4}, Wm^{-2}K^{-4},θ_d = 1.047198 rad., V_w, = 0.74ms^{-1}, (for 23-03-02, T_{a0} = 298.83499K, T_{a1} = -5.367E-5 Ks^{-1}, T_{a2} = 3.1721E-08 Ks^{-2}, T_{a3} = -8.571E-13 Ks^{-3}, T_{a4} = 5.3463E-18 Ks^{-4}).

4.3 Results and Discussion

The output parameters were calculated at 1800.0s intervals using a computer programme written for that purpose. The interval was chosen to lie between the two time constants m_1^{-1}, and m_2^{-1}at time t = 0s. The model outputs for four days: 14-03-02, 16-03-02, 21-03-04, and 23- 03-04 are shown in figs. 2a and b; 3a and b; 4a and b; and 5a and b respectively. For each day considered, the first graph shows the ambient (fluid inlet) Ta = $T_{fi,}$ cover $T_{c,}$ absorber $T_{p,}$ and fluid outlet T_{fr}, temperatures; collector energy delivery rate Q_{uD}, and global radiation I_T, on its plane computed from eqns. (91), (33), (34), (5V1/52), (58) and (23) respectively for the indicated measured daily global radiation H on a horizontal plane. The second graph for each day considered shows the optical efficiencies on the cover η_{oc}, and absorber $\eta_{op,}$ fluid outlet velocity V_f, and mass flow rate, m_f; combined plates coefficient of performance F_p, collector heat removal factor F_R, and instantaneous efficiency η_{ci} calculated respectively from eqns (66), (67), (63), (64), (55), (59) and (61). In each case considered, the global radiation and absorber plate optical efficiency are symmetrical about the solar noon where they have their maximum. Though the optical efficiency of the cover plate is symmetrical about the solar noon (t 21600.05), it has local minimum there. All other parameter also pass through maximums but at points, which depend on the parameter. For the fluid velocity and collector energy delivery rate and generally for the fluid mass flow rate, their maximums are at solar noon. For the plates', and outlet fluid temperatures and collector instantaneous efficiency, the points at which they possess maximums are displaced from solar noon by as much as 1800.0-3600.0s.

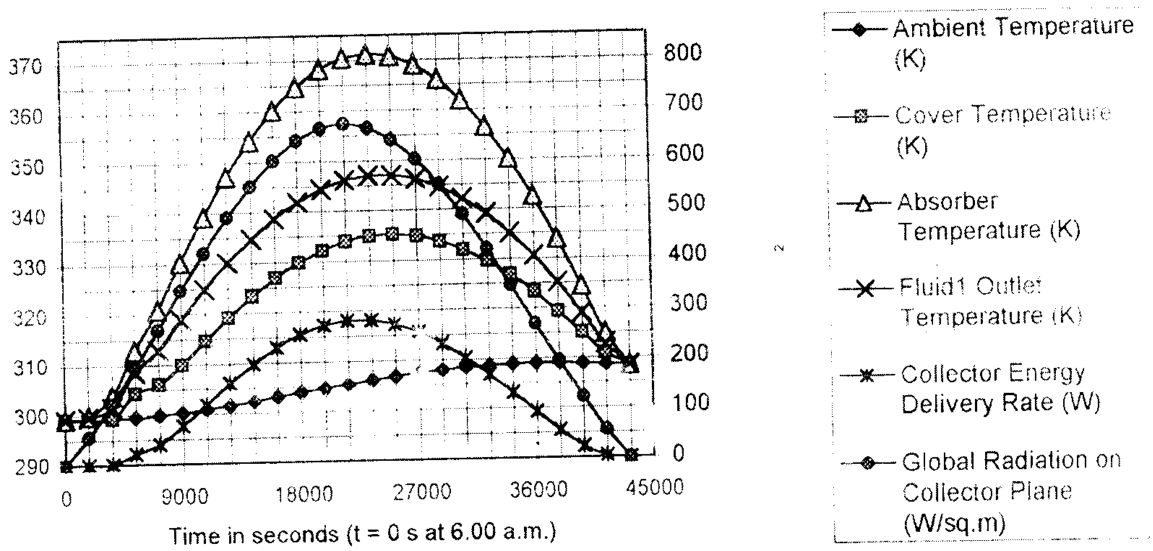

Fig 2a SSSCA Flat Plate Collector Model Input and Output Parameters on 14-03-02

$T_{a} = T_{fi} = 299.09883 - 1.323E\text{-}04t + 3.596E\text{-}08t^2 - 8.711E\text{-}13t^3 + 5.1751E\text{-}18t^4$ K, $H = 17.0$ MJm^{-2}

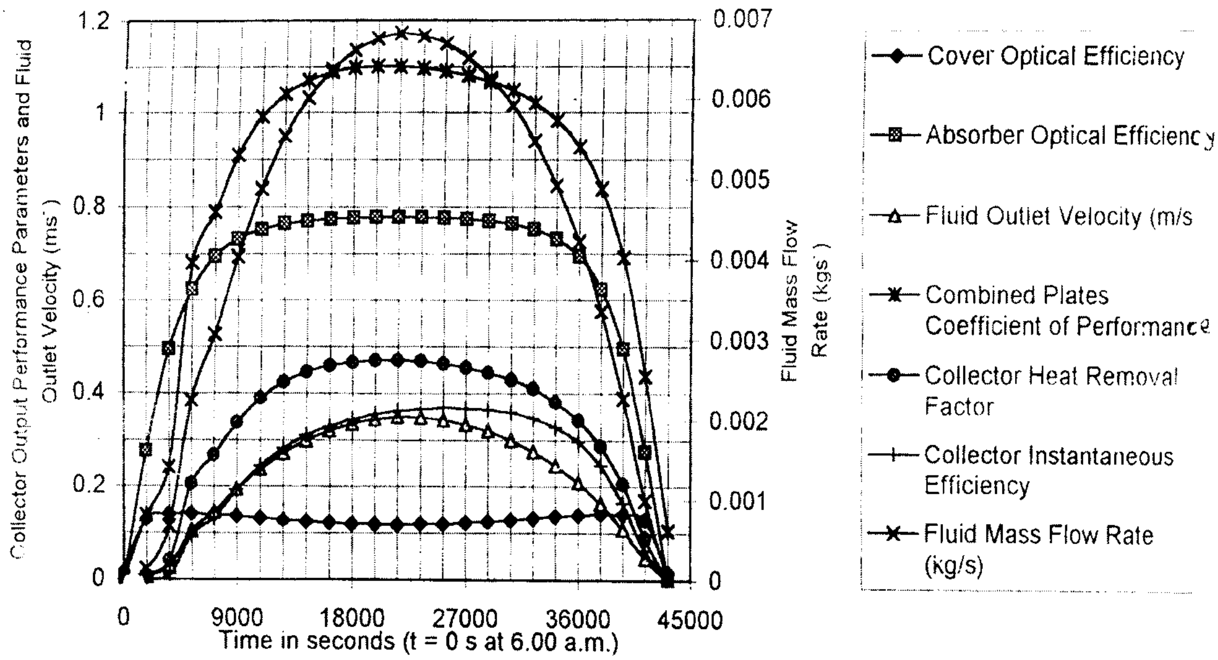

Fig. 2b SSSCA Flat Plate Collector Model Output Performance Parameters and Fluid Mass Flow Rate and Outlet Velocity on 14-03-02

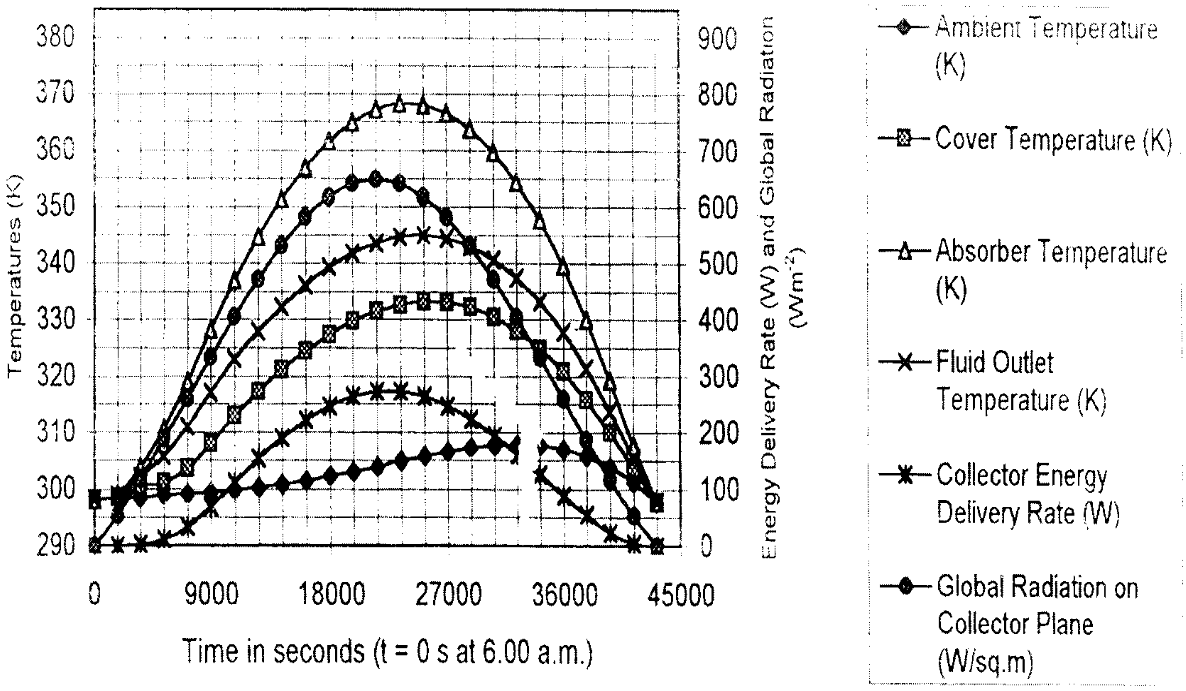

Fig. 3a SSSCA Flat Plate Collector Model Input and Output Parameters on 16-03-02

$T_a = T_{fi} = 298.052408 + 2.262E\text{-}04t - 2.187E\text{-}08t^2 + 1.8432E\text{-}12t^3 - 3.385E\text{-}17t^4$ K, H =16.4 MJm^{-2}

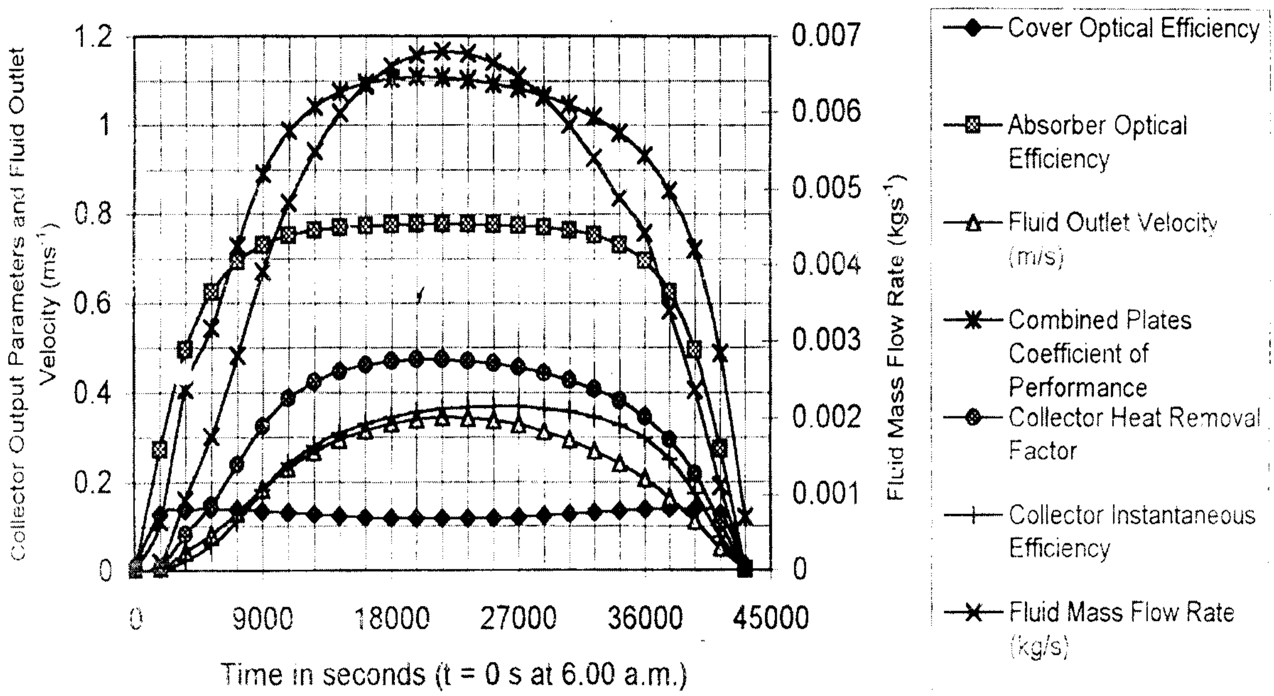

Fig. 3b. SSSCA Flat Plate Collector Model Output Performance Parameters and Fluid Mass Flow Rate and Outlet Velocity on 16-03-02

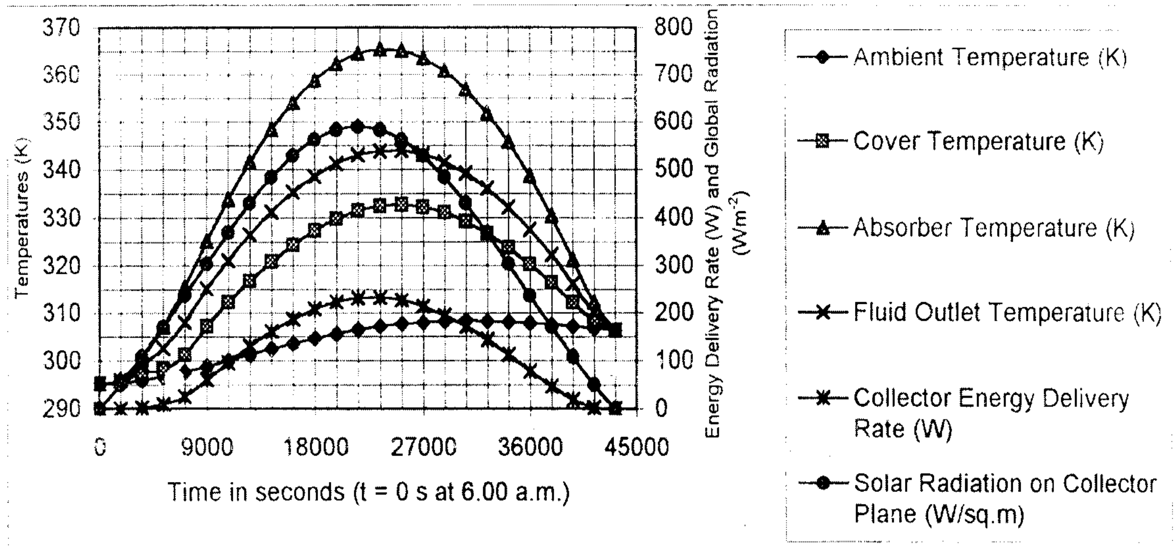

Fig 4a SSSCA Flat Plate Solar Collector Input and Output Parameters on 21-03-02

$T_a = T_{fi} = 295.247134 + 2.0434E\text{-}05t + 5.9981E\text{-}08t^2 -$ [illegible]$163E\text{-}12t^3 + 2.0873E\text{-}17t^4$ K, H = 15.0 MJm^{-2}

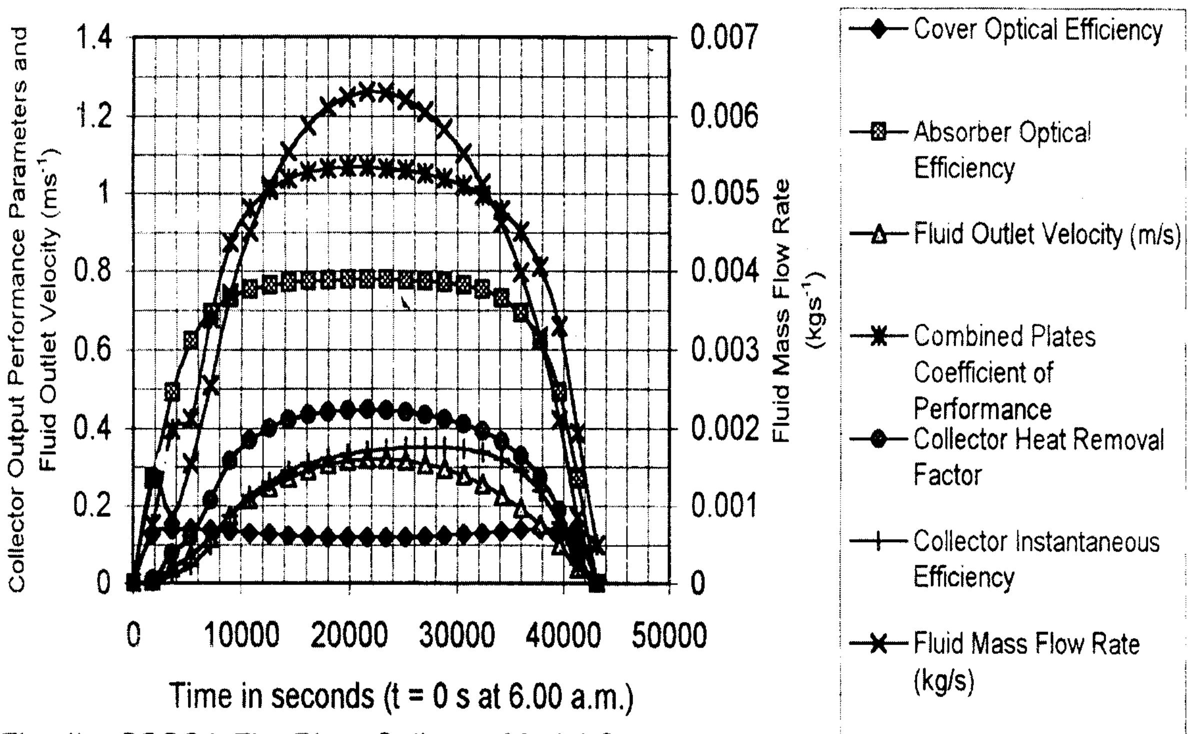

Fig. 4b. SSSCA Flat Plate Collector Model Output Performance Parameters and Fluid Mass Flow Rate and Outlet Velocity on 21-03-02

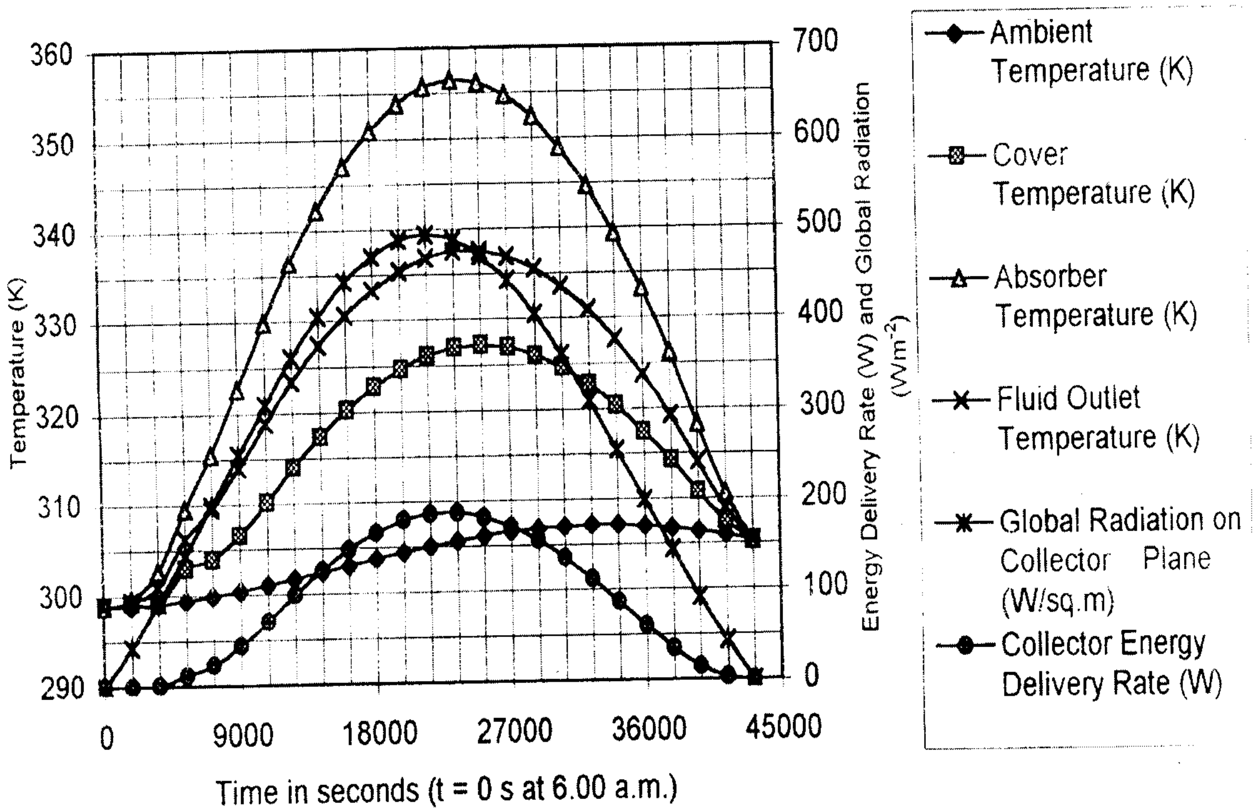

Fig. 5a SSSCA Flat Plate Collector Model Input and Output Parameters on 23-03-02

$T_a = T_{fi} = 298.83499 - 5.367E\text{-}05t + 3.1721E\text{-}08t^2 - 8.571E\text{-}13t^3 + 5.3463E\text{-}18t^4$ K, H = 12.6 MJm^{-2}

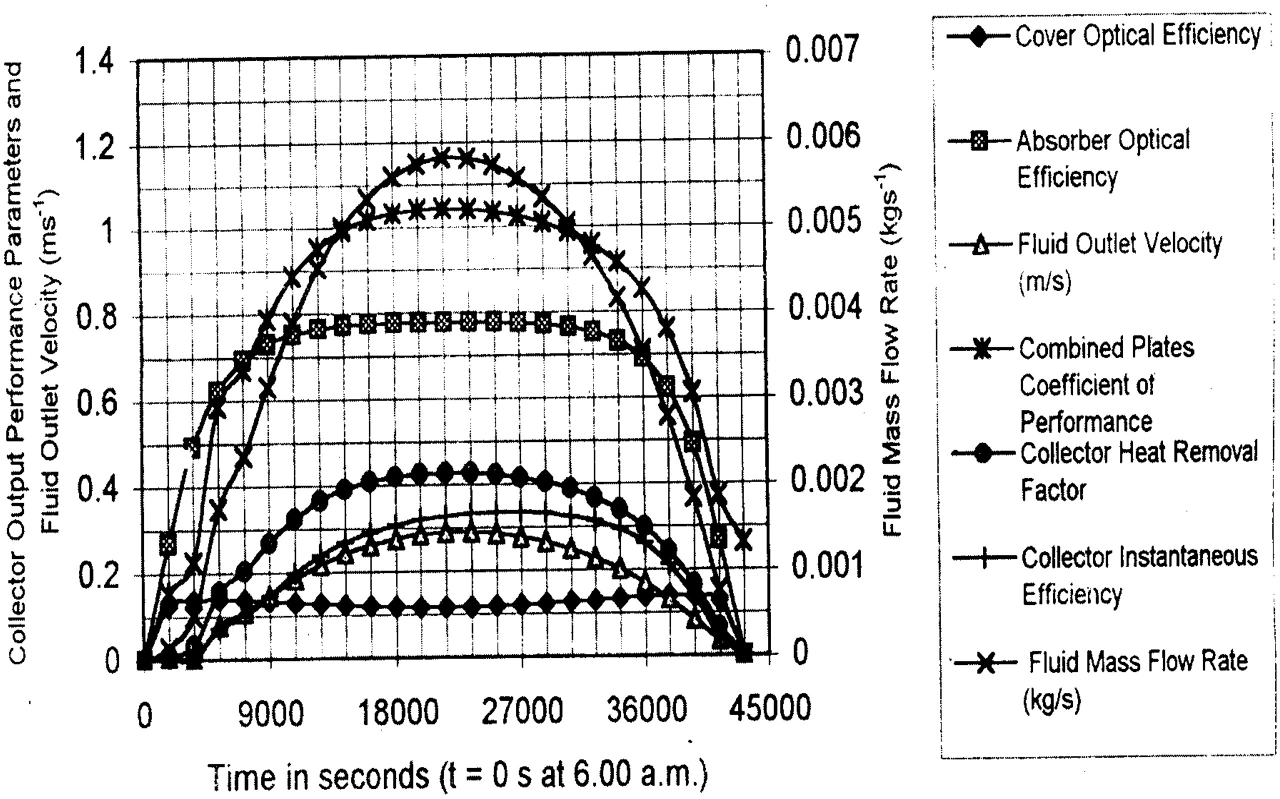

Fig. 5b SSSCA Flat Plate Collector Model Output Performance Parameters and Fluid Mass Flow Rate and Outlet Velocity on 23-03-02

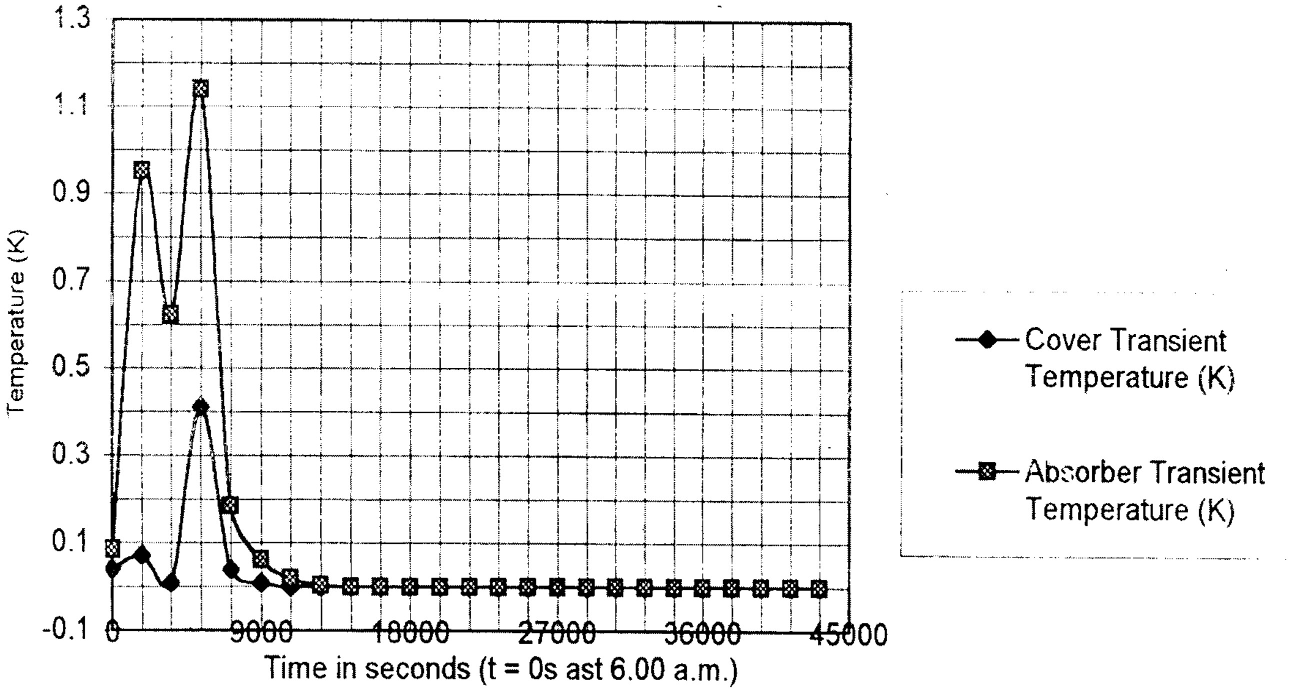

Fig. 6a SSSCA Flat Plate Collector Model Plates' Transient Tempertures on 14-03-02

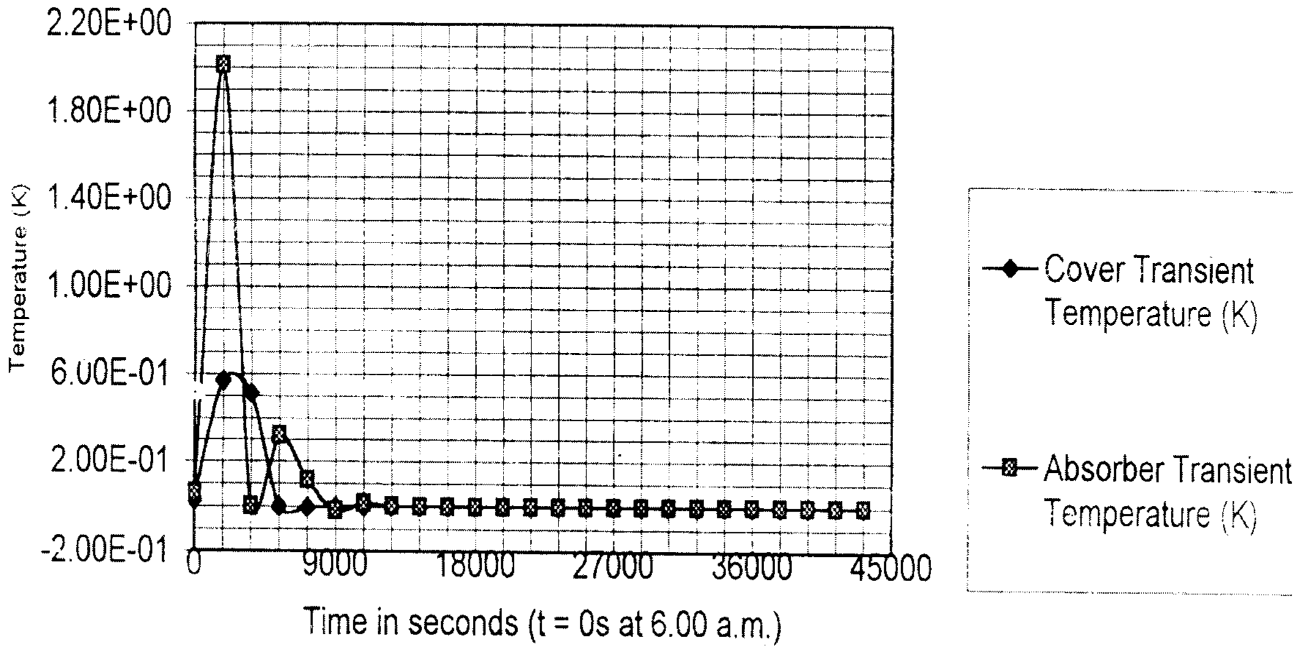

Fig. 6b SSSCA Flat Plate Collector Model Plates' Transient Temperatures on 16-03-02

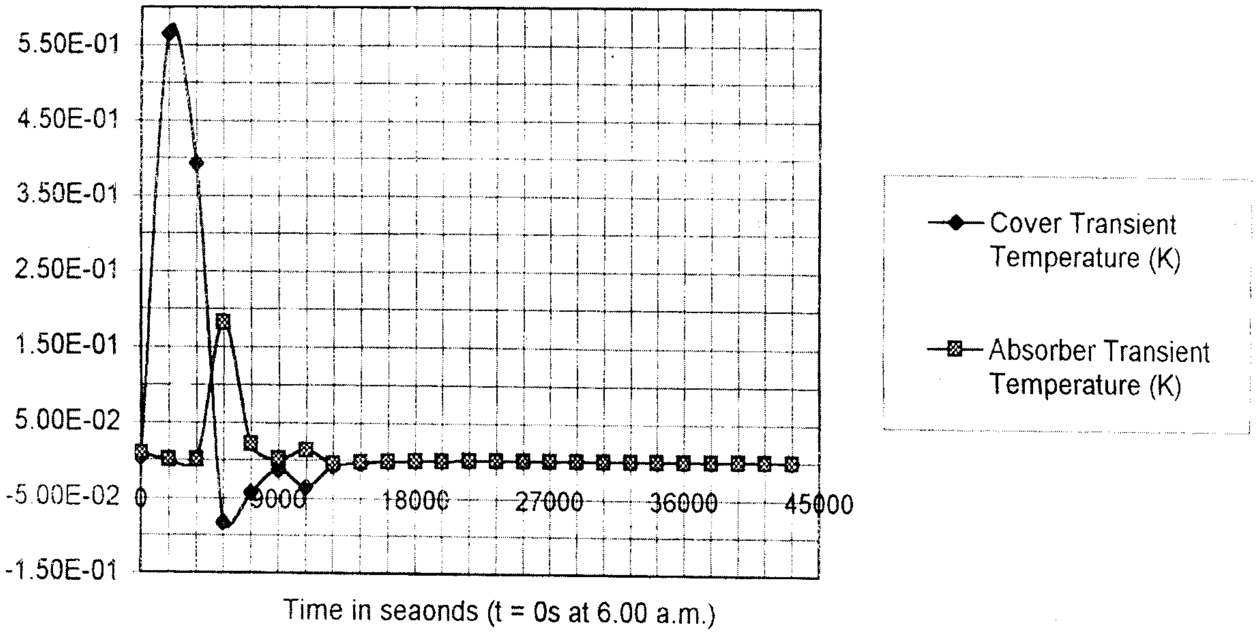

Fig. 7a SSSCA Flat Plate Collector Model Plates' Transient Temperatures on 21-03-02

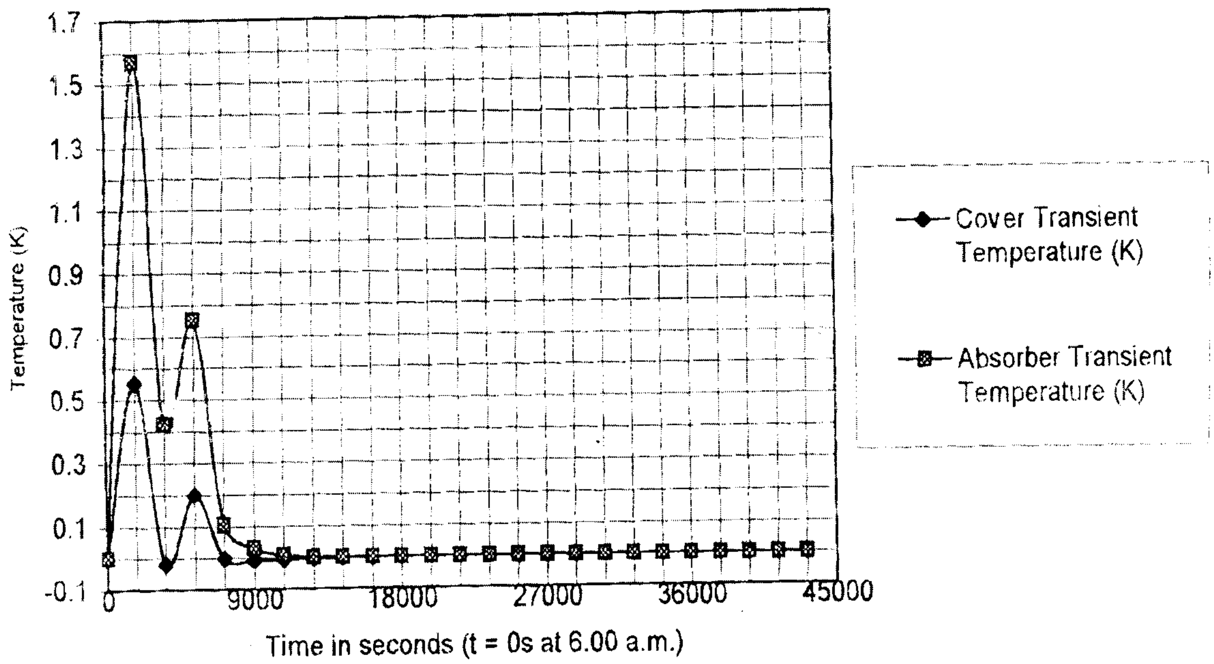

Fig. 7b SSSCA Flat Plate Collector Model Plates' Transient Temperatures on 23-03-02

The time lag for the attainment of the maximum temperatures after that of global radiation is attributable to the heat capacity of the materials of the solar collector and the fluid. These, together with the collector total energy delivery and overall efficiency are summarized in table l.

TABLE 1; MODEL COLLECTOR OUTPUT FOR FOUR DIFFERENT DAYS

DATE & DAY NO.	14-03-02, 73	16-03-02, 73	21-03-02,80	23-03-02,82
H, MJm^{-2}	17.0	16.4	15.0	12.6
ηco	0.3114	0.3101	0.2926	0.2757
$Q_{uD \cdot T}$' MJ	6.06	5.81	5.00	3.95
$I_{T,max}$' Wm^{-2} At time, s	673.71 21600	648.32 21600	589.69 21600	493.90 21600
$T_{c,max}$'K At time, s	335.24 25200	333.24 25200	332.67 25200	327.15 25200
$T_{p,max}$'K At time, s	37l.17 23400	368.23 23400	365.54 23400	356.49 23400
$T_{fo,max}$'K At time, s	347.04 25200	344.89 25200	343.92 23400	337.59 25200
$V_{f,max}$' ms^{-1} At time, s	0.34924 21600	0.34497 21600	0.32032 21600	0.29027 23400
$^{mf,}{}_{max}$,kgs^{-1} At time, s	0.00684 21600	0.00680 21600	0.00632 21600	0.0058 23400
Q_{uD}, max, W At time, s	280.87 23400	271.40 21600	233.52 23400	187.02 23400
$\eta_{Ci,max}$ At time, s	0.36854 25200	0.36905 25200	0.35930 26100	0.33543 25200

NOTE: H = *daily global radiation on a horizontal surface (or its monthly average,* H, *ηco = overall collector efficiency,* Q_{uDT}, = *collector total energy delivery* ,$I_{T,mass}$ = *maximum instantaneous global radiation on a tilted surface,* T_{c1max}= *maximum cover plate temperature,* $T_{p,mass}$ = *maximum absorber plate temperature,* $T_{fo,max}$ = *maximum outlet, fluid temperature,* $V_{t,max}$ = *maximum fluid velocity,* $m_{j,max}$ = *maximum fluid mass flow rate,* $Q_{uD,max}$ = *maximum fluid energy delivery rate,* $\eta_{Ci,max}$ = *maximum collector instantaneous efficiency. Time is measured from 6.00a.m., taken as time, t = Os.*

The first two terms of eqns. (33) and (34) are exponential in character and constitute the transient solution to eqns. (16) and (17) and are displayed in figs. (6a, b) for 14-03-02 and 16-03-02 respectively and (7a, b) for 21-03-02 and 23-03-02 respectively. The remaining terms constitute the steady state solution. With

$$X = 1/2(m_1 + m_2) = 1/2(u_{cp}/e_c + u_{pp}/e_p) \quad (92)$$

as the damping ratio, $\omega_o = (m_1 + m_2)^{1/2} = (u_{cp}u_{pp} - {h_f}^2)^{1/2}$ (93)

as the frequency of natural vibration and

$$X^2{\omega_o}^2 = 1/4(u_{cp}/e_c + u_{pp}/e_p)^2 + {h_f}^2 > 0 \quad (94)$$

the solar collector is an overdamped vibrator. Separation of the transient from the steady-state terms showed that the transient terms loose significance after 10800s from start. Entire deletion of the transient terms did not much affect the output and the maxima and minima of the parameters as discussed above were not affected in any way. Hence, the steady-state solutions accurately represent the operation of the solar collector. With the transient terms deleted, we have:

$$T_c = C_{AC}\cos 2\omega t + C_{AS}\sin 2\omega t + C_{BC}\cos\omega t + C_{BS}\sin\omega t + \lambda_c/e_m \quad (95)$$

$$\text{and } Tp = P_{AC}\cos 2\omega t + P_{AS}\sin 2\omega t + P_{BC}\cos\omega t + P_{BS}\sin\omega t + \lambda_p/e_m \quad (96)$$

Making use of eqn. (106) and with $T_{fi} = T_a$, eqns. (78) - 84) become:-

$$T_{fo} = T_{fi} + [(1 - \psi)/U_T]\{[f(t)/F_p\eta_{op} + C]\eta_{op}\} \quad (97)$$

where $F_p = h_f[u_{cp} + h_r + (u_{pp} + h_r)\eta_{op}/\eta_{op}]/e_m$ (98)

Also, $U_L = h_f[u_{pp}u_c + h_r u_p + u_{cp}u_p + h_r u_c]/F_p e_m$ (99)

$$Q_{uD} = m^f c_{pf}(T_{fc} - T_{fi}) = F_R A_c[f(t)/F_p\eta_{op} + C]\eta_{op} \quad (100)$$

where $F_R = (m^f c_{pf}/u_T A_c)\{1 - \exp[-F_p U_T A_c/m^f c_{pf}]\}$ (101)

and $U_T = 2h_f/F_p$ (102)

$$\eta_{ci} = Q_{uD}/I_T A_c = (F_R/I_T)[f(t)/F_p\eta_{op} + C]\eta_{op} \quad (103)$$

where $f(t) = h_f\{T_c + T_p - (\lambda_c + \lambda_p)/e_m$ (104)

and all are as previously defined.

6.1 CONCLUSION

Equations have been developed that can be used to predict the output parameters of a natural convection, single glazing, single pass, single flow, air heating flat plat solar energy collector with flow between the cover and the absorber plates. The usefulness of the derived equations lie in their enabling a direct calculation of the output and performance parameters of the solar collector analyzed without recourse to iterations and successive approximation in the numerical analysis which is always time consuming. The study itself has afforded a better understanding of the operation of solar collectors in the sense of the former plate's efficiency which is now defined as a combined plates' coefficient of performance, F_p as this performance parameter is greater than unity for a greater part of the time of operation of solar collectors. The definition is due to the incorporation of the optical efficiency of the glazing material (cover plate) in the analysis, which appears in its expression and which is often neglected by past investigators using numerical techniques. The knowledge of the functional relationship of the parameters involved in the solar collector operation as given by the analytical dynamic analysis will make Optimization studies simpler, and easier, thereby leading to a better design of good and efficient sola, collectors *in* future. The validation of the derived model solution equation *is* the subject of the companion paper, in the volume (Onuoha, A).

NOMENCLATURE (see [1] below)

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