INFLUENCE OF OFF-TAKE ANGLES ON FLOW DISTRIBUTION PATTERN AT CONCAVE CHANNEL BIFURCATION

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ABSTRACT
A physical model with meandering features was constructed and used to investigate the effect of off-take angles on the flow distribution at a concave channel bifurcation. Seven different off-take angles with varied main channel flow rates were used for the study. Predicting equations for the off-take discharge dependent on the off-take angles, main channel discharges, dispersion coefficients and Reynolds numbers were developed and calibrated statistically. Results of the study and predicting equations showed that the off-take discharge increased positively with increases in off-take angles as well as main channel discharges. The developed empirical predicting equations for the off-take discharge gave correlation coefficient values of 9.9974E-1 for both model equations with corresponding standard errors of 9.754E-5 and 9.42E-5, respectively. It was observed that the predicted off-take discharge values from the model equations compared closely with those of the study suggesting that off-take discharges for concave channel bifurcations could be fairly predicted with the established model equations.

Keywords: Concave section, Off-take angles, Channel bifurcation, Flow rates.

NOTATIONS
D_1, D_2, D_3 Dispersion Coefficients for main and off-take channels.
T_i Time after introduction of tracer
C_i Tracer response concentration at selected points.
θ Average flow time
U Mean velocity
L Distance traveled by tracer material before sample collection.
s Normalized variance.
Q_1, Q_3 Main and off-take channel discharges
α_0, α_1, α_2, α_3, α_4 Regression constants; R - Regression correlation coefficient
θ Off-take angle; Re Reynolds Number
Q_m, Q_{p1}, Q_{p2} Measured and predicted off-take discharges

INTRODUCTION
Leopold and Wolman [1] reported that bifurcations are typical features in alluvial rivers as well as in estuaries and the morphological behavior of rivers at bifurcations is not yet a properly understood phenomenon. A bifurcation occurs when a river or stream splits into two branches and naturally it occurs when a middle bar forms in a channel or a distributary carries flow from the main river.

In a meandering river/stream, the outer part of the curve is referred to as the concave section of the channel curvature. If an intake is sited on the concave part of the curve as is the case of the Obinna river intake supplying water to Adani rice farm at Nsukka in Enugu State, Nigeria, it is said to have a concave channel bifurcation. The angle it makes with the outer part of the main channel is called the off-take angle.

According to Shettar and Murthy [2], at any channel bifurcation with a known upstream discharge (Q_1), it is usually difficult to predict correctly the downstream discharges (Q_2 and Q_3) because they depend mostly on factors such as off-take angle, cross sectional areas, slope of downstream channels and shape of inlet section. They equally proposed that the flow which results when a channel bifurcates from the main channel has many complex features such as transverse motion accompanying the main flow, extensive separation zone that develops in the branch channel and re-circulation zone that appears in the main channel at higher discharge ratios.

Research on division of flows has received the attention of hydraulic engineers like [3-8] but most of their works were restricted to finding out the discharge distribution downstream of the bifurcation when the flow conditions upstream and downstream of the bifurcation are known. Law and Reynolds [9] noted that both the theoretical and experimental study of a dividing flow in open channel reveals that the walls of the channels are mostly not the effective boundaries of the flow region resulting to the occurrence of separation zone. Taylor [10] conducted the first comprehensive study on a dividing flow in open channel and suggested from the results obtained that for a given angle of intersection of the branch channel, it was possible to correlate the discharge and depth ratios with the ratios of the kinetic energy head to depth in the main channel.

Pattabhiramiah and Rajaratnam [11, 12] studied the branch channel problem by treating the branch flow as flow through a side weir of zero sill height while Law [13] used both analytical and experimental investigations to study the features of the dividing flow.

Many fluid flow problems cannot be solved mathematically due to the complex nature of the equations and boundary conditions inherent in them. It has been highlighted in most literature that even with modern computing facilities; many complex fluid problems still defy complete theoretical analysis. In such situations, physical models seem to be the most appropriate research tool to be employed. Essentially, physical models are constructed to mirror the actual physical behaviour of the original phenomenon or prototype. The aim of this study is therefore to ascertain the relative influence of the off-take angles on the flow distribution by using the physical model to investigate the flow distribution pattern at a concave channel bifurcation.

Model Construction and Experimental Setup

The model was constructed using the scale ratios of 1:50 for horizontal and 1:15 for the vertical. These scale ratios produced a distortion of 3.30. The model covered about 200m of the river channel. The detailed plan views of the model and its cross section are shown in Figure 1. Three structural metal sheets having a width of 30cm each were welded together to form the base and side walls of the rectangular main channel which represents fixed banks and bed of a natural river/stream. The main channel was 850cm long with straight and meandering sections. Similarly three structural metal sheets having a width of 10cm and length of 90cm each were welded together to form the base and side walls of the branch rectangular channel. Seven branch channels were constructed for each channel curvature and each of them was cut at the fitting end to be welded to the main channel in a manner that it represented one of the off take angles to be investigated in the study. The off take angle, θ was defined as the angle between the outer bank of the main channel and the inner bank of the branching channel at the bifurcation point. According to this definition the suggested off take angles used in the study were10°, 15°, 20°, 30°, 5°, 60° and 90°. The upstream and downstream ends of the main channel were covered with 30cm by 30cm sheet metals by welding to retain the water in the main channel during the experimental work. The downstream end was provided with a spillway to allow excess water to flow out of the channel. At the bifurcation point a rectangular opening was created on the main channel at the concave section of the model. A difference in invert levels of –15cm between the main and branching channels was created to resemble a natural situation. The meandering section of the model has an outer radius of 135cm and inner radius of 105cm indicating a mild bend that would not require future channelization.

The experimental materials consisted of the main channel, branch channels, two elevated tanks, sediment supplier, pump, current meter kit, water, stop watch, measuring tape, beakers, plastic containers, electronic weighing balance, electric oven, Silver Nitrate, Sodium Chloride and Potassium Chromate. The experimental set up may be sub divided into three major components namely the water supply unit, the sediment supply unit and the regulatory/measuring unit. The circulation of water within the model was a closed system with a sediment trap provided at downstream where the flow passed through a filter medium before being diverted back to the sump.

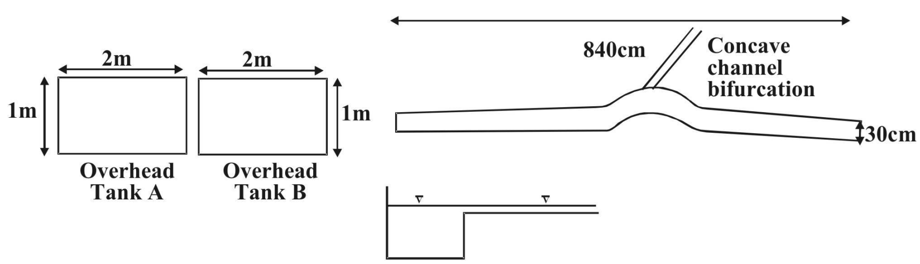

Fig. 1: Plan views of main channel, off-take channel and cross- section.

Experimental procedure

The experiment was carried out at the hydraulic laboratory of the Civil Engineering Department, University of Nigeria, Nsukka in Enugu State of Nigeria. This study concentrated mainly on the investigation of the discharge distribution patterns at concave channel bifurcation by varying the off-take angles and the main channel flow rates. Four flow rates with corresponding flow depths in the main channel of 16cm, 17cm, 18cm and 19cm and were represented by Q_{16}, Q_{17}, Q_{18}, Q_{19} were used throughout the experimental work. The flow depths corresponding to each of the flow rates was established and marked on the walls of the main channel as bench marks to ascertain each of the flow rates required The water in the sump was pumped to the first elevated tank that supplied water to the second overhead tank. The second overhead tank supplied water directly to the model at controlled rates in line with the established points on the walls of the main channel while the weir and spillway provided at the downstream maintained the required water level in the main channel. The flow rates in the main channel were obtained by measuring the mean velocities of the flow with the current meter apparatus at the marked points on the channel walls and multiplying them with their corresponding wetted cross sectional areas of the channel.

Flow distribution measurement at concave channel bifurcation.

Each of the seven off-take channels representing the recommended off-take angles of 10°, 15°, 20°, 30°, 5°, 60° and 90° was fixed at the opening created on the concave side of the main channel under each of the main channel flow rate conditions. Because the resulting off-take discharges were low when compared with the main channel flows, its discharge measurement was determined volumetrically by collecting the flow discharging freely into the air at the end of the channel in plastic containers at a specified time. The volumes of the collected water were measured with a graduated measuring cylinder and recorded against both the main channel flow rates and the off-take angles as presented in Figure 2.

Dispersion experiment conducted at concave channel bifurcation.

The dispersion coefficient was determined by carrying out tracer studies using the constant distance variable time method. Twenty grams of Sodium Chloride (Nacl) was taken and dissolved in 200ml of water. For each of the specified main channel flow rates, a detention time was determined for the concave channel bifurcation. For each of the off-take angles the solution of the sodium chloride was introduced at the beginning of the channel for every flow rate and samples of the tracer were collected at the off-take channel and downstream of the bifurcation at regular time intervals of 20 seconds. This was repeated for the seven off-take angles and four main channel flow rates. The samples collected were titrated with Silver Nitrate solution after the addition of Potassium Chromate. The data generated by the tracer experiment were evaluated analytically to determine the variance that is related to the dispersion coefficient from the statistical moment method according to Levenspiel et al [14] thus;

$$D = \frac{UL}{8}\left[\sqrt{8\sigma^2 + 1} - 1\right]$$

$$\sigma^2 = \frac{1}{\bar{\theta}^2}\left[\frac{\sum_{i=1}^{n} c_i t_1}{\sum_{i=1}^{n} c_i} - \bar{\theta}^2\right]$$

According to Marecos de Monte and Mara [15]) the average flow time could be determined from the expression

$$\bar{\theta} = \frac{\sum_{i=1}^{n} c_i t_1}{\sum_{i=1}^{n} c_i} \qquad 3$$

Flow distribution pattern at concave

channel bifurcation

Figure 2 shows the graphical representation of the specific discharge ratios and the off-take angles for concave channel bifurcation. Indications are that the off-take discharges gradually increased with increase in off-take angles. It was also observed that an increase in the parent channel discharge also resulted in an increase in the off-take discharge for any of the off-take angles considered and the least off-take angle yielded minimum off-take discharge while the highest off-take angle yielded maximum off-take discharge. This is because for small off-take angles, the fluid particles had the tendency to hit the right wall of the off-take channel thereby creating a separation zone after the bifurcation point while some of the fluid particles returned back to the main channel. Also, the sharp edged point at the bifurcation point served as energy dissipator thereby restricting the rate of the fluid motion to the off-take channel. It was, however, noticed that the width of the separation zone increased with decrease in both off-take angles and main channel discharge. The knowledge of flow distribution at channel bifurcation with respect to the off-take angles is very important in the selection and design of water intakes for irrigation and water supply schemes particularly in the sizing of the off-take channel dimensions based on proper knowledge of the off-take discharge. The specific discharge ratios (q_3/q_1) for the minimum and maximum discharges at the off-take angle of 10^o are 0.18 and 0.57 respectively while for 90^o off-take angle they are 1.19 and 2.51 respectively.

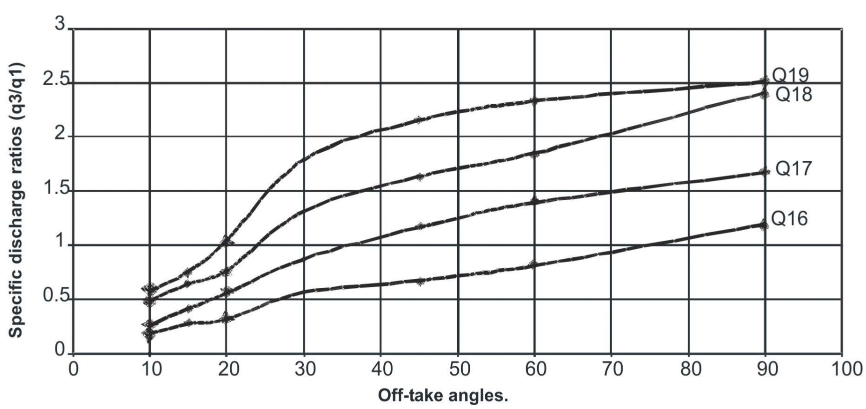

Fig. 2: Effect of off-take angles on specific discharge ratios for flows at concave channel bifurcation

Formulation of empirical expression.

It has been proposed that the off-take discharge could be proportional to the off-take angle (0), main channel flow (Q), dispersion coefficients for main and off-take channels (D_1, D_3) and the Reynolds number (Re) and may be expressed in the forms presented below:

$$Q_3 = \alpha_0 \theta^{\alpha_2} \left(\frac{D_3}{D_1}\right)^{\alpha_3} (RE)^{\alpha_4} (Q_1)^{\alpha_5}$$

Where α_0, α_2, α_3, α_4, and α_5 are constants. Linearlizing this equation by taking the semi log and log of both sides of the equation gave the following expressions thus:

$$\log Q_3 = \alpha_0 \theta^{\alpha_2} \left(\frac{D_3}{D_1}\right)^{\alpha_3} (RE)^{\alpha_4} (Q_1)^{\alpha_5}$$

$$Q_3 = \log\alpha_0 + \alpha_2 \log\theta + \alpha_3 \log\left(\frac{D_3}{D_1}\right) + \alpha_4 \log(ER) + \alpha_5 \log Q_1$$

$$\log Q_3 = \log\alpha_0 + \alpha_2 \log\theta + \alpha_3 \log\left(\frac{D_3}{D_1}\right) + \alpha_4 \log(ER) + \alpha_5 \log Q_1$$

Introducing a sine function in the above equation could lead to the modified expression of the off-take discharge, thus:

$$Q_3 = \alpha_0 (\sin\theta)^{\alpha_2} \left(\frac{D_3}{D_1}\right)^{\alpha_3} (RE)^{\alpha_4} (Q_1)^{\alpha_5}$$

Similarly, this expression can be linearlized by taking the semi log and log of both sides of the equation, thus:

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$$\log Q_3 = \alpha_0 (\sin\theta)^{\alpha_2} \left(\frac{D_3}{D_1}\right)^{\alpha_3} (RE)^{\alpha_4} (Q_1)^{\alpha_5}$$

6

$$Q_3 = \log\alpha_0 + \alpha_2 \log(\sin\theta) + \alpha_3 \log\left(\frac{D_3}{D_1}\right) + \alpha_4 \log(RE) + \alpha_5 \log Q_1$$

7

$$\log Q_3 = \log\alpha_0 + \alpha_2 \log(\sin\theta) + \alpha_3 \log\left(\frac{D_3}{D_1}\right) + \alpha_4 \log(RE) + \alpha_5 \log Q_1$$

The regression model constants were determined with the statistical package for the social sciences (SPSS + PC) and the results for each model 8equation are presented in Tables 1 and 2.

Table 1: Predicting equation for flow distribution at concave channel bifurcation.

S/N	Model equation		Regression Coefficients				Regression correlation coefficient R	Standard Error
		α_0	α_2	α_3	α_4	α_5		
1	$\log Q_3 = \alpha_0 \theta^{\alpha_2} \left(\frac{D_3}{D_1}\right)^{\alpha_3} (RE)^{\alpha_4} (Q_1)^{\alpha_5}$	-4.695950	0.004835	-0.105147	-1.949734×10	339.279524	0.98166	0.07821
2	$Q_3 = \log^{\alpha_2} \log\theta + \alpha_3 \log\left(\frac{D_3}{D_1}\right) + \alpha_4 \log(RE) + \alpha_5 \log(Q_1)$	0.029791	7.471976×10^{4}	0.002543	-0.003501	0.005792	0.98151	9.048931×10^{-5}
3	$\log Q_3 = \log^{\alpha_2} \log\theta + \alpha_3 \log\left(\frac{D_3}{D_1}\right) + \alpha_4 \log(RE) + \alpha_5 \log(Q_1)$	1.205168	-0.052569	1.247833	-0.393355	1.057387	0.99974	9.418394×10^{-3}

Table 2: Modified equation for predicting flow distribution at concave channel bifurcation.

S/N	Model equation	Regression Coefficients					Regression correlation coefficient R	Standard Error
		α_0	α_2	α_3	α_4	α_5		
1	$\log Q_3 = \alpha_0 (\sin\theta)^{\alpha_2} \left(\frac{D_3}{D_1}\right)^{\alpha_3} (RE)^{\alpha_4} (Q_1)^{\alpha_5}$	-4.430350	01222500	0.108935	1.639304×10^{-5}	152.035465	0.97277	0.09508
2	$Q_3 = \log^{\alpha_0} + \alpha_2 \log(\sin\theta) + \alpha_3 \log\left(\frac{D_3}{D_1}\right) + \alpha_4 \log(RE) + \alpha_5 \log(Q_1)$	0.030678	-2.43217×10^{-5}	0.004028	-0.004415	0.004200	0.97840	9.772729×10^{-5}
3	$\log Q_3 = \log^{\alpha_0} + \alpha_2 \log(\sin\theta) + \alpha_3 \log\left(\frac{D_3}{D_1}\right) + \alpha_4 \log(RE) + \alpha_5 \log(Q_1)$	1.152662	0.009816	1.1424493	-0.329293	1.172536	0.99974	9.753988×10^{-5}

Discussion of Results

The results of the flow distribution were used to evaluate the existing correlation between the off-take discharge and the associated flow parameters for all the proposed expressions by the authors.

Tables 1 and 2 show that all the proposed expressions yielded high regression correlation coefficient values ranging between 9.8151E-1 to 9.9974E-1 with standard error values ranging from 9.754E-5 to 7.821E-2. The high regression correlation coefficient values recoded in the regression analysis suggests that there is a high degree of correlation between the off-take discharge and the proposed hydraulic parameters. Table 1 shows that the semi log expression of the off-take discharge indicates that the dispersion coefficients are inversely proportional to the off-take discharge while the second equation of semi logs on the hydraulic parameters show that the off-take discharge is inversely proportional to only the Reynolds numbers. These two expressions have regression correlation coefficient values of 9.8166E-1 and 9.8151E-1 with standard errors of 7.821E-2 and 9.049E-5 respectively. However, the third expression with the log on both sides of the expression (Table 1) as well, shows that the off-take discharge is inversely proportional to both the off-take angle and the Reynolds numbers with a regression correlation coefficient value of 9.9974E-1 and standard error value of 9.4184E-5. The third expression with the highest value of regression correlation coefficient and the least standard error value could be adopted as the model predicting equation for the off-take discharge, thus:

$$\log Q_3 = \log 1.21 + 1.25 \log\left(\frac{D_3}{D_1}\right) + 1.06 \log(Q_1) - 0.053 \log\theta - 0.393 \log RE$$

$$Q_3 = \frac{12.1 (D_3/D_1)^{1.25} (Q_1)^{1.06}}{(\theta)^{0.053} (Re)^{0.393}}$$

Considering the expressions in Table 2, the result of the semi log expression of the off-take discharge is theoretically in agreement with the proposed expression stating that the off-take discharge is proportional to the hydraulic parameters except that it has the lowest regression correlation coefficient value of 9.7277E-1 and the highest standard error value of 9.508E-2. The result of the second expression having semi log on the hydraulic parameters is similar to that of the model equation with log on both sides of the expression indicating that the off-take discharge is inversely proportional to the off-take angle and Reynolds numbers with the second highest regression correlation coefficient value of 9.7840E-1 and standard error value of 9.773E-5. Finally, the expression with the log on both sides of the modified equation shows that the off-take discharge is inversely proportional to the Reynolds numbers and has the highest regression correlation coefficient value of 9.9974E-1 with a standard error value of 9.754E-5. It can be rightly suggested that the expression with the highest regression correlation coefficient and lowest standard error values may predict fairly the off-take discharge for the concave channel bifurcation. For the modified equation, it could be assumed that the off-take discharge is inversely proportional to the Reynolds numbers as shown by the expression in Table 2, thus:

$$\log Q_3 = \log 1.15 + 0.0098 \log(\sin\theta) + 1.141 \log\left(\frac{D_3}{D_1}\right) + 1.17 \log(Q_1) - 0.33 \log RE$$

$$Q_3 = \frac{11.5 (\sin\theta)^{0.01} (D_3/D_1)^{1.14} (Q_1)^{1.17}}{(Re)^{0.393}}$$

It can be generally seen from Tables 1 and 2 that the sine function introduced in the model equation has significant effect on the proposed pattern of relationship between the off-take discharge and the hydraulic parameters and its effect on the predicted values will be confirmed by the result of the model verification. Both equations produced precise coefficients correlation very close to unity, though the model equation could be preferred due to its lower standard error value of 9.418394 $\times 10^{-3}$ as against 9.753988 $\times 10^{-3}$ for modified equation.

Model Verification.

Both model equations were verified with experimental data at each specified off-take angles. Figures 2 - 5 show the plots of the off-take angles against the measured and predicted off-take discharges for different main channel flows and the results show that the off-take discharge gradually increases as the off-take angles increase which is in agreement with the result of the experimental study. Considering the model results under the minimum main channel discharge of Q_{16}, the divergence in result values for both model and laboratory tests was above 30% for the minimum off-take angle of 10° while that of 45° off-take angle was about 12%. The divergence of result values for the off-take discharges obtained for the other off-take angles were within 20%. The predicted values for all the off-take angles show a divergence in values less than 20% while only that of 10^0 showed a divergence of more than 30%. This could be attributed to the errors inherent in measurements carried out particularly in low flows as noticed in Q_{16} and Q_{17}. The model results for the main channel discharge of Q_{17} showed also that the divergence in results for 10° off-take angle is about 30% while the results of the other off-take angles showed values less or equal to 20%. For the minimum main channel discharge (Q_{16}), the off-take discharge values predicted by the model equation were less than those obtained from the laboratory test for off-take angles between 10° and 30° with corresponding percentage difference in values between 10 and 28. At 45° off-take angle the predicted value was equal to that obtained from the laboratory test while the predicted valued for off-take angles of between 60° and 90° were above those obtained from the laboratory tests with a maximum of about 17%.

For the modified model equation, the predicted values were mostly lower than those measured values except for the 90° off-take angle that was slightly above the measured values by 2.3%. The trend displayed by the modified model equation was that the degree of divergence was highest for the minimum off-take angle of 10° which gradually decreased with increase in off-take angles. It could be seen that the 90° off-take angle showed the minimum divergence in values of about 2.3%. Considering the main channel discharge of (Q_{17}), the off-take discharge values predicted by the model equation were less than the values obtained from the laboratory tests for the off-take angles between 10° and 20° while those for the off-take angles between 30° and 90° were greater than the laboratory measured discharges. The highest percentage divergence value of 24% was recorded for the 10° and 90° off-take angles while the minimum percentage divergence value of 4.2% was recorded for 20° off-take angle. The difference in values between the modified and model predicted values were less than the measured laboratory values and were seen to be decreasing between 30 to 2% for the off-take angles of 10° to 30° while the predicted values for off-take angles of 45° to 90° were higher than the measured laboratory values. The minimum off-take angle of 10° showed the highest percentage divergence in values of 30 while the minimum percentage divergence of 20 was recorded for the 45° off-take angle. Both model and modified equations did not show any definite pattern in the variation of predicted and measured values.

Conclusion

A comparative study of the various main channel flows with different off-take angles showed that as the off-take angles and main channel flow increased, the off-take discharge equally increased. The quantitative estimates of errors obtained in the measured and predicted values is within the range of less than 2.4% to 15% indicating that the theoretical models developed could reasonably be applied in the determination of the off-take discharges at concave channel bifurcations. This study is useful for the determination of the distribution flow pattern at concave channel bifurcation.

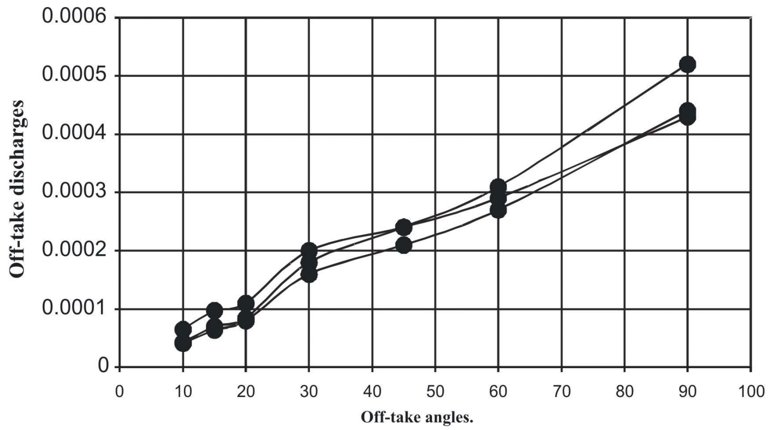

Fig.3: Plot of measured and predicted off-take discharges against off-take angles for Q16

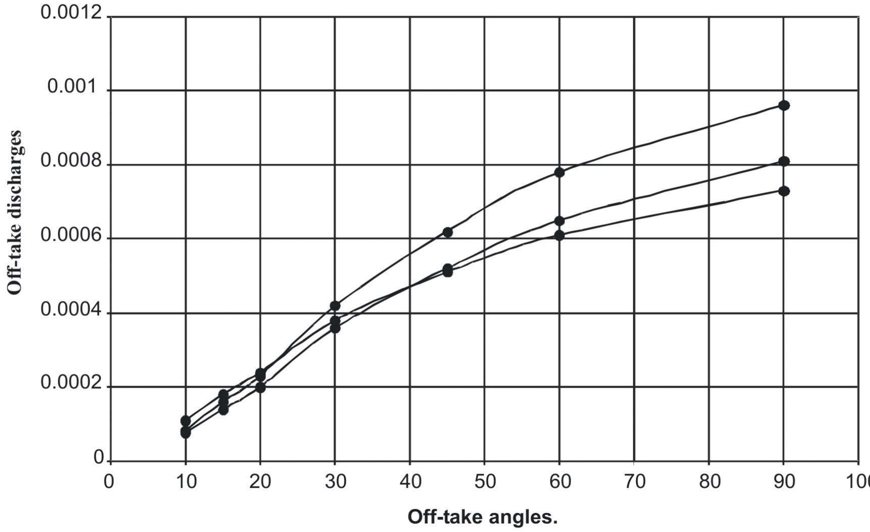

Fig.4: Plot of measured and predicted off-take discharges against off-take angles for Q17.

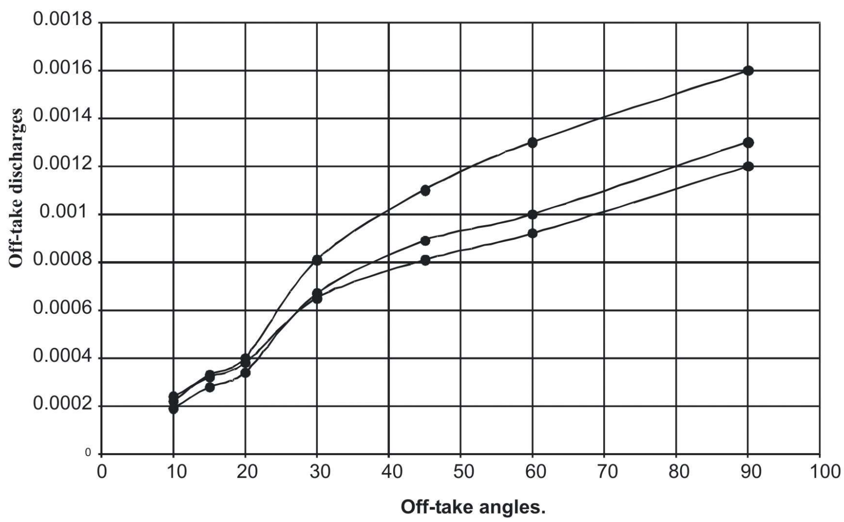

Fig.5: Plot of measured and predicted off-take discharges against off-take angles for Q18.

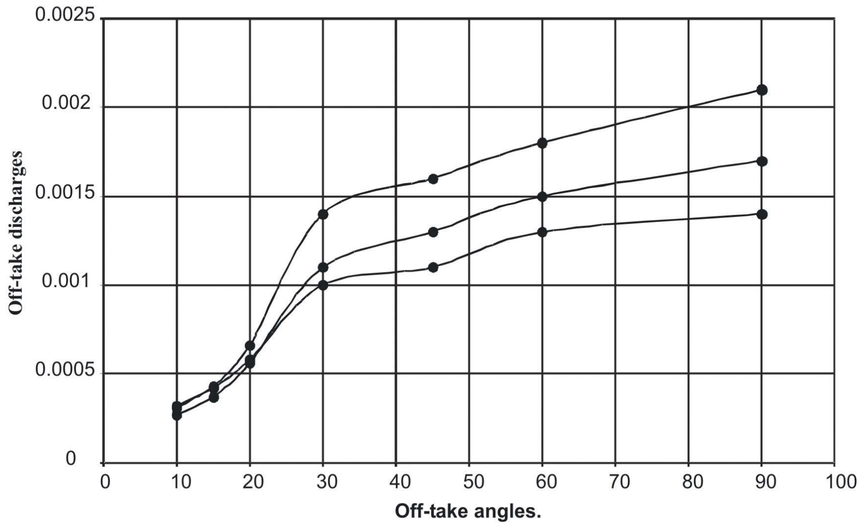

Fig.6: Plot of measured and predicted off-takes against off-take angles for Q19.

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