A FINITE ELEMENT MODEL FOR THE ANALYSIS OF BRIDGE DECKS

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ABSTRACT

The analysis of bridge decks is at present effected using mostly the method of distribution coefficients. The method involves the use of coefficients obtained from charts for the approximate determination of bending moments in simply supported right concrete bridge decks. However, the method is limited to only simply supported decks. Also, reading the charts and interpolating between curves can be very tiresome and can easily introduce errors in the analysis. This paper therefore proposes and develops a finite element model as a more versatile alternative for the analysis of bridge decks for all support conditions. The results show that the proposed model is sufficiently accurate compared to solutions obtained using the method of distribution coefficients.

NOTATION

$\{\Delta^e\}$	Nodal displacement vector
$[K^e]$	Element stiffness matrix
$\varepsilon(x, y)$	State of stress at a point
$\sigma(x, y)$	State of stress at a point
$[D]$	Elasticity matrix
θ_x, θ_y	Rotations about x and y-axis respectively
w	Transverse deflection
D_x	Longitudinal flexural rigidity per unit width of bridge
D_y	Transverse flexural rigidity per unit span of bridge
D_{xy}	Torsional rigidity per unit width of bridge
D_{yx}	Torsional rigidity per unit span of bridge
D_1, D_2	Coupling rigidities in the x and y-directions respectively
q	Distributed load per unit area

INTRODUCTION

In the past, analysts have attempted to solve problems of the beam and slab type bridge decks using various approaches. Ganga Rao, et al [1] proposed a method, the macro-approach, which involves splitting the beam and slab system into separate components. The slab is separately analysed and all the interactive forces and displacements are determined. These are imposed on the beams which are analysed while ensuring that compatibility and orthogonality conditions are satisfied. The method involves the use of Kernel's identity function and complicated mathematical manipulations. But attempts to analyse the beam and slab deck as a continuous slab have been criticized on the grounds that the underlying assumption that the girders and cross beams constitute non-yielding supports simply does not hold for the beam and slab deck. The beams actually yield to pressure in the transverse direction. The longitudinal and transverse beams can be viewed as ribs that serve to stiffen the bridge deck and are deformable in the transverse direction. At best, the system can be treated as a deck that is continuous over deformable or elastic beams, [2].

The method of distribution coefficients, often employed by structural engineers for the analysis of beam and slab decks, is an approximate method that replaces the structure under study by an equivalent elastic system. This equivalent system is obtained by transforming the stiffnesses of a number of beams into a uniformly distributed system of the same overall stiffness. The width of the distributed system is given by the number of original beams multiplied by their spacing. This equivalent width may be different from the original width of the structure.

Cussens and Pama [3] prepared design curves based on a series solution of the partial differential equation for the deflection w of an orthotropic plate:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = p(x, y) \quad (1)$$

where 2H is the torsional rigidity of the bridge deck given by:

$$2H = D_{xy} + D_{yx} + D_1 + D_2 (2)$$

They showed that if values of the distribution coefficients K_{mx} and K_{my} can be found from their design curves, the moments may be determined as follows:

$$longitudinal\ moment\ M_x = K_{mx} M_{xmean} \quad (3)$$

$$Transcerse\ moment\ M_y = \left(\sqrt{\frac{D_y}{D_x}}\right) K_{my} M_{xmean} \quad (4)$$

so that both bending moments M_x and M_y are now expressed in terms of the product of a distribution coefficient and the mean longitudinal moment.

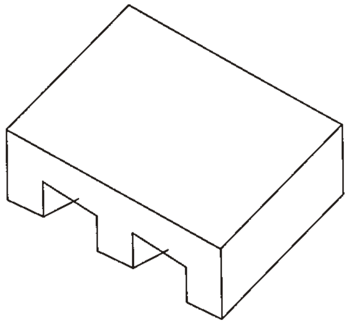
The method of distribution coefficients is approximate, since the charts were prepared on the basis of nine terms of the series solution of the partial differential equation (1) stated above, [3].

The method is limited to simply supported decks. Also, reading the charts and interpolating between curves can be very tiresome and can easily introduce errors in the analysis.

This paper therefore aims at the development and application of a more versatile approach to bridge deck analysis using a finite element model.

A Finite Element Model

The proposed model involves the representation of the slab by rectangular plate finite elements and the beams by undimensional beam elements. In order to simulate the T-beam action of the deck, the centroidal surfaces of the plate elements and the beam elements are considered joined together at the corresponding nodes by members that are assumed to be of infinite stiffness, so that no curvature of these connections can occur. (See Fig. 1). This assumption implies that at these nodes, in addition to plane sections remaining plane, the displacements of the plate and the displacements of the beam may be related to each other, so that at the plate-beam connections there is only one independent node. Therefore the number of independent nodes, and thus equations, is the same as that in a grillage representation. Thus the simplicity of the grillage representation is combined with the sophistication of the finite element approach.



(a) Beam-plate system

(b) T-beam structural model

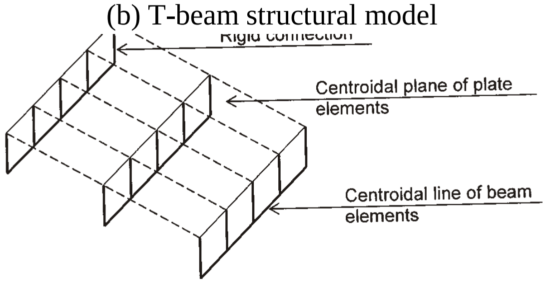


Fig. 1 Beam-plate structure idealisations

The matrix displacement method of analysis is used. The continuum structure is divided into a number of sub-regions, called finite elements, which are assumed to be interconnected at the nodal points only. Approximate displacement functions are assumed over each finite element. Displacement compatibility conditions are satisfied and the governing equilibrium equations that are generated are solved to yield the unknown nodal displacements. Once the displacements are known, the strains may then be evaluated from the strain-displacement relations, and finally the stresses are determined from the stress-strain relations.

The slab of the T-beam bridge deck is represented by non-conforming but complete rectangular plate bending elements with three degrees of freedom per node (w, θ_x, θ_y) and a cubic displacement model, (Fig. 2). Studies by Gallagher [4] showed that this element is efficient and yields solutions of acceptable accuracy.

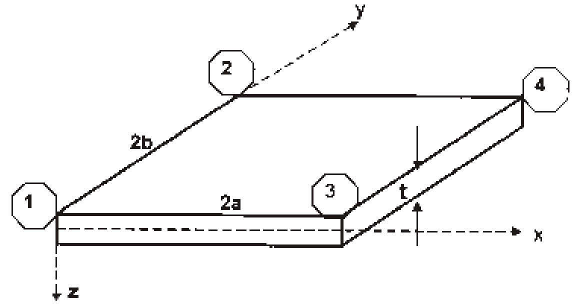


Fig. 2 Co-ordinate system for the plate element

A suitable displacement function chosen for the rectangular plate bending element is:

$$W = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2 + a_7x^3 + a_8x^2y + a_9xy^2 + a_{10}y^3 + a_{11}x^3y + a_{12}xy^3 \quad (5)$$

Although the chosen displacement function involves a discontinuity of cross slope at inter-element boundaries, and is said to be non-conforming, it is nevertheless used for the rectangular plate bending element since it has been found to exhibit good convergence, [5].

The beams of a T-beam bridge-deck are idealized by one-dimensional beam elements, (Fig. 3).

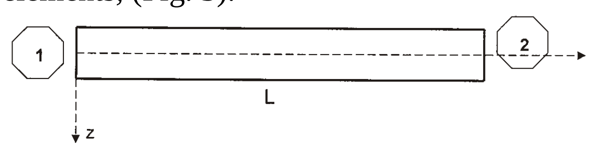


Fig. 3 Co-ordinate system for the beam element

Element Stiffness Matrix

The above displacement function defines displacement Δ(x, y) at any point in the rectangular finite element:

$$\{\Delta(x, y)\} = \begin{Bmatrix} W \\ \theta_x \\ \theta_y \end{Bmatrix} \quad (6)$$

The nodal displacement $\{\Delta^e\}$ can be obtained by substituting the coordinates of the element in $\{\Delta(x, y)\}$.

$$\{\Delta^e\} = \begin{Bmatrix} \Delta(x_1, y_1) \\ \Delta(x_2, y_2) \\ \Delta(x_3, y_3) \\ \Delta(x_4, y_4) \end{Bmatrix} \quad (7)$$

(7)

The state of strain at any point ε(x, y) is expressed by the curvatures and twist:

$$\{\varepsilon(x, y)\} = \begin{Bmatrix} -\partial^2 w/\partial x^2 \\ -\partial^2 w/\partial y^2 \\ 2\partial^2 w/(\partial x \partial y) \end{Bmatrix} \quad (8)$$

{ε(x, y)} can be related to nodal displacements $\{\Delta^e\}$:

$$\{\varepsilon(x, y)\} = [B]\{\Delta^{\mathbf{e}}\} \quad (9)$$

where [B] is (3×13) matrix for the rectangular plate element, [6].

The internal 'stresses' are the bending and twisting moments. Thus, the state of stress can be represented by:

$$\{\sigma(x, y)\} = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} \quad (10)$$

From the constitutive relationship:

$$\{\sigma(x, y)\} = [D]\{\varepsilon(x, y)\} \quad (11)$$

where [D] is the elasticity matrix.

$$[D] = \begin{Bmatrix} D_x & D_1 & 0 \\ D_1 & D_y & 0 \\ 0 & 0 & D_{xy} \end{Bmatrix} \quad (12)$$

The element stiffness matrix $[K^e]$ can be obtained by replacing the internal stresses {σ(x, y)} with statically equivalent nodal forces $[F^{\mathbf{e}}]$.

By applying the principles of virtual work, we derive the element stiffness matrix to be:

$$[K^e] = \int_v [B]^T [D][B] dvol \quad (13)$$

or, for the plate bending finite element, we have that:

$$[K^e] = \int_A [B]^T [D][B] dA \quad (14)$$

The nodal forces are obtained using the same principle. (9)

Explicit expressions for the element stiffness matrix for an orthotropic material and the element load vector for the plate bending finite element have been evaluated by Zienkiewicz [5].

Stiffness matrix for the beam element may be obtained directly from the well-known slope deflection equation, [7]

Numerical Example

A T-beam bridge deck is simply supported (11) at opposite ends. There are 4 longitudinal girders of size 250mm × 300mm deep and 4 transverse ribs of size 250mm × 200mm deep, spaced as shown in Fig. 4

(12)

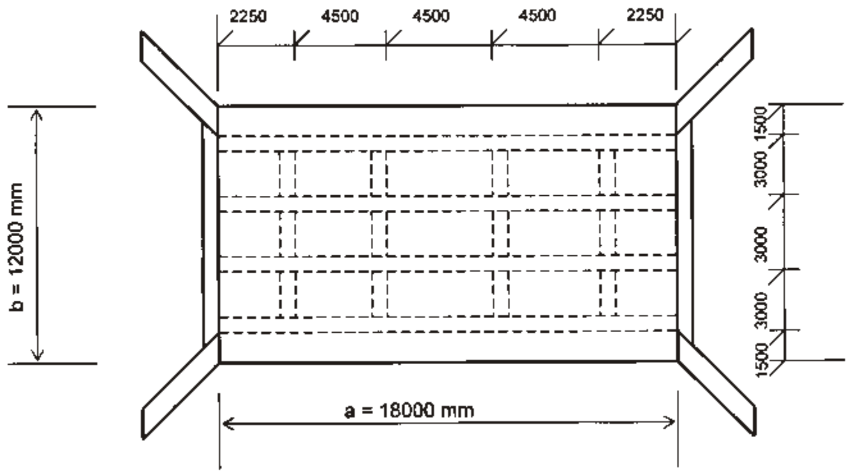


Fig. 4 T-beam bridge deck-plan

The bridge deck was analysed using uniformly distributed loads and a central point load as
(a) a simple slab deck, and
(b) a T-beam bridge deck (i.e. with girders and cross beams)

The results are presented in the Table below:

Table 1: Simple slab deck (Uniform load, q)

Method	Deflection, W at centre span (x qa^4/D)	Moment, M_x at centre of span (x qa^2)	Moment, M_y at centre of span (x qa^2)	Deflection, W at centre of edges (x qa^4/D)	Moment, M_x at centre of edges (x qa^2)
Finite Element	0.01302	0.12507	0.01043	0.01359	0.12904
Distribution Coefficients	0.01306	0.12509	0.01045	0.01361	0.12907
% Difference	0.31	0.02	0.19	0.15	0.02

Table 2: Simple slab deck (Central Point Load, p)

Method	Deflection, W at centre span (x qa^2/D)	Moment, M_x at centre of span (x p)	Moment, M_y at centre of span (x p)	Deflection, W at centre of edges (x qa^2/D)	Moment, M_x at centre of edges (x p)
Finite Element	0.03238	0.52062	0.29016	0.03123	0.03037
Distribution Coefficients	0.03242	0.52074	0.29060	0.03127	0.03039
% Difference	0.12	0.02	0.15	0.13	0.07

Table 3: T-beam Bridge deck (Uniform load, q)

Method	Deflection, W at centre span (x qa^4/D)	Moment, M_x at centre of span (x qa^2)	Moment, M_y at centre of span (x qa^2)	Deflection, W at centre of edges (x qa^4/D)	Moment, M_x at centre of edges (x qa^2)

Finite Element	0.01145	0.11020	0.01641	0.01166	0.11223
Distribution Coefficients	0.01155	0.11090	0.01640	0.01170	0.11250
% Difference	0.87	0.63	0.06	0.34	0.24

Table 4: T-beam Bridge deck (Central Point Load, p)

Method	Deflection, W at centre of span (x qa^2/D)	Moment, M_x at centre of span (x p)	Moment, M_y at centre of span (x p)	Deflection, W at centre of edges (x qa^2/D)	Moment, M_x at centre of edges (x p)
Finite Element	0.02817	0.47503	0.24727	0.02719	0.27591
Distribution Coefficients	0.02840	0.47600	0.24748	0.02729	0.27664
% Difference	0.82	0.20	0.08	0.37	0.26

DISCUSSION OF RESULTS AND CONCLUSION

The Tables show that the finite element solutions of the bridge deck problem agree reasonably with the solutions obtained by the method of distribution coefficients, (less than 0.4% maximum mean difference). The proposed finite element model is therefore acceptable and clearly offers more attractions than the chart-based method of distribution coefficients presently in use in many design offices. Reading the charts and interpolating between curves can be very tiresome and can easily introduce errors in the analysis. On the other hand, the finite element method, being computer-based, is incomparably faster and less prone to errors. Again, it is not limited to only simple supports as in the method of distribution coefficients. It can analyse the deck for more complex support and loading conditions.

The versatility of the proposed model can be improved by including shear deformation in the formulation in order to cater for T-beam bridge decks with deep beams.

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