



THE STEADY-STATE PERFORMANCE CHARACTERISTICS OF SINGLE PHASE TRANSFER FIELD MACHINE OPERATING IN THE ASYNCHRONOUS MODE

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Abstract

The paper reports the derivation of the steady-state equivalent circuit of a single phase transfer field (SPTF) machine operating in the asynchronous mode from which the performance characteristics could be predicted. The equivalent circuit of the machine was derived using the concept of double field revolving theory cum symmetrical components. It is shown that the machine displays torque-slip characteristics similar to those of a single phase induction motor. The performance characteristics of the SPTF machine using the derived equivalent circuit are in tune with the theoretical predictions and validate the equivalent circuit.

Keywords: symmetrical components, asynchronous mode, double field revolving theory

1. Introduction

The primitive poly-phase transfer field (TF) machine was first presented as a flux bridge machine with a two air gap three element construction [1,2]. Following a careful study of the equations describing the air-gap flux density distribution, it was found that a topology manipulation yields an equivalent single air-gap two-element machine of much simpler mechanical construction [1-3]. The equivalent circuit of the poly-phase TF machine operating in the asynchronous mode has been reported in [2] while the dynamic performance of the machine was reported in [4]. Thus far, studies on the TF machine have been limited to the poly-phase TF machine and to the best of the knowledge of the authors; no researcher has considered the single phase operation of the TF machine. The single phase transfer field (SPTF) machine is expected to have wider applications than the poly-phase version because most residential houses, offices and rural areas are supplied with single phase ac rather than 3-phase as power requirements of individual load items are rather small. Furthermore, single phase ac machines are invariably used in home appliances such as fans, refrigerators, vacuum cleaners, washing machines, mixers, grinding machines, portable tools etc.

This paper re-examines some aspects of the poly-phase TF machine operating in the asynchronous

mode and develops the steady state equivalent circuit of the SPTF machine operating in the asynchronous mode, from which the steady-state performance characteristics can be predicted. The single phase equivalent circuit of the TF machine operating in the asynchronous mode is derived in a manner which is consistent with the derivation of a single phase induction motor from a poly-phase induction motor by disconnecting one of its supply lines and using the concept of symmetrical components [5].

2. Physical Arrangement of the Polyphase TF Machine Operating in the Asynchronous Mode

The polyphase TF machine comprises two identical stack machines whose stators are typical induction motor types. The rotors are, however, of the salient-pole type. The two rotors are mechanically coupled together such that their pole axes are in space quadrature and housed in their respective induction motor stators. Each unit machine comprising the TF machine has two sets of windings known as the main and auxiliary windings respectively. The two windings are electrically isolated but magnetically coupled and corresponding phases occupy the same stator slots for maximum coupling. The two windings are integrally wound for the same number of poles as the ro-

tor poles. The main windings of the respective halves are connected in series and energized from the utility supply, while the auxiliary windings are connected in series opposition between the two halves of the machine as shown in fig 1. The machine is brushless and there is no windings on the rotor, including ar-motisseur windings; the main and auxiliary windings being located on the stator side only. The machine is self-starting and has a torque-slip curve similar to that of a polyphase induction machine except that its synchronous speed is $\omega_o/2$ instead of ω_o as obtains in induction machines. Furthermore, the pull-out torque of TF machine is low compared to that of an induction machine of the same size on account of higher leakage-reactance associated with the machine. Analogously to an induction machine the relationship between the frequency of the current in the main and auxiliary windings is $\omega_o : [(\omega_o - 2\omega_r) = (2s - 1)\omega_o]$ [1-4]. In this novel configuration, the auxiliary windings mimic the role of the rotor windings in an induction machine. The roles of the main and auxiliary windings can be interchanged and will produce the same result.

3. Review of Dynamic Model of the Polyphase TF Machine Operating in the Asynchronous Mode

The voltage and flux linkage equations in d-q reference frame of the poly-phase TF machine in the asynchronous mode were derived in arbitrary reference frame in references 2 and 4 as:

$$\begin{aligned} V_Q &= R_S I_Q + \omega \lambda_D + p \lambda_Q \\ V_D &= R_S I_D - \omega \lambda_Q + p \lambda_D \\ V_O &= R_S I_O + p \lambda_O \end{aligned} \quad (1)$$

$$\begin{aligned} V_q &= R_r I_d + (\omega - 2\omega_r) \lambda_D + p \lambda_d \\ V_d &= R_r I_d - (\omega - 2\omega_r) \lambda_q + p \lambda_d \\ V_o &= R_r I_o + p \lambda_o \end{aligned} \quad (2)$$

$$\begin{aligned} \lambda_Q &= (2L_L + Lmq + Lmd)I_Q - (Lmd - Lmq)I'_q \\ \lambda_D &= (2L_L + Lmq + Lmd)I_D + (Lmd - Lmq)I'_d \\ \lambda_o &= 2L_L I_o \\ \lambda'_q &= (2L_L + Lmq + Lmd)I'_q - (Lmd - Lmq)I_Q \\ \lambda'_d &= (2L_L + Lmq + Lmd)I'_d + (Lmd - Lmq)I_D \\ \lambda'_o &= 2L'_L I_o' \end{aligned} \quad (3)$$

The upper case subscripts Q, D, O and lower case subscripts q, d, o refer to the main and auxiliary winding parameters respectively. R_s is the sum of the resistance of the main windings in both halves of the machine and R_r is the sum of the resistances of the auxiliary windings in both halves of the machine. The torque of the machine is given by [4] as:

$$\begin{aligned} T_e &= I^T \frac{\partial(L_{abc.ABC})}{\partial \theta_{rm}} I \\ &= \frac{3}{4} p (Lmd - Lmq) (I_Q I'_d + I'_q I_D) \end{aligned} \quad (4)$$

where $I^T = [I_a \ I_b \ I_c \ I_A \ I_B \ I_C]^T$, θ_{rm} = rotor mechanical angle displacement, $L_{abc.ABC}$ is the mutual inductance between the main and auxiliary windings. The dynamic equivalent circuit of the poly-phase TF machine can be obtained if the flux linkage expressions of 3 are re-written as:

$$\begin{aligned} \lambda_Q &= 2(L_{Ls} + Lmd)I_Q + (Lmq - Lmd)(I_Q + I'_q) \\ \lambda_D &= 2(L_{Ls} + Lmq)I_D + (Lmd - Lmq)(I_D + I'_d) \\ \lambda_o &= 2L_{Ls}I_o \\ \lambda'_q &= 2(L_{Lr} + Lmq)I'_q + (Lmq - Lmd)(I_Q + I'_q) \\ \lambda'_d &= 2(L_{Lr} + Lmq)I'_d + (Lmd - Lmq)(I_D + I'_d) \\ \lambda'_o &= 2L_{Lr}I_o' \end{aligned} \quad (5)$$

Equations (1) and (5) lead to the dynamic equivalent circuit of the polyphase TF machine as shown in fig 2

All primed quantities are auxiliary winding parameters referred to the main winding. However, with turns ratio of unity between main and auxiliary windings, the auxiliary windings parameters retain same values irrespective of being referred to the main winding.

4. Steadystate Equivalent Circuit of the Polyphase TF Machine

The steady-state equivalent circuit of the polyphase TF machine may be derived from either the q-axis or d-axis dynamic equivalent circuit of fig 2 by setting all derivative terms ($\frac{d}{dt}$) to zero and by assuming that the d -axis or q -axis is aligned with the A-phase of the three phase ABC system and by using the operator j to transform q -axis parameters to d -axis in the main winding and $-j$ in the auxiliary winding. This follows from the fact that the mmfs of the main and auxiliary windings rotate in opposite directions [1,3]. That is, $F_D = jF_Q$ (main winding circuit), $F_d = -jF_q$ (auxiliary winding circuit), where F may be current, voltage or flux linkage. If the q -axis is aligned with the A-phase of the three phase ABC system, then phase voltage V_A is obtained by substitution of eqn.5 in eqn 1. Thus,

$$V_A = (R_S + j2X_q)I_A + j(Xmd - Xmq)(I_A + I'_a) \quad (6)$$

where $X_q = X_{ls} + Xmq$. Similarly, for the auxiliary winding

$$V'_a = (R'_r + j2X'_q(2s-1)I'_a) + j(Xmd - Xmq)(2s-1)(I_A + I'_a) \quad (7)$$

where $(\omega_o - 2\omega_r) = (2s - 1)\omega_o$ and s is the p. u. slip. Dividing by $2s - 1$ gives

$$\frac{V'_a}{(2s - 1)} = \frac{R'_r}{(2s - 1)} I'_a + j2X'_q I'_a + j(Xmd - Xmq)(I_A + I'_a) \quad (8)$$

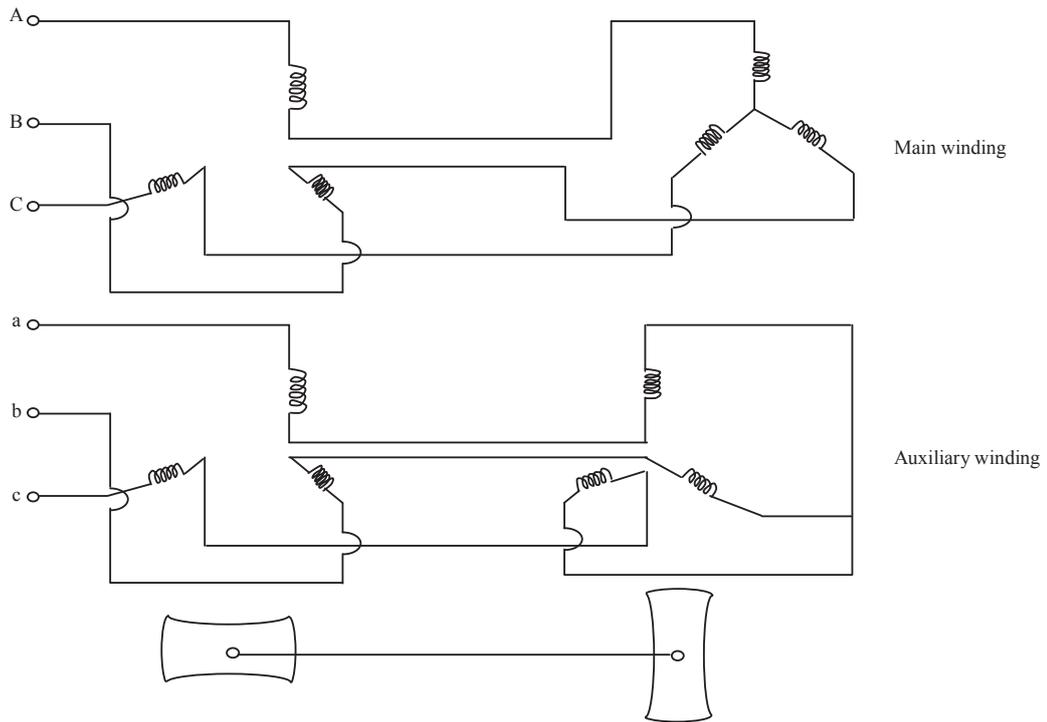


Figure 1: Physical configuration of the polyphase TF machine.

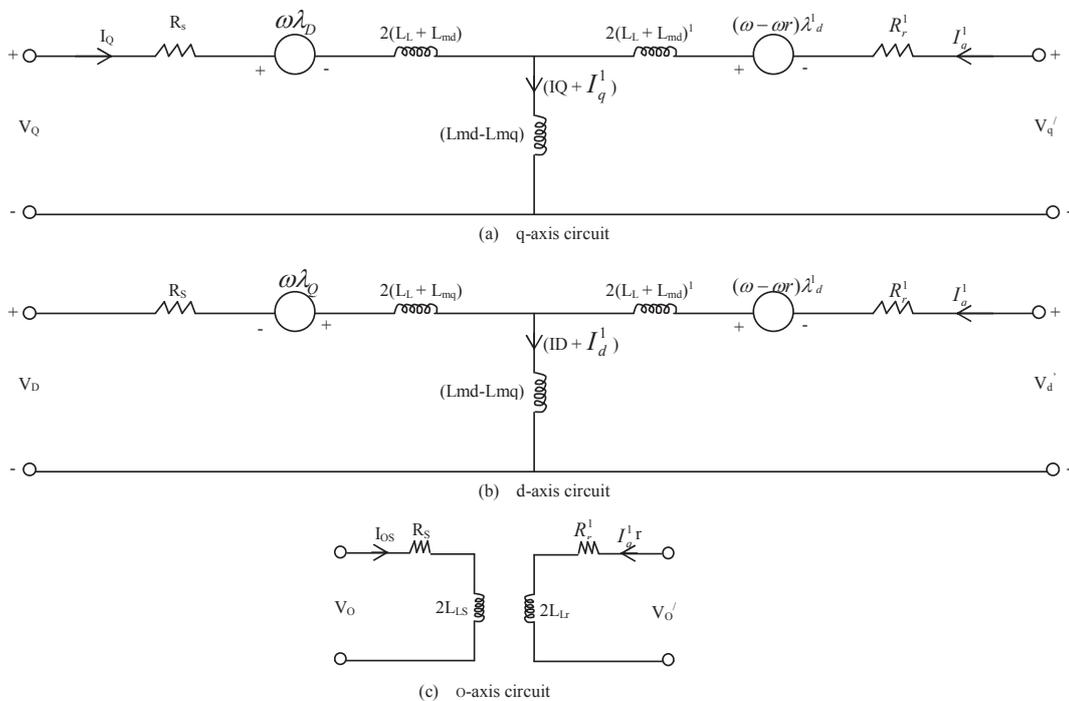


Figure 2: Dynamic equivalent circuits of the polyphase TF machine, (a) q-axis circuit (b) d-axis circuit (c) o-axis circuit.

Using equations 6 and 7 and noting that the auxiliary winding is usually short-circuited in a manner akin to the rotor winding of an induction motor, it therefore follows that $\frac{V'_a}{(2s-1)} = 0$. Furthermore, the slip dependent resistance $\frac{R'_r}{(2s-1)}$ can be split into R'_r and $2R'_r \frac{(1-s)}{(2s-1)}$.

R'_r accounts for the copper loss in the auxiliary winding, while $2R'_r \frac{(1-s)}{(2s-1)}$ accounts for the gross output power of the machine. The resulting equivalent circuit is as shown in Fig3.

The reactance $j(X_{md} - X_{mq})$ is the magnetizing reactance of the machine and is associated with the current required to provide the necessary magnetization of the machine. Since the main and auxiliary windings are identical, and considering unity turns ratio of main and auxiliary windings $R_s = R'_r = R_r$.

5. Single-phase Transfer Field (SPTF) Machine Concept

If one line of a three phase TF machine is opened while the machine is running with moderate or light load, the machine maintains running although at a slower speed. This condition is known as single phase operation and implies that the three phase TF machine has become a single phase TF machine. An analogous situation obtains in an induction machine.

The single-phase TF machine is similar to the polyphase type in which a single phase winding replaces the 3-phase winding. A single-phase current in a single phase winding produces a pulsating, not a rotating magnetic field. The pulsating field is resolved into two components of half its amplitude and rotating in opposite directions with synchronous speed N_s . The concept of double field revolving theory can be used to assess the qualitative and quantitative performance of single phase motors.

When the main winding carries a sinusoidal current, it produces a sinusoidally space distributed mmf whose peak value pulsates with time. The mmf wave at any time t may be expressed as

$$F = F_{peak} \cos \theta \quad (9)$$

where θ is the angle measured from the winding axis and F_{peak} may be expressed as

$$F_{peak} = F_{max} \cos t$$

by substituting into 8, it follows that the mmf wave is a function of time and space and may be expressed as:

$$F = F_{max} \cos \omega_o t \cos \theta \\ = \frac{1}{2} F_{max} \cos(\theta - \omega_o t) + \frac{1}{2} F_{max} \cos(\theta + \omega_o t) \quad (10)$$

Thus, the pulsating stator mmf may be represented by two oppositely rotating cosinusoidally distributed mmfs each traveling at synchronous speed. Each of these components of main winding mmf distribution gives rise to corresponding auxiliary winding mmf distributions.

The resulting instantaneous torque developed has 4 components:

a. A torque due to the interaction of the forward traveling main winding and auxiliary winding mmf distributions.

b. A torque due to the interaction of the backward traveling main and auxiliary winding mmf distributions.

c. A torque due to the forward traveling main winding mmf distribution and the backward traveling auxiliary winding mmf distribution .

d. A torque due to the backward-traveling main winding mmf distribution and the forward traveling auxiliary winding mmf distribution.

The first component gives steady non-pulsating torque acting on the rotor in the forward direction and gives rise to a component torque/slip characteristic of the form obtained from a poly-phase TF machine [2-4]. The second component gives rise to a similar torque/slip characteristic, the torque acting in the opposite, backward direction. The third and fourth components give rise to torques which pulsate at twice supply frequency and do not contribute to the mean torque of the machine. That is, oppositely traveling mmf distributions do not contribute to the mean torque.

6. Slip of SPTF Machine

TF machine is a half-speed machine. That is, its synchronous speed is $\frac{\omega_o}{2}$. The slip of SPTF machine when the rotor runs in the forward direction is given by

$$s_f = \frac{\omega_o/2 - \omega_r}{\omega_o/2} = 2s - 1$$

Similarly, when the rotor rotates in the opposite direction the backward slip S_b is given by

$$S_b = \frac{\omega_o/2 - (-\omega_r)}{\omega_o/2} = 3 - 2s \quad (11)$$

7. Equivalent Circuit of a SPTF Machine

An equivalent circuit of the single phase TF machine may be obtained by considering it as a 3-phase machine with one of its supply lines disconnected, line c say as shown in fig 4.

From fig 4, $I_A = I$, $I_B = -I$, $I_C = 0$, $V_{AN} = \frac{1}{2}V$, $V_{BN} = -\frac{1}{2}V$. Also $V_{CN} = 0$, since voltages induced in the C phase due to its couplings with the A and

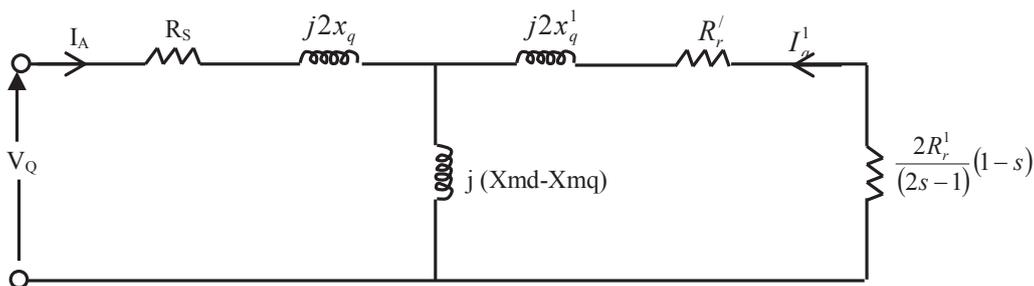


Figure 3: Per-phase steady-state equivalent circuit of polyphase TF machine operating in the asynchronous mode.

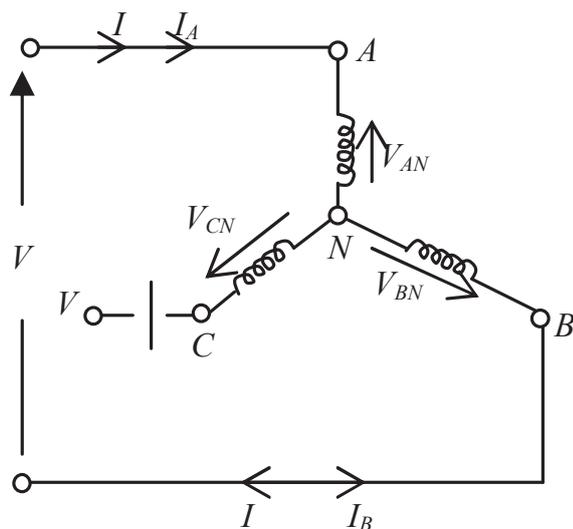


Figure 4: Single-phase operation of a 3-phase TF machine with line C disconnected.

B phases cancel out. Using the method of symmetrical components [6], the zero-sequence voltage of the applied voltage is

$$V_{AO} = \frac{1}{3}(V_{AN} + V_{BN} + V_{CN}) = 0 \quad (12)$$

So that $V_{AO} = V_{BO} = V_{CO} = 0$.

The applied voltage is given by

$$V = V_{AN} - V_{BN} \quad (13)$$

$$= (V_{A+}V_{A-} + V_{AO}) - (V_{B+}V_{B-} + V_{BO}) \quad (14)$$

$$= (1 - a^2)V_{A+} + (1 - a)V_{A-} \quad (15)$$

where $a = 1/120^\circ$

Considering the symmetrical components of the current, the zero-sequence current is

$$I_{AO} = \frac{1}{3}(I_A + I_B + I_C) = 0 \quad (16)$$

The positive phase-sequence current is

$$I_{A+} = \frac{1}{3}(I_A + aI_B + a^2I_C) = \frac{1}{3}I(1 - a)$$

Similarly, the negative phase-sequence current is

$$I_{A-} = \frac{1}{3}(I_A + a^2I_B + aI_C) = \frac{1}{3}I(1 - a^2) \quad (17)$$

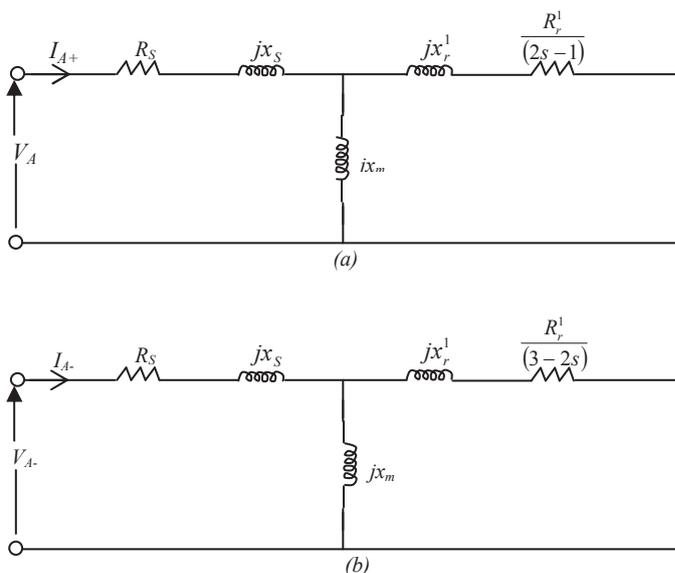


Figure 5: Per phase-sequence networks of the 3-phase TF machine (a) Positive (b) Negative.

From (16) and (17)

$$I = \frac{3I_{A+}}{1 - a} = \frac{3I_{A-}}{1 - a^2}$$

The total input impedance is

$$\begin{aligned} Z &= \frac{V}{I} = \frac{(1 - a^2)V_{A+}}{\frac{3I_{A+}}{1 - a}} + \frac{(1 - a)V_{A-}}{\frac{3I_{A-}}{1 - a^2}} \\ &= \frac{V_{A+}(1 - a^2)(1 - a)}{I_{A+} \cdot 3} + \frac{V_{A-}(1 - a)(1 - a^2)}{I_{A-} \cdot 3} \\ &= \frac{V_{A+}}{I_{A+}} + \frac{V_{A-}}{I_{A-}} \\ &= Z_+ + Z_- \end{aligned} \quad (18)$$

where Z_+ and Z_- is the positive phase-sequence and negative phase sequence operators of the 3-phase TF machine respectively.

The positive phase sequence equivalent circuit of the polyphase TF machine is evidently of the same form as the normal polyphase TF machine equivalent

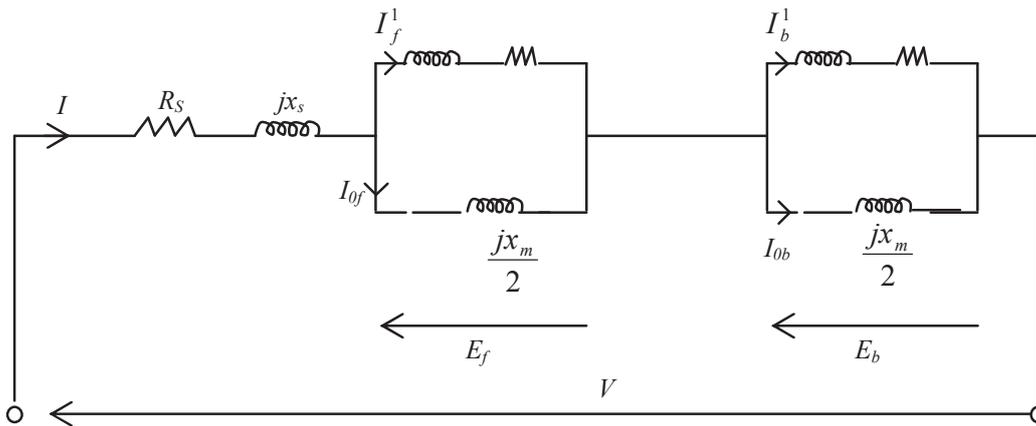
Figure 6: Equivalent circuit of a single phase transfer field machine for any slip s .

Table 1: Machine data.

Parameter	Value
L_{md}	140mH
L_{mq}	28mH
L_{ls}	7mH
$R_s = R'_r$	4Ω
J	$1.98 \times 10^{-3} \text{kgm}^3$
V	220V
Frequency	50Hz
No of Poles	2

circuit per phase. However, the magnetizing branch will have different values for single phase excitation than for 3-phase excitation owing to different mutual effects.

In fig 5, $X_s = 2X_q$, $X'_r = 2X'_q$, $Xm = (Xmd - Xmq)$. The final equivalent circuit of the SPTF machine is obtained by series connection of the positive and negative sequence networks of fig 5 as predicted by eqn. 18 and shown in fig 6 and recognizing that one phase of the 3-phase machine corresponds to half of the single-phase winding.

The pertinent data of the SPTF machine are given in table 1

8. Performance Characteristics of SPTF Machine

The performance characteristics of a SPTF machine can be determined from its equivalent circuit of fig 6 for different values of slip. Impedance due to forward field

$$Z_f = R_f + jX_f = \frac{R'_r}{2(2s-1)} + \frac{jx'_r}{2} \quad (19)$$

in parallel with $\frac{jx_m}{2}$.

Impedance due to backward field

$$Z_b = R_b + jX_b = \frac{R'_r}{2(3-2s)} + \frac{jx'_r}{2} \quad (20)$$

in parallel with $\frac{jx_m}{2}$.

$$E_f = IZ_f \quad (21)$$

$$E_b = IZ_b \quad (22)$$

$$I'_f = \frac{E_f}{\frac{R'_r}{2(2s-1)} + j\frac{X'_r}{2}} = \frac{IZ_f}{\sqrt{\left[\frac{R'_r}{2(2s-1)}\right]^2 + \left(\frac{X'_r}{2}\right)^2}} \quad (23)$$

$$I'_b = \frac{E_b}{\frac{R'_r}{2(3-2s)} + j\frac{X'_r}{2}} = \frac{IZ_b}{\sqrt{\left[\frac{R'_r}{2(3-2s)}\right]^2 + \left(\frac{X'_r}{2}\right)^2}} \quad (24)$$

Gross power transferred to the auxiliary winding by the forward field

$$P_{gf} \text{ (watts)} = (I'_f)^2 \frac{R'_r}{2(2s-1)} \quad (25)$$

Gross power transferred to the auxiliary winding by the backward field

$$P_{gb} \text{ (watts)} = (I'_b)^2 \frac{R'_r}{2(3-2s)} \quad (26)$$

Mechanical power output for the forward field

$$\begin{aligned} P_{mech,f} \text{ (watts)} &= [1 - (2s - 1)]P_{gf} = 2(1 - s)P_{gf} \\ &= (I'_f)^2 \frac{R'_r}{2(2s-1)}(1 - s) \end{aligned} \quad (27)$$

Torque due to forward field

$$T_f \text{ (Nm)} = \frac{(I'_f)^2 R'_r}{\omega_s(2s-1)} \quad (28)$$

A plot of the forward torque T_f for various values of slip is shown in fig7a.

Mechanical power output for the backward field,

$$\begin{aligned} P_{mech,b} \text{ (watts)} &= [1 - (3 - 2s)]P_{gb} = -2(1 - s)P_{gb} \\ &= \frac{-(I'_b)^2 R'_r [1 - s]}{(3 - 2s)} \end{aligned} \quad (29)$$

Torque due to backward field

$$T_b \text{ (Nm)} = \frac{-(I'_b)^2 R'_r}{(3 - 2s)\omega_s} \tag{30}$$

A plot of the backward torque T_b for various values of slip is shown in fig 7b.

Net mechanical power output is the sum of equations 27 and 29

$$\begin{aligned} P_{mech,net} &= 2(1 - s)P_{gf} + [-2(1 - s)]P_{gb} \\ &= 2(1 - s)[P_{gf} - P_{gb}] = P_{mech,f} - P_{mech,b} \end{aligned} \tag{31}$$

Resultant torque developed

$$\begin{aligned} T &= \frac{2(1 - s)[P_{gf} - P_{gb}]}{\omega_s(1 - s)} \\ &= \frac{2(P_{gf} - P_{gb})}{\omega_s} \end{aligned} \tag{32}$$

where $\omega_s = \frac{2\pi}{60}Ns$ and $Ns = \frac{120f}{P}$.

A superimposition of the forward, backward and resultant torque against slip is shown in fig 7c. Motor efficiency

$$\eta = \frac{\text{output power}}{\text{input power}} \times 100 \tag{33}$$

where input power is given by $VI \cos \phi$ and output power is given by eqn 31, neglecting mechanical losses. It is evident from fig 6 that the motor current is given by:

$$I = \frac{V}{Z_{eq}} + \frac{V}{Z_s + Z_f + Z_b} \tag{34}$$

where $Z_s = R_s + jx_s$, Z_f is the parallel combination of $(\frac{R'_r}{2(2s-1)} + \frac{jx'_r}{2})$ and $\frac{jx_m}{2}$ and Z_b is the parallel combination of $(\frac{R'_r}{2(3-2s)} + \frac{jx'_r}{2})$ and $\frac{jx_m}{2}$. The Power factor,

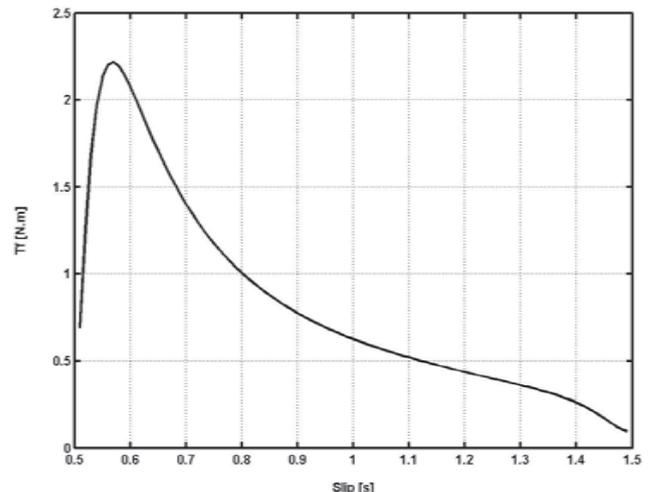
$$\cos \phi = \frac{R_{eq}}{Z_{eq}} = \frac{R_s + R_f + R_b}{Z_s + Z_f + Z_b} \tag{35}$$

where R_f and R_b are the real parts of Z_f and Z_b , respectively.

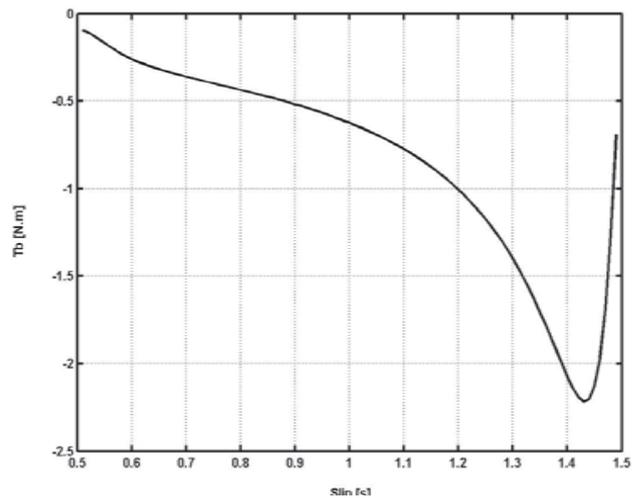
A plot of the efficiency η against slip for motoring operation is shown in fig 8. It is seen that the maximum efficiency is very low about 36%.

9. Conclusion

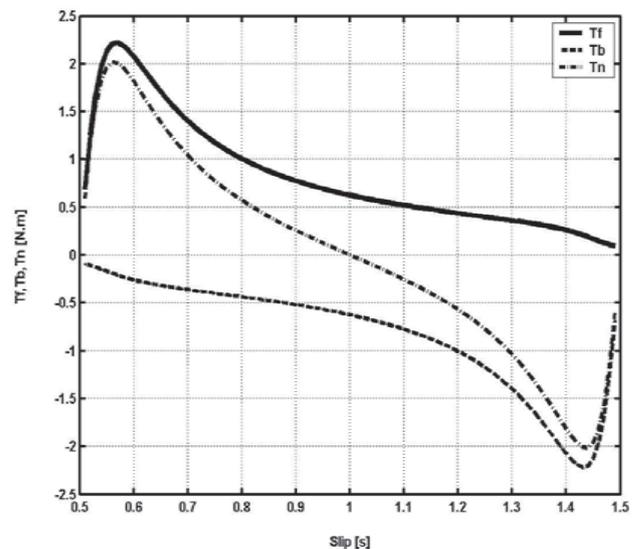
The equivalent circuit of a SPTF machine from which the performance characteristics can be predicted has been presented. The torque-slip curve of the SPTF machine shows that at standstill ($s = 1, Nr = 0$) the net torque of the machine passes through zero which implies that the machine unlike the polyphase TF version is not self-starting. For other values of slip; however, the motor develops a



(a) Torque/slip curve due to forward field.



(b) Torque/slip curve due to backward field.



(c) Torque/slip curves due to forward, backward and net fields

Figure 7: Torque/slip curves.

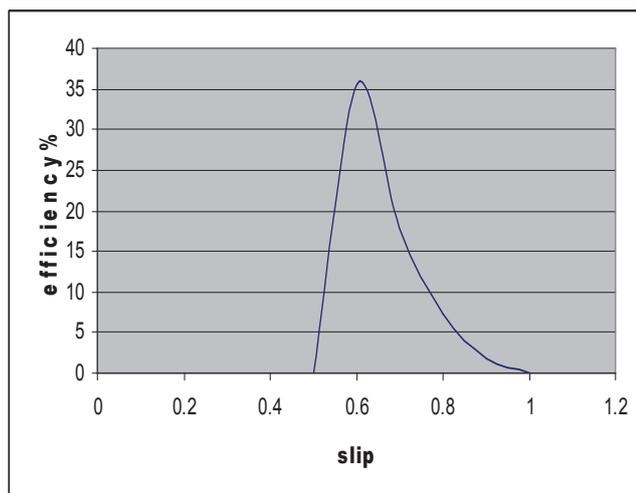


Figure 8: Efficiency/slip curve of SPTF machine.

net torque in the direction of rotation and the operating conditions with the rotor running in either direction at a given speed are identical. Additionally, unlike polyphase type, the torque developed in SPTF machine drops to zero at a slip slightly below synchronous on account of negative torque developed by the backward field. This phenomenon is absent in a polyphase TF machine. The performance characteristics of the SPTF machine in general are inferior to those of polyphase TF machine of the same size in terms of maximum output, power factor, efficiency etc on account of the backward rotating field. As described in section 5.0, c and d, the torque due to forward travelling main winding mmf distribution and the backward travelling auxiliary winding mmf distributions and the torque due to the backward travelling main winding mmf distribution and the forward travelling auxiliary winding mmf distribution move in opposite direction with relative speeds of double the synchronous speed and develop second harmonic pulsating torque with zero average value. As a consequence, the SPTF machine is noisier than the polyphase type which has no such pulsating torques. This characteristic is, however, common to most single phase motors. The effect of the pulsating torque can be minimized by using elastic mounting, rubber pads etc.

The efficiency of the machine is very low compared to an equivalent single phase induction motor on account of excessive leakage reactance of the machine. The excessive leakage reactance is predicated on the two machines comprising the SPTF machine which are connected in cascade thus giving rise to double end leakage reactance. Additionally, the inter-segment conductors between the two machine sections contributes to leakage reactance and does not contribute to energy transfer in the machine. More fundamentally, the leakage reactance of the machine is a

combination of the normal leakage reactance and the quadrature axis reactance [1-4]. Enhancement of efficiency of the SPTF machine is being studied by the authors and will be reported soon.

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