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Tiragatso Ya Itlhagiso Ya Setlhare Se Se Okeditsweng Ka Kgetsi Mo Bothateng Jwa Popo Ya Metato Ya Dipeipi Tsa Oli

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Kakaretso

Go na le mathata a mantsi a ditshwetso tsa tiriso tse di welang mo mathateng a a mo setlhopheng sa kelelo ya kgokagano le palo e kgolo ya dikai tsa tiragatso tse di ka bonwang mo dikgaolong jaaka tsa neeletsanyokgakala, thwalo, boenjineri, saense ya dikhomphutara jalo le jalo. Mo pampiring e, kgonagalo ya go tlhagisa mmotlele wa kelelo ya kgokagano o o leng mmotlele wa kgokagano ya setlhare mme morago re e rarabolole ka go dirisa itlhagiso ya setlhare se se okeditsweng ka kgetsi ka go e batlisisa. Go bapisa le go tlhwatlhwafatsa thekeniki e e tlhagisiwang, thuto ya nnete e e totobetseng e e dirilweng (bothata jwa popo ya motato wa dipeipi tsa oli) e tlhophilwe go tswa mo dikwalong gore e dirisiwe go nna motheo wa porojeke e ya patlisiso. Ka go latela pono ya bothata jwa popo ya metato ya dipeipi, tlhabololo ya sekao sa setlhare se se okeditsweng ka kgetsi se tlaa tlhagisiwa. Tiragatso ya mokgwa o mo bothateng jwa popo ya metato ya dipeipi tsa oli e tlaa tlhagisiwa morago. Maduo a a bonwang a tlaa tlhagisiwa mme a bontsha gore go na le boleng jwa go ka dirisa itlhagiso ya setlhare se se okeditsweng ka kgetsi go ka rarabolola tse dingwe tsa mathata a kelelo ya dikgokagano.

Mareo: Dikao tsa kelelo dikgokagano, tlhamomananeo ya ka mela ya intejere, sekao sa setlhare se se okeditsweng ka kgetsi.

Abstract

There are many practical decision problems that fall in the category of network flow problems and numerous examples of applications can be found in areas such as telecommunication, logistics, engineering and computer science. In this paper, the feasibility of representing a network flow model as a tree network model and subsequently solving it using an extended tree knapsack approach is investigated. To compare and validate the proposed technique, a

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specific case study (an oil pipeline design problem) was chosen from the literature that can be used as a basis for the paper. Following on an overview of the pipeline design problem, the extended tree knapsack model is developed. The application of this approach to the oil pipeline design problem is then presented. Results indicate that it is feasible to apply an extended tree knapsack approach to solve certain network flow problems.

Key words: Network flow models, integer linear programming, extended tree knapsack model

Extended abstract: Applying an extended tree knapsack approach to an oil pipeline design problem

There are many practical decision problems that fall in the category of network flow problems and numerous applications can be found in areas such as telecommunication, logistics, distribution, engineering and computer science. An important aspect relevant to the discussion of network models is network design. Many network models and examples usually describe problems related to existing networks. For example, determine the shortest path, or maximum flow through an existing network. Network design problems usually involve decisions regarding network topology and capacity planning to satisfy certain demands or requirements. Some of the major issues in network design include justification of a network, scope, manageability, architecture, topology, sizing, routing *etc.* [3]. Apart from these issues, another challenge that researchers have to deal with is to constantly attempt to enhance the models or improve on the computational times to solve these types of models. Recent studies by Van der Merwe and Hattingh [6, 7] applied an extended tree knapsack approach to local area telecommunication networks in order to address these challenges.

In this paper, the feasibility of representing a network design model in a tree network model and subsequently solving it using an extended tree knapsack approach is investigated. To compare and validate the proposed technique, a specific case study (an oil pipeline design problem) was chosen from the literature that can be used as a basis for the research project [1].

The remainder of this extended abstract is organized as follows. In the first section a brief overview of the oil pipeline design problem is given. This is followed by the model development for the design problem in the second section. The third section briefly presents the results and the fourth section concludes the extended abstract.

The oil pipeline design problem

To investigate the feasibility of solving a network design problem using an extended tree knapsack approach, a specific case in the literature was chosen that could be used to test the proposed approach. The selected case describes an optimal oil pipeline design for the South Gabon oil field in Africa [1]. The project considers a set of offshore platforms and onshore wells, each producing a known or estimated amount of oil that needs to be connected to a port. These connections may take place directly between platforms, well sites and the port, or may go through connection points at given locations. The objective of the pipeline system is to reduce the cost of transporting oil to a specific port in order to allow for expansion of production to enable increased profitability this implies that the configuration of the network and sizes of pipes must be chosen to minimize construction cost. The South Gabon oil field network consists of 33 nodes (distances between the nodes are known). These represent the offshore platforms, onshore wells, seven connection points and one port called Gamba. There is 129 potential arcs and all the oil production in this region is transported to Gamba, from where it is then exported by sea.

Model development for the oil pipeline design problem

An ordinary tree knapsack problem may be regarded as choosing a sub tree of a tree. A complete description and formulation can be found in Van der Merwe & Hattingh [7]. The extended tree knapsack model is a more general form of the tree knapsack model. In the extended tree knapsack model there is also a cost involved in transmitting y_i units from node i to predecessor p_i, say $f_i(y_i)$ where f_i is an arbitrary function that satisfies the condition that $f_i(0) = 0$. This model is discussed in detail in Shaw [4] and Van der Merwe [6].

The methodology followed in this study comprises two main steps. In the first step, the network representation of the pipeline design problem was converted into a tree structure to facilitate the use of a tree knapsack method as a solution approach. During the second, a mathematical programming model based on an extended tree knapsack model was formulated and solved in order to be able to express an opinion on the feasibility of the proposed methodology. The following paragraphs describe the two steps.

Converting the pipeline network into a tree network structure

Prior to model development, the South Gabon oil field network had to be converted into a tree structure. This process involves a series of steps *i.e.* identification of the root node, creating adjacent node lists for each node in the network, and finally building a tree network structure by creating paths based on the adjacency lists.

The root node was given as the Port of Gamba. Next, adjacent node lists were created; an adjacent node list for a specific node is a set of nodes that are directly connected to that specific node. Following this, a tree network can be build. This is done in a breadth first manner adding child nodes level by level to the tree.

Model development

The next step in the proposed methodology is to formulate a mathematical programming model which is based on an extended tree knapsack model, and which will be used to solve the tree structure constructed above.

The objective function of the extended tree knapsack model was based on a fixed charge cost model and is interpreted as the minimization of the sum of selected pipe links plus a variable cost determined by the flow above capacity level. The following binary variables are used. Let

$$x_i = \begin{cases} 1 & \text{if node } i \text{ is selected} \\ 0 & \text{otherwise for all nodes} \end{cases}$$

and

$$\delta_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is selected} \\ 0 & \text{otherwise for all arcs.} \end{cases}$$

The model was formulated to

$$\text{minimise} \sum_{(i,j)} E_{ij}\delta_{ij} + \sum_{(i,j)} a_{ij}(f_{ij_1} + f_{ij_2}) \tag{1}$$

$$\text{subject to } x_j - x_{p_j} \leq 0 \qquad j = 1, 2, \ldots, n-1, \tag{2}$$

$$\sum_{i \in S_j} x_i = 1 \qquad j = 1, 2, \ldots, n, \quad S_j = \{i | N(i) = j\}, \tag{3}$$

$$D\boldsymbol{x} - B(\boldsymbol{f}_1 + \boldsymbol{f}_2) = \boldsymbol{0} \qquad j = 1, 2, \ldots, n, \tag{4}$$

$$f_{ij_1} + f_{ij_2} \leq \delta_{ij} C \qquad \text{for all } (i,j), \tag{5}$$

$$0 \leq f_{ij_1} \leq P_{ij} \qquad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, n, \tag{6}$$

$$f_{ij_2} \geq 0 \qquad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, n, \tag{7}$$

$$\delta_{ij} \in \{0, 1\} \qquad \text{for all } (i,j), \tag{8}$$

$$x_i \in \{0, 1\} \qquad i = 1, 2, \ldots, n. \tag{9}$$

where E_{ij} is the fixed cost associated with each arc (i,j), a_{ij} is the cost incurred per unit of flow f_{ij}, D is a diagonal matrix with diagonals d_j giving the production at node j. B is a node-arc incidence matrix, C is the total capacity at the root of the tree network, P_{ij} is the existing pipe capacity on arc (i,j) (or small positive ε_{ij} when no pipeline exists), $\boldsymbol{x}$ is a variable vector of the binary variables x_i for $i = 1, 2, \ldots, n$ and p_j is the parent node of node j. If f_{ij} represents the total flow between node i and its predecessor node j, flow is partitioned into two parts such that $f_{ij} = f_{ij1} + f_{ij2}$ where f_{ij1} represents the flow which is less than ε_{ij} and f_{ij2} represents the flow greater than ε_{ij} (a small positive number). The vectors $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ have elements f_{ij1} representing the flow below existing pipe capacity and f_{ij2} representing the flow above existing pipe capacity respectively. If no pipeline exists between i and j both these flows must be zero. In the case of a network expansion problem one may visualize f_{ij1} as a flow that is available at zero cost (up to the capacity). If no pipeline exists, this capacity is zero.

Results

A program was coded in C++ to perform the tree generation. This resulted in an indexed tree consisting of 7 030 nodes and 4 183 paths. The tree was then solved by the extended tree knapsack model using CPLEX (version 10.1) which employs Concert Technology from

ILOG [2]. A solution was obtained after 70.15 seconds utilising a HP Beowulf Cluster with Gbit Interconnect. The cluster consists of one DL385 control node (2GB RAM) and ten DL145G2 computing nodes (4GB RAM each), and uses a RedHat Enterprise operating system. The solution obtained showed that in 6 instances the extended tree knapsack model has chosen different arcs from those in the original problem in the literature. The difference in the two solutions can be attributed to the approximated cost function used in the extended tree knapsack model. This approximation may result in an over or under estimation of the costs.

To perform a meaningful comparison between this study's results and those of Brimberg *et al.* [1], the cost for both solutions, was calculated using the given pipe costs. This resulted in a cost of 1423 units for the Brimberg *et al.*, study and 1 461 units for the extended tree knapsack model resulting in a deviation of 2.6%.

The results of the study may be summarized as follows.

1. The extended tree knapsack model produced a result that was within 2.6% of that in the original study. The low percentage deviation proves that it is definitely feasible to use an extended tree knapsack approach to solve certain network design problems. By refining the cost function used in the tree knapsack model the 2.6% gap could be further reduced – an approximation was used in this paper to represent the objective function. A closer approximation may be introduced at the expense of more discrete variables but does not fall within the scope of the problem considered here.
2. The feasibility is further proved by the relatively short computational time to solve the model (70.15 seconds using CPLEX).
3. The oil pipeline design problem was a fairly large network comprising 33 nodes and 129 arcs. With modern software, *e.g.* CPLEX, large network design problems may be solved in a reasonable time using the extended tree knapsack approach.

Conclusion

This paper considered a network design problem and investigated the feasibility of representing a network flow model as a tree network model that may be solved by an extended tree knapsack approach.

A specific case study (an oil pipeline design problem) from the literature was selected in order to test the proposed solution method. The original oil pipeline design problem was transformed into a tree structure and the tree was then solved using an extended tree knapsack model.

The results show a small deviation from the solution presented in the literature which may be attributed to an approximation used to represent the objective function in the extended tree knapsack model. A solution was obtained after a relatively short time for a fairly large network. Based on these results it was concluded that the extended tree knapsack model is a feasible alternative to solve certain network design problems.

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1 Matseno

Kgokagano ya naga ya oli (jaaka e tlhalositswe mo karolong ya bobedi) e e agilweng ka palo ya dipolateforomo tsa mo lewatleng le didiba tse di leng mo lewatleng tse di tlhokang go gokaganngwa le poroto e tlaa elwa tlhoko mo thutong e. Bothata jwa mo nageng ya oli le sekai sa mofuta wa bothata jwa popo ya kgokagano e e dirang setlhopha se segolo e bile e le se se kgethegileng sa dimmotlele tsa tlhamomananeo ya mela ya intejere tse di kgolo tse di ka mofuta o o kgethegileng.

Kgokagano ke thulaganyo ya ditsela tse di tshwaraganwang le mafelo a mantsi a mo go yone go fetang mofuta o le mongwe kampo e mentsi ya dilo go tswa mo lefelong le le nngwe go ya kwa go tse dingwe [5]. Mefuta e ya dimmotlele e ratiwa thata mme e dirisiwa mo mefuteng e mentsi ya tiriso jaaka mo phitisong ya tshedimosetso, thwalo ya batho, phatlhalatso ya dithoto jj. Ka tshwanelo, kgokagano e ka ranolwa ka go re ke sete ya dilo tse di gokaganweng ka digopo. Sediriso sengwe le sengwe se bidiwa ntlhana kampo noutu mme go ya ka diteori tsa dikerafo e ka tlhagisiwa jaaka para (V, E), fa V e leng sete ya dintlhana, mme E e leng sete ya disi mo gare ga dintlhana gore $E \subseteq \{(u, v)|u, v \in V\}$ [3].

Ntlha e e botlhokwa e e maleba mo puisanong ya dikao tsa dikgokagano ke popo ya dikgokagano. Dimmotlele tse dintsi tsa dikgokagano le dikai ka gale di tlhalosa mathata a a amaneng le dikgokagano tse di leng teng. Sekai e ka nna, batla tsela e khutshwane go feta, kampo batla kelelo e kgolo go feta e e ka kgonagalang go feta mo kgokaganong e e leng teng. Mefuta e ya mathata mo dikgokaganong e ka rarabololwa ka dikao tsa tlhamomananeo ya ka mela kampo ka dialekoreteme le diheuresetiki jaaka ka alekoreteme ya ga Dijkstra e e itsiweng thata [3]. Mathata a go bopa dikgokagano fa di farologanngwa ka gale a tlisa dikakanyo ka ga popego ya tikologo le go loga maano a motlhamo go ka itumedisa dikopo kampo ditlhokego tse di rileng. Fa go le jaana, ditekeniki tsa tlhamomananeo ya ka mela ya intejere ka gale di a dirisiwa. Mefuta e ya mathata nako e ntsi e thata go rarabolola mme nako e ntsi batsayadikgato ba gatelelwa go amogela ditharabololo tse di atametseng fela. Mathata a popo ya kgokagano e kgatlha thata mo thutong e, foo go tshwanetseng gore go diriwe tlhopho go tswa mo mefuteng ya mathamo ya dipeipi go nna dikopantsho mo nageng ya oli.

Kwa ntle ga se se fa godimo, kgwetlho e nngwe e babatlisisi ba tshwanetseng go lebagana le yone ke gore ka dinako tsotlhe ba tshwanetse go leka go tlhatlhosa dimmotlele kampo ba leke go tokafatsa nako e e dirisiwang go rarabolola mefuta e ya dimmotlele. Dithuto tsa maloba tsa ga Van der Merwe le Hattingh [7, 8] di dirisitse mokgwa wa itlhagiso ya setlhare se se okeditsweng ka kgetsi mo dithulaganyong tsa tlhaeletsano ya megala ya kgaolo ya selegae go leka go rarabolola dikgwetlho tse.

Ka maiteko a go tshwaela mo dikganngeng dingwe tse go builweng katsone fa godimo, pampiri e e sekaseka kamogelo ya go baya mmotlele wa kgokagano ya kelelo (bothata jwa popo ya dipeipi tsa oli) go nna mmotlele wa kgokagano ya setlhare mme morago bothata bo ka rarabololwa ka go dirisa itlhagiso ya setlhare se se okeditsweng ka kgetsi ka mokgwa o o tshwanang le mokgwa o Van der Merwe le Hattingh [7, 8] ba o dirisitseng go rarabolola bothata jwa dithulaganyong tsa tlhaeletsano ya mogala ya kgaolo ya selegae. Go bapisa le go rurifatsa thekeniki e e akantsweng, bothata jo bo totobetseng jwa popo ya metato ya dipeipi bo tlhophilwe go tswa mo dikwalong gore bo dirisiwe go nna motheo wa porojeke e ya patlisiso [1].

Karolo e e setseng ya pampiri e e ka rulagangwa ka mokgwa o o latelang. Mo karolong ya 2 ya pampiri go na le tlhaloso e khutshwane ya bothata jwa popo ya metato ya dipeipi tsa oli. Se se tlaa latelwa ke tlhagiso ya popo ya ka kakaretso ya mmotlele wa setlhare se se okeditsweng ka kgetsi mo karolong ya 3 fa tlhabololo ya mmotlele wa popo ya bothata e tlaa tlhagisiwa mo karolong ya 4. Karolo ya 5 e tlhalosa maduo mme karolo ya 6 e feleletsa pampiri ka dikakanyo tsa bofelo.

2 Bothata jwa popo ya metato ya dipeipi tsa oli

Go batlisisa kgonagalo ya go ka rarabolola bothata jwa popo ya kgokagano ka go dirisa itlhagiso ya setlhare se se okeditsweng ka kgetsi, go tlhopilwe kgang e e totobetseng mo dikwalong e e ka dirisiwang go leka itlhagiso e e tshitshingwang. Kgang e e tlhopilweng e tlhalosa popo ya metato ya dipeipi ya oli e e dirang sentle kwa nageng ya oli ya kwa Borwa jwa Gabon mo Aforika [1]. Karolo e e tlaa naya tlhaloso e e khutshwane ya kgang e e tlhopilweng.

Porojeke e lebelela palo ya dipolateforomo tse di mo lewatleng le didiba tse di mo lewatleng, tse nngwe le nngwe ya tsone e e ntshang selekanyo sa oli se se itseweng kgots se se fopholediwang se se tshwanetseng go tlisiwa lefatsheng mo thoko ga noka. Dikgokelelo tse di ka diragala ka tlhamalalo fa gare ga dipolateforomo, ga mafelo a didib, le phote, kgotsa di ka nna tsa tsamaya ka dintlha tsa kgokelelo tse di mo mafelong a a filweng. Maikaelelo a thulaganyo ya metato ya dipeipi ke go leka go fokotsa ditshenyegelo tsa thwalo ya oli go ya kwa boemadikepeng jo bo rileng gore go tle go kgonwe go ka godisa ntshokumo go kgontshas gore go nne le morokotso o o oketsegileng — se se rayang gore sebopego sa kgokagano le bogolo jwa dipeipi bo tshwanetse go tlhopiwa sentle go ka fokotsa ditshenyegelo tsa kago jaaka go ka kgonagala. Sethalo 2.1 ke setshwantsho sa kgokagano ya naga ya oli ya kwa Borwa jwa Gabon.

Go tswa mo Sethalong sa 2.1 go bonala sentle gore kgokagano ya naga ya oli ya kwa Borwa jwa Gabon e bopilwe ka dinoutu di le 33. Tse di balela dipolateforomo tsa mo lefatsheng thoko ga lewatle le didiba tsa mo lewatleng (ka bobedi di bontshiwa ka didiko mo sethalong sa 2.1), dintlha di le 7 tsa tshwaragano (di bontshiwa ka dikhutlonne mo sethalong sa 2.1) le boemadikepe bo le nngwe jwa Gamba (noutu ya 33). Palo mo teng ga sediko se sengwe le se sengwe le mo khutlonneng nngwe le nngwe le raya noutu mme palo e e bapileng le didiko ke palo ya kumo mo mafelong ao. Go na le palo ya 129 ya digopo tse di ka bo di leng le palo ya mo digopong e supa sekgala magareng a dinoutu. Kumo yotlhe ya oli mo tikologong e e isiwa kwa Gamba mme go tswa moo e isiwa kwa ntle ka tsela ya lewatle.

Kgokagano le data, jaaka go bontshiwa mo sethalong sa 2.1 di tlaa dirisiwa mo thutong e go atlhola tsiamo ya go ka dirisa itlhagiso ya setlhare se se okeditsweng ka kgetsi go ka rarabolola bothata. Tshoboko e khutshwane ya mekgwa ya go rarabolola le dikgwetlho tsa bothata jwa kwa tshimologong e tlaa fiwa go tswa mo dikwalong mo temeng e e latelang; tlhaloso e e tseneletseng e ka bonwa mo Brimberg *et al.* [1].

Methamo e e farologaneng ya dipeipi e ne ya akanngwa ke Brimberg *et al.* [1] mme tlhotlhwa yotlhe ya karolo ya peipi e amogetswe ka go atisa sekgala sa digopo ka tlhotlhwa ya diyuniti mo mothalong wa peipi nngwe le nngwe. Mo tshimologong go dirisitswe mekgwa e mebedi ya go rarabolola bothata jwa popo jwa metato ya dipeipi (patlo ya tabu le patlo ya boagisani jo bo fetogang). Mekgwa e ka bobedi e ile ya fana ka tharabololo ya ditshenyegelo tse di lekanang le diyuniti di le 1423. Brimberg *et al.* [1] gape o tlhalositse bothata go nna mmotlele wa porokeramo ya ka mela ya intejere e e tlhakaneng ka maikaelelo a go fokotsa tlhotlhwa yotlhe ya dipeipi tsotlhe tse di tsengwang. Ka go bopa sewa kgokagano ya naga ya oli go nna dikgokagano di le pedi ka go dirisa tsamaiso ya dikala le dikgole, gammogo le dintlha tse dingwe go ka laola dikgole tsa bothata, bothata jwa popo ya kgokagano e ne ya akangwa mme go ne go kgonwa gore ba bontshe ka bopaki gore ditharabololo tsa heuresetiki di tlisa ditharabololo tsa nnete tsa maemo a a kwa godimo. Ba lemogile gore mo popong ya bone ya MILP, tharabololo ya nnete ya maemo a a kwa godimo e thata go lekanngwa ka tlhamalalo. Se se nnile tlhotlheletso e nngwe gape mo itlhagisong ya rona mo bothateng jwa popo ya metato ya dipeipi tsa oli.

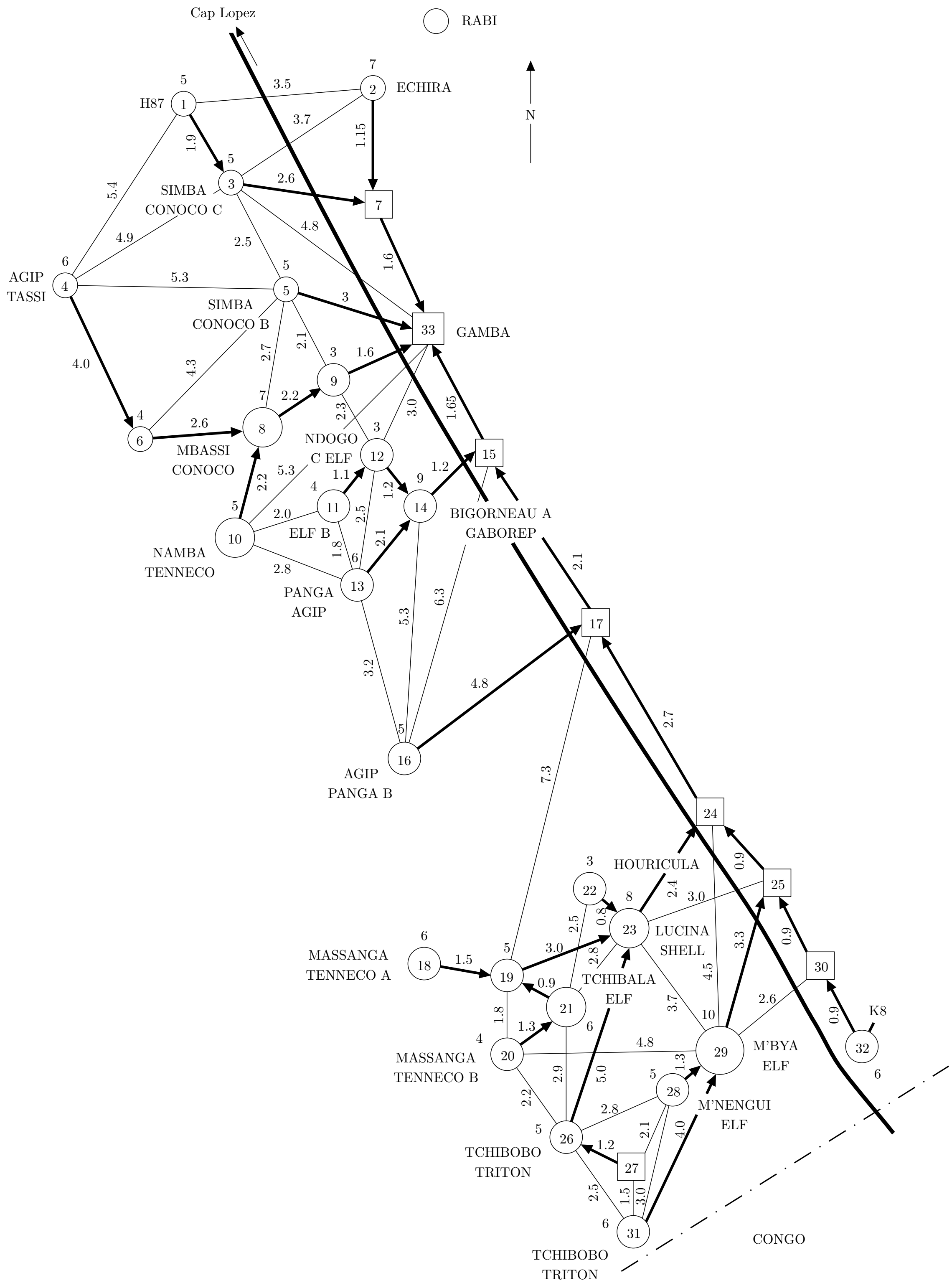
3 Setlhare se se okeditsweng ka kgetsi

Karolo e e simolola ka tlhaloso e e khutswane ya bothata jwa setlhare ka kgetsi ya tlwaelo ka gore se se tlhokega thata mo pampiring e.

Bothata jwa setlhare ka kgetsi e ka bonwa e le go tlhopha setlhare tlase ga setlhare mme e ka tlhaloswa ke Van der Merwe le Hattingh ka tsela e e latelang [7]. Fa o fiwa setlhare se se sa kaelwang $T = (V, E)$ ka n dinoutu tse di meletseng mo noutung ya 0, $V = \{0, 1, \ldots, n-1\}$ ke sete ya dinoutu tse di ka beiwang maina, a ke ka boteng le ka bophara tsa feene ya ntlha le fa E go diriwa didinoutu tsa mafelo a a tlhalositsweng. Kwalonnoto e e latelang e a amogelwa,

d_i	kopo e e dirisiwang ka go akaretsa le noutu i mo setlhareng se se kwa tlase,
c_i	poelo e e amogelwang ka go akaretsa noutu i mo setlhareng se se tlase,
p_i	ketelopele kampo noutu ya motsadi i, mme
H	palogotlhe ya mothamo wa kgetsi.

Tiro jaanong ke go bona setlhare tlase $T' = (V', E')$ ya T e e meletseng mo noutung ya 0 jaaka $\sum_{i\in V'} d_i \leq H$ le $\sum_{i\in V'} c_i$ di oketswa go nna kgolo jaaka go ka kgonagala. A jaanong $x_j = 1$ fa noutu ya j e tlhophiwa, 0 mo go tse dingwe.



Sethalo 1: *Naga ya oli ya Borwa jwa Gabon [1].*

Bothata jaanong bo ka bewa e le bothata jwa tlhamomananeo ya ka mela ya intejere

$$\text{Godisetsa kgolo } Z = \sum_{j=0}^{n} c_j x_j \tag{10}$$

$$\text{ka go baya taolo go } x_j - x_{p_j} \leq 0 \qquad j = 1, 2, \ldots, n, \tag{11}$$

$$\sum_{j=0}^{n} d_j x_j \leq H \tag{12}$$

$$x_j \in \{0, 1\} \qquad j = 1, 2, \ldots, n. \tag{13}$$

Mo setlhareng sa tlase noutu e ka akaretswa fela fa e le gore noutu ya motsadi e akaretswa mo setlhareng sa tlase (lebelela tshwenyego ya ntlha fa godimo). Se le sone se ka beiwa ka mokgwa o o rileng gore fa noutu j e ka akaretswa, dinoutu tsotlhe mo tseleng e e leng nngwe ka mofuta magareng a noutu j le noutu ya modi 0 di tshwanetse go akaretswa — se se bidiwa e le tumelo ya tse di atumetsaneng. E ka kwadiwa gore kgetsi ya nnoto-ya-ntlha ke kgang e e kgethegileng foo setlhare se se bopilweng fela ka noutu ya modi le maemo a le mangwe a matlhare a setlhare. Ka gale go na le ditlhare tlase tse dintsi tse di ka nnang teng tse di tshwaragantshitsweng ka noutu ya modi ya 0. Tlhopho ya gore o dirisa setlhare tlase sefe go tswa mo gore maitlhomo a bothata jwa kgetsi mme ka tlwaelo e theetswe mo go godisang ga poelo kampo go nnyenyefatsa ditshenyegelo.

Setlhare se se okeditsweng ka kgetsi ke mofuta wa ka gale go feta mmotlele wa setlhare ka kgetsi. Go buiwa ka botlalo ka mmotlele mo go Shaw [4] mme o tlhaloswa fela ka bokhutshwane fa ka gore ke metheo e e tshwanang e e ka dirisiwang go araba bothata jwa popo ya metato ya dipeipi tsa oli.

Mo sekaong sa setlhare se se okeditsweng ka kgetsi go na le ditshenyegelo tse di leng teng mo tsamaisong ya y_i diyuniti go tswa go noutu ya i go ya pele p_i, e re $f_i(y_i)$ e ne e le f_i ke tiro e le nngwe e e arabelang lebaka la gore $f_i(0) = 0$. Van der Merwe [6] o tlhalosa sekao jaaka go latela.

A $T = (V, E)$ e nne setlhare se se sa kaelwang ka n mme dinoutu di teiwe maina ka boteng pele mme ka tatelano e e meletsweng mo go 0 mme ka $V = \{0, 1, \ldots, n-1\}$. A $\hat{T} = (V, E)$ mo se se tswang mo setlhareng e le se se fanang ka T, ditaetso mo dikarolong tsa mo thoko tsa setlhare sa mo tshimologong. A setlhopha sa A se tlhaloswe jaaka go latela, $A = \{(p_i, i) | i \in V\}$. Ranola B go nna $n \times n$ materikisi ya tiragalo ya segopo sa noutu $\hat{T}$ ya go ntsha mola wa go tsamaisana le noutu ya modi (noutu 0). Mo go B, mola o mongwe le o mongwe o tsamaisana le noutu mme kholomo nngwe le nngwe e tsamaisana le segopo. Se se raya gore kholomo ya i ya B e na le dilo mo teng, tsa phapogo ya nnoto mo moleng wa i le ya p_i ($\neq 0$), tse di nang le boleng jwa 1 le -1 ka tatelano eo. A y_i e nne palo ya pharakano e e romelwang go tswa go noutung i go ya go motsading wa yone p_i mme e letlelela $x_i = 1$ fa noutu i e e tlhophilweng go direlwa, 0 ka mokgwa o mongwe, mme letlelela $\boldsymbol{y} = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n$. Tlhalosa materikisi D go nna, $D = \mathrm{diag(d_j)}$. Bothata jwa setlhare se se okeditsweng ka kgetsi jaanong bo ka tlhaloswa jaaka go latela, mo H

mothamo wa kgetsi e leng:

$$\text{Godisetsa kgolo } Z = \sum_{j=0}^{n-1} c_j x_j - \sum_{j=0}^{n-1} f_i(y_i), \tag{14}$$

$$\text{ka go baya taolo go } x_j \leq x_{p_j} \qquad j = 1, 2, \ldots, n-1, \tag{15}$$

$$D\boldsymbol{x} - B\boldsymbol{y} = \boldsymbol{0}, \tag{16}$$

$$\sum_{j=0}^{n-1} d_j x_j \leq H, \tag{17}$$

$$\boldsymbol{y} \geq \boldsymbol{0}. \tag{18}$$

4 Tlhabololo ya Mmotlele wa bothata jwa popo ya metato ya dipeipi tsa oli

Thutatsela e e tsamailweng mo pampiring e e bopilwe ka dikgato tse pedi. Kgato ya ntlha, tlhagiso ya kgokagano ya bothata jwa popo ya metato ya dipeipi e fetotswe go nna popego ya setlhare go nolofatsa tiriso ya mokgwa wa setlhare ka kgetsi go nna tharabololo. Sa bobedi, mmotlele wa tlhomamananeo ya mmetshe e e theetsweng mo mmotleleng wa sekao sa setlhare se se okeditsweng ka kgetsi, o bopilwe mme o rarabolotswe go ka kgona go thadisa kakanyo ka ga bonnete jwa mokgwa o o tshitshingwang. Ditema tse pedi tse di latelang di tlhalosa dikgato tse pedi.

4.1 Go fetola kgokagano ya metato ya dipeipi go nna popego ya kgokagano ya setlhare.

Pele ga tlhabololo ya mmotlele, kgokagano ya naga ya oli ya Borwa jwa Gabon (bona Sethalo sa 2.1) e ne e etshwanetse go fetolelwa kwa popegong ya setlhare. Tirego e e balela tlhatlhamano ya dikgato tse di leng temogo ya noutu ya modi, go bopa manaane a dinoutu tse di bapileng mo noutung nngwe le nngwe mo kgokaganong mme mo bofelong go agiwe popego ya kgokagano ka go bopa ditsela tse di theetsweng mo dinaaneng tse di bapileng.

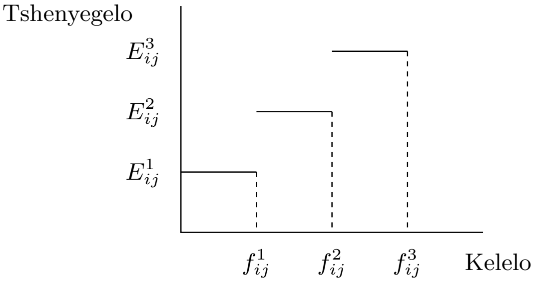
Noutu ya modi e ne e filwe e le noutu ya 33 (Poroto ya Gamba). Morago, manaane a dinoutu tse di bapileng a ne a bopiwa; lenaane la dinoutu tse di bapileng foo noutu e e totobetseng e leng sete ya dinoutu tse di tshwaragantshitsweng le noutu e e totobetseng. Go latela se jaanong go ka agiwa kgokagano ya setlhare. Se se diriwa mo bophareng pele ka go tlhakanya dinoutu tse dinnye mo karolong nngwe le nngwe go tsenya dikalanyana maemo ka maemo mo setlhareng.

Go tshwanetse go elwe tlhoko gore tlhagiso e e dumela gore ga go ketla go na le kgabetlelo ya kelelo kampo dilupu mo kgokaganong.

4.2 Tlhabololo ya Mmotlele

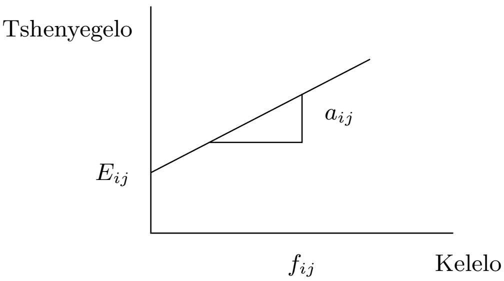
Kgato e e latelang mo thutatselang e e tlhagisiwang ke go bopa mmotlele wa tlhomamo-naneo ya mmetshe o o theetsweng mo sekaong sa setlhare se se okeditsweng ka kgetsi mme o tlaa dirisiwa go rarabolola popego ya setlhare se se bopilweng fa godimo .

Tiro ya maitlhomo mo sekaong sa mmetshe e e tshitshingwang ke Brimberg *et al.* [1] e ne e theetswe mo ditshenyegelong tse di tsamiasanang le methomo e e farologaneng ya dipeipi jaaka mokgwa wa kelelo wa f_{ij}^1 o ka tlhoka tshenyegelo ya E_{ij}^1 (mothamo $k = 1$). Fa kelelo e oketsega go feta mothamo wa 1, go tlaa tlhokega mothamo o mowa ($k = 2$) wa mofuta wa peipi e a tlhokaga. Se se raya gore mo kelelong ya f_{ij}^2 tshenyegelo ya E_{ij}^2 e e tlhokega. Tirego e jaanong e ipoeletsa nako nngwe le nngwe fa kelelo e feta kwa godimo ga mothamo. Sethalo 2 e bontsha tlhagisosekerafo ya tiro e ya ditshenyegelo.



Sethalo 2: *Tlhagisosekerafo ya tiro ya ditshenyegelo e e mo go Brimberg et al. [1].*

Tiro ya maitlhomo ya sekao sa setlhare se se okeditsweng ka kgetsi, mo pampiring e theetswe mo thepisong ya tiro ya ditshenyegelo jaaka go tshitshintswe ke Brimberg *et al.* [1]. Go dirisiwa mmotlele wa go kopa ditshenyegelo se se sa fetogeng mme e ka tlhaloswa jaaka go latela. Tsaya gore kelelo ya f_{ij} e romelwa go tswa mo noutung ya i go noutu ya j (e e leng mo pele ga noutu ya i). Pele ga gore kelelo e nngwe kampo e nngwe e ka romelwa mo tshwaraganong, go tshwanetse gore go duelwe ditshenyegelo tse di rileng. Fa ka nako e nngwe kelelo e ka oketsega ka yuniti e le nngwe, tshenyegelo e e fetogang, a_{ij} e a duelwa. Fa e le gore ditshenyegelo tse di golegilweng di emelwa ke E_{ij} mme morago sekao sa dikopo tsa ditshenyegelo tse di golegilweng se ka bontshiwang ka dikerafo jaaka go latela mo sethalong sa 3.



Sethalo 3: *Tlhagisosekerafo ya sekao sa kopo ya ditshenyegelo tse di golegilweng.*

Thulamo ya kerafo e bontsha ditshenyegelo tse di fetogang. Tlhaloso ya gore dipalo tse di totobetseng tsa a_{ij} tse di lemogilweng di tlaa fiwa mo karolong mo maduo a a tlaa tlhagisiwang.

Tiro ya maitlhomo ya bothata jwa popo ya metato ya dipeipi tsa oli e bopilwe jaaka go latela.

Nyenyefatsa $\sum_{(i,j)} E_{ij}\delta_{ij} + \sum_{(i,j)} a_{ij}(f_{ij_1} + f_{ij_2})$ mo E_{ij} ke ditshenyegelo tse di sa fetogeng tse di amanang le segopo sengwe le sengwe (i,j), a_{ij} ke ditshenyegelo tse di nnileng teng tsa yuniti e nngwe gape ya kelelo f_{ij} (thulamo ya molatlhamalalo mo mmotleleng o o dulang o tšhajitswe), f_{ij} e emetse kelelo magareng a noutu ya i le noutu e e tlang pele ga yone ya j. Kelelo e e bopilwe ka dikarolo tse pedi gore $f_{ij} = f_{ij_1} + f_{ij_2}$ foo f_{ij_1} e emetseng kelelo e e leng kwa tlase ga ε_{ij} mme f_{ij_2} e emetse kelelo e e kwa godimo ga ε_{ij} (palokoketso e e nnye). Popo e e tlhophilwe go ka letlelela go oketsa mmotlole (mo nakong e e tlang) mo mathata a katoloso a kgokagano e e leng teng a akangwang. (Mo mabakeng a f_{ij_1} e ka nna kelelo e e se nang tšhatšhe epe go ya kwa mothamong o o rileng mme f_{ij_2} ke kelelo e e kwa godimo ga mothamo o o nang le tshenyegelo e e rileng). $\delta_{ij} = 1$ fa segopo i, j mo setlhareng se tlhophilwe, go seng jalo ke 0.

Modiro o wa maitlhomo o o fa godimo o bonwa o le go isa tlase tlhakano ya ditshenyegelo ya ditshwaragano tsa dipeipi tse di tlhaotsweng di tlhakane le ditshenyegelo tse di farologanang jaaka di bonwa ka kelelo e e kwa godimo ga bogodimo jwa mathamo.

Go tlhomamisa gore fa noutu ya j e tsenngwa mo setlhareng dinoutu tsotlhe tse di mo tseleng e e rileng magareng a noutu ya j le noutu ya modi e tsenngwa le yone, go ne ga tsenngwa tshegetsho ya kontikuiti e e latelang.

$$x_j \leq x_{p_j}, \qquad j = 1, \ldots, n-1,$$

foo x_{p_j} ke noutu ya pele ga kampo ya botsadi ya j mme x_j ke pontshi ka palo ya 1 fa noutu ya j e tlhophiwa mme e le nnoto fa go se jalo. Sete ya maparego e tlhokega go netefatsa gore dinoutu tse di mo popegong ya setlhare di tlhophiwa gangwe fela go tlhokomologa gore ditsela tse di pedi tse di tshwanang di diragale. Go ka kgona se, go tsenngwa dipharologanyo tsa 0-1 jaaka go latela. Fa e le gore $x_i = 0$ kampo 1 mo $i = 1, 2, \ldots, n$. Tshegetsho $x_1 + x_2 + \ldots + x_n = k$ e raya gore mo godimo ga tlhogo k dipharologantsho tsa n dintlha tse di ka diragalang di ka tlhophiwa.

Go netefatsa kelelo ya tekatekano mo kgokaganong ya setlhare, go tlhokega disete tsa maparego tsa tekatekano tsa kelelo. Maparego a ke a mofuta wa kelelo yotlhe go tswa mo noutung ya j = palo yotlhe ya kelelo ya go tsena mo noutung ya j + kumo ya mo noutung ya j.

Sete e e latelang ya maparego a botha jwa popo ya metato ya dipeipi a tlaa tlhomamisa tekatekano ya kelelo mo kgokaganong ya setlhare.

$D\boldsymbol{x} - B\boldsymbol{f}_1 + \boldsymbol{f}_2 = \mathbf{0}$, mo D e leng materikisi ya phatsakhutlo ka diphatsakhutlo tsa d_j fa kumo mo noutung e le j. B ke materikisi ya tiragalo ya noutu ya segopo. Mo diweketorong tsa $\boldsymbol{f}_1$ le $\boldsymbol{f}_2$, f_{ij_1} di emela kelelo fa tlase ga mothamo wa peipi mme f_{ij_2} e emela kelelo kwa godimo ga mothamo wa peipi o o leng teng. Fa e le gore ga go na metato ya dipeipi magareng a i le j dikelelo tse ka bobedi di tshwanetse go nna nnoto. Ka nako ya bothata jwa go katolosa kgokagano motho a ka akanya go bona f_{ij_1} go nna kelelo e e ka nnang teng ka tshenyegelo ya nnoto (go ya kwa mothamong). Fa go se na metato ya dipeipi, mothamo ke nnoto.

Sete ya maparego e tlhokega go tlhomamisa gore kelelo e e diragadiwang kwa dinoutung ga e tlole palo ya mothamo ya kgokagano ya setlhare. Maparego ano a bopiwa go nna

$$f_{ij_1} + f_{ij_2} \leq \delta_{ij} C \quad \text{mo go tsotlhe } (i,j).$$

Foo C e leng mothamo otlhe wa kgokagano ya setlhare mme $\delta_{ij} \in \{0,1\}$. Go tlhomamisa gore kelelo e dula mo teng ga mothamo maparego tse di latelang di dirisitswe

$$0 \leq f_{ij_1} \leq P_{ij},$$

foo P_{ij} e leng mothamo wa peipi wa jaanong mo segopong i,j mme f_{ij_1} e le kelelo tlase ga mothamo. Mo lebakeng le go se nang metato ya dipeipi, P_{ij} e ka tseiwa e le palo e nnye ya palokoketso (gaufi le nnoto).

Sete ya bobedi ya maparego a kelelo e kgolo go na le mothamo e bopiwa e le

$$f_{ij_2} \geq 0.$$

Mo bofelong, sete ya maparegopedi e beilwe jaaka go latela Mo tlhophong ya segopo:

$$\delta_{ij} = 1 \text{ fa segopo } (i,j) \text{ se tlhophiwa, fa go se jalo ke } 0, \text{ mo digopong tsotlhe.}$$

Mo tlhophong ya noutu:

$$x_i = 1 \text{ fa noutu e tlhophiwa, } 0 \text{ ka mekgwa e mengwe, } i = 1,2,\ldots,n.$$

Karolo e e latelang e tlaa tlhalosa maduo a mmotlele wa setlhare se se okeditsweng ka kgetsi se se dirisitsweng mo bothateng jwa popo ya metato ya dipeipi.

5 Maduo le dipuisano

Data e e latelang ya ntlha e e tlhokiwang ke mmotlele e amogetswe go tswa mo sethalong sa 2.1.

- Naga ya oli e bopilwe ka dinoutu di le 33 ka digopo di ka nna 129.
- Noutu ya modi (mo popegong ya setlhare) ke noutu ya 33 e e leng poroto e e bidiwang Gamba.
- Bokgakala magareng a para nngwe le nngwe ya dinoutu tse di tshwaragantsweng e bontshiwa ka dinomoro mo digorong, jk bokgakala go tswa go noutu ya 1 go ya go noutu ya 2 ke 3.5. Bokgakala jo ke jwa boleele jwa dipeipi jo bo tlhokegang go gokaganya dipolateforomo le didiba.
- Kumo ya oli mo lefelong lengwe le lengwe (noutu) e bontshiwa ka nomoro e e bapileng noutu nngwe le nngwe jk. kumo ya noutu ya 1 ke 5.

Ga go bonale sentle go tswa mo dikwalong tse di tlhalosang bothata jwa popo ya metato ya dipeipi gore diyuniti tsa go lekanya di ne di le eng go lekanya digopo le kumo ya oli. Go arabela ditlhokego tsa thuto e lereo la "diyuniti" le tlaa dirisiwa go raya bokgakala jwa digopo le kumo ya oli: jk. bokgakala jwa noutu 1 go 2 ke diyuniti di le 3.5 le gone kumo ya oli mo noutung ya 1 ke diyuniti di le 5.

Mo godimo ga data e e builweng fa godimo, gape go ne go na le ditshenyegelo tse di tsamaisanang le kopanyo nngwe le nngwe ya dipeipi tsa dinoutu tse di farologaneng. Tshenyegelo yotlhe ya karolo ya peipi e bonwa ka go atisa boleele jwa segopo ka tlhwatlhwa ya yuniti ya mothamo wa peipi nngwe le nngwe. Diyuniti tsa dithelete le methalo e e tsamaisanang ya dipeipi di ne tsa fiwa mo nageng nngwe le nngwe ya oli.

Bogolo jwa mothamo	Diyuniti tsa Madi
5	10
10	15
25	25
50	40
10	65

Lenaneo 1: *Methamo ya dipeipi le diyuniti tsa dithelete tse di tsamaisanang.*

Dipalo tsa a_{ij} le E_{ij} (ditshenyegelo tse di farologanang le tse di sa fetogeng) tse di dirisitsweng mo medirong ya maitlhomo di baletswe jaaka go latela. Sekaseka lenaneo la 2 le le fa tlase e e nang le data ya segopo (1-2). Palo ya tlhwatlhwa ya ditshenyegelo tsa a_{12} di ka balelwa ka go tsenya molatlhamalalo, wa mofuta wa $y = mx + c$, go ya ka mothamo go tsamaisana le palo ya tlhwatlhwa ya ditshenyegelo jaaka di baletswe mme di beilwe mo lenaneong la 2. Se se tlisa palo tekatekano ya: $y = 1.9938x+32736$. Tlhwatlhwa ya $m = 1.9938$ jaanong e raya tlhwatlhwa e e farologanang ya ditshenyegelo ya a_{12}. Palo ya 32.736 e bonwa e le tshenyegelo e e bofeletsweng ya E_{12} go ka tsenya metato ya dipeipi mo segopong (1-2). Go tshwanetse gore go elwe tlhoko gore mokgwa o o fana ka phokoletso ya mathata mo ditshenyegelo tsa nnete di fetolwang go nna modiro wa ditshenyegelo tse di sa fetogeng jaaka di tlhaloseditswe mo karolong ya 4.

Ditshenyegelo tsa segopo (1-2) ka boleele jwa segopo sa 3.5		
Bogolo jwa mothamo	Diyuniti tsa Madi	Tshenyegelo
5	10	35
10	15	32.5
25	25	87.5
50	40	140
10	65	227.5

Lenaneo 2: *Palo ya ditshenyegelo ya segopo (1-2).*

Ka go dirisa noutu ya 33 (poroto ya Gamba) jaaka noutu ya modi le go latela dikgato jaaka di tlhalositswe mo kgaolong ya 4, kgokaganyo ya oli ya Borwa jwa Gabon e fetotswe go nna kgokaganyo ya setlhare. Porokeramo e ne ya kwadiwa mo C++ go ka diragatsa phetogo go ya kwa setlhareng. Maduo a kgato e e ne ya nna setlhare se se filweng maina se se nang le dinoutu di le 7030 le ditsela di le 4183. Setlhare se se ne sa rarabololwa mo bofelong ke mmotlele wa setlhare se se okeditsweng ka kgetsi ka go dirisa “Cplex (version 10.1)” e e dirisang “Concert Technology” go tswa go ILOG [2]. Tharabololo e ne ya bonwa morago ga metsotswana e e 70.15. Khomputara e e dirisitsweng go rarabolola mmotlele o ne o le wa HP Beowulf Cluster ka Gbit Interconnect. E bopilwe ka noutu e le nngwe ya taolo ya DL385 (2GB RAM) le dinoutu tse di khomputang di le lesome tsa DL145G2 (4GB RAM). Kopanyetso e yotlhe e dirisa thulaganyo ya tsamaiso ya RedHat Enterprise.

Tharabololo e e bonweng e bontshitse gore mo makgetlong a le 6 mo mmotleleng wa setlhare se se okeditsweng ka kgetsi se tlhophileng digopo tse di farologanang le bothata jwa ntlha jaaka bo ntsemo dikwalong. Dipharologano tse di fiwa fa tlase mo lenaneong la 3.

Tharabololo ya setlhare se se okeditswe ka kgetsi	Tharabololo ya Heuresetiki (Brimberg*et al.* [1])
segopo (5–9)	segopo (5–33)
segopo (16–13)	segopo (16–17)
segopo (21–23)	segopo (21–19)
segopo (26–20)	segopo (26–23)
segopo (29–30)	segopo (29–25)
segopo (31–28)	segopo (25–29)

Lenaneo 3: *Papiso ya Tharabololo.*

Pharologano ya ditharabololong tseno tse pedi e ka bewa mo mokgweng wa go repisa wa modirong wa maitlhomo o o dirisitsweng mo mmotleleng wa mo setlhare se se okeditsweng ka kgetsi. Thepiso ka kakaretso e ka tlisa maduo a gore palo ya kakanyo ya ditshenyegelo e nne kwa godimo kampo kwa tlase ga phopholetso ya ditshenyegelo.

Go ka bapisa maduo a thuto e ka boleng le maduo a ga Brimberg *et al.* [1], ditshenyegelo di baletswe ka go dirisa ditshenyegelo tse di leng teng tsa methamo ya dipeipi tse di farologaneng (bona lenaneo la 1). Sekao e ka nna, kelelo go tswa go noutu ya 1 go ya kwa go ya 3 ke 5 (bona sethalo sa 2.1) e e tlhokang mothalo wa peipi wa e e seng tlase ga 5. Ditshenyegelo tsa peipi ka mothalo wa 5 di ne di filwe e le diyuniti di le 10 mme boleele jwa peipi bo ne bo le 1.9. Tshenyegelo ya segopo (1,3) ka jalo ke $10x1.9$ = diyuniiti di le 19. Ditshenyegelo tsa digopo tsotlhe tse di tlhophilweng mo ditharabololong di le pedi di baletswe ka mokgwa o. Se se nnile maduo a tshenyegelo ya diyuniti di le 1423 mo thutong ya ga Brimberg *et al.* [1] mmediyuniti di le 1461 mo mmotleleng wa setlhare se se okeditsweng ka kgetsi pharologano ke ya 2.6%.

Maduo a thuto di ka sobokiwa jaaka go latela.

- Mmotlele wa setlhare se se okeditsweng ka kgetsi se ntshitse maduo a a neng a le mo teng ga 2.6% go nna gaufi le thuto ya ga Brimberg *et al.* [1] Pharologano e e kwa tlase ya diphesente e netefatsa gore ruri go siame go ka dirisa itlhagiso ya setlhare se e okeditsweng ka kgetsi go ka rarabolola mathata a mangwe a dipopo tsa dikgokagano. Ka go tlhalosa le go ela tlhoko mediro ya ditshenyegelo e e dirisitsweng mo mmotleleng wa setlhare se se okeditsweng ka kgetsi pharologano ya 2.6% e ka fokotswa go feta - phopholetso e ne ya dirisiwa mo pampiring e go emela modiro wa maitlhomo. Phopholetso e ka diriwa gore e nepagale go feta ka go dirisa dipharologatsho tse di botlhale go feta mme se ga se ise se diriwe fa.

- Kgonagalo e netefaditswe go feta ka nako e nnye e e dirisitsweng go ka rarabolola mmotlele (metsotswana e e 70.15 ka go dirisa CPLEX).

- Bothata jwa popo ya metato ya dipeipi tsa oli bo ne bo le kgokagano e kgolo ka e ne e na le dinoutu di le 33 le digopo di le 129. Ka dirweboleta tsa segompieno, jk. CPLEX, mathata a popo ya dikgokagano a magolo a ka rarabololwa mo teng ga nako e e seng telele ka go dirisa itlhagiso ya setlhare se se okeditsweng ka kgetsi.

6 Bokhutlo

Pampiring e e lebeletse bothata jwa popo ya kgokagano mme ga batlisisia kgonagalo ya go baya mmotlele wa kelelo ya kgokagano go nna mmotlele wa kgkagano ya setlhare e e ka rarabololwang ka go dirisa tsela ya setlhare se se okeditsweng ka kgetsi.

Thuto ya sekao se se rileng (bothata jwa popo ya metato ya dipeipi) go tswa mo makwalong, e ne ya tlhophiwa gore go tle go lekelediwe mokgwa o o tshitshintsweng wa go tla ka tharabololo. Bothata jwa popo ya metato ya dipeipi tsa oli ya kwa tshimologong, bo bo nnang le dinoutu di le 33 le digopo di ka nna di le 129, bo fetotswe go tsena mo popegong ya setlhare ya dinoutu di le 7030 le ditsela tse di ka nnang 4183. Setlhare se ne sa rarabololwa ka go dirisa mmotlele wa setlhare se se okeditsweng ka kgetsi ka thepiso ya modiro wa maitlhomo.

Maduo a bontsha pharologano e nnye fa e bapisiwa le tharabololo e e fiwang ke makwalo mme se se nnile jalo ka go dirisitswe katametso go emela modiro wa maitlhomo mo mmotleleng wa kgetsi e e okeditsweng ka kgetsi. Tharabololo e ne ya bonwa morago ga nako e re ka reng e khutswane e e dirisitweng mo kgokaganong e re ka re e kgolo. Ka go latela maduo go tliwa ka tshwetso ya gore mmotlele wa setlhare se se okeditsweng ka kgetsi o bontshitse gore o ka nna karabo ya go ka rarabolola dimmotlele tse di farologaneng tsa dipopo tsa dikgokagano.

Mmotlele o o tshitshingwang e ka tokafatswa ka go dira ditekelelo tse dingwe ka mediro e e farologaneng ya ditshenyegelo mo modirong wa maitlhomo. Mekgwa e mengwe ya go ka tokafatsa le go oketsa tiro ya mmotlele le yone e ka akarediwa go ka fana ka ditharabololo mo dimmotleleng tse di kgolo go feta tsa dikgokagano tsa kelelo.

Metswedi

[1] Brimberg J, Hansen P, Lih K, Mladenovic N & Breton M, 2003, *A oil pipeline design problem*, Operations Research, **51(2)**, pp. 228–239.

[2] ILOG, 2006, ILOG Cplex 10.1, [Online], [Cited December 5$^{\text{th}}$, 2011], Available from `http://www.ieor.berkeley.edu/Labs/ilog_docs/html/index.html`.

[3] Kershenbaum A, 1993, *Telecommunication network design algorithms*, McGraw-Hill, New York.

[4] Shaw DX, 1997, *A depth-first dynamic programming procedure for the extended tree knapsack problem in local access network design telecommunication systems*, Journal on Computing, **27**, pp. 29–43.

[5] Taylor BW, 2002, *Introduction to Management Science.*, 7$^{\text{th}}$ Edition, Princeton Hall, Upper Saddle River (NJ).

[6] Van der Merwe DJ, 2002, *Solving tree knapsack problems for local access network design applications*, MSc Thesis, North-West University, Potchefstroom.

[7] Van der Merwe DJ & Hattingh G, 2006, *Tree knapsack approaches for local access network design*, European Journal of Operational Research, **174**, pp. 1968–1978.

[8] Van der Merwe DJ & Hattingh G, 2010, *Solving the extended tree knapsack problem with fixed cost flow expansion functions*, ORiON, **26(1)**, pp. 1–16.