



Decision support for open-air irrigation reservoir control

JC van der Walt*

JH van Vuuren†

Received: 7 December 2016; Revised: 23 July 2017; Accepted: 10 April 2018

Abstract

The availability of irrigation water greatly impacts on the profitability of the agricultural sector in South Africa and is largely determined by prudent decisions related to water release strategies at open-air irrigation reservoirs. The selection of such release strategies is difficult, since the objectives that should be pursued are not generally agreed upon and unpredictable weather patterns cause reservoir inflows to vary substantially between hydrological years. In this paper, a decision support system is proposed for the selection of suitable water release strategies. The system is based on a mathematical model which generates a probability distribution of the reservoir volume at the end of a hydrological year based on historical reservoir inflows. A release strategy is then computed which centres the expected hydrological year-end reservoir volume on some user-specified target value subject to user-specified weight factors representing demand satisfaction importance during the various decision periods of the hydrological year. The probability of water shortage for a given year-end transition volume may be determined by the decision support system, which allows for the computation of acceptable trade-off decisions between the fulfilment of current demand and the future repeatability of a release strategy. The system is implemented as a computerised concept demonstrator which is validated in a special case study involving Keerom Dam, an open-air reservoir in the Nuy agricultural district near Worcester in the South African Western Cape. The system's strategy suggestions are compared to historically employed strategies and the suggested strategies are found to fare better in maintaining reservoir storage levels whilst still fulfilling irrigation demands.

Key words: Decision support, reservoir releases.

*Stellenbosch Unit for Operations Research in Engineering, Department of Industrial Engineering, Stellenbosch University, Private Bag X1, Matieland, 7602, South Africa, email: 17124891@sun.ac.za

†Corresponding author: (**Fellow of the Operations Research Society of South Africa**) Stellenbosch Unit for Operations Research in Engineering, Department of Industrial Engineering, Stellenbosch University, Private Bag X1, Matieland, 7602, South Africa, email: vuuren@sun.ac.za

1 Introduction

South Africa is classified as a semi-arid, water-stressed region, with an average annual rainfall of 450 mm — almost half the global average of 860 mm [21]. The limited and erratic water supply resulting from precipitation necessitates irrigation in most instances of crop farming. Open-air reservoirs are most commonly used to store irrigation water in South Africa, because precipitation periods and river flows are dynamic, in some cases volatile and not necessarily overlapping with demand periods. There are approximately 1.3 million hectares of agricultural land under irrigation in South Africa and agriculture accounts for roughly fifty percent of South Africa's total annual water usage [21].

If irrigation reservoir levels are not carefully controlled, water shortages or flood damage may occur downstream from the reservoirs with disastrous effects for farmers in the region. An effective release strategy must therefore be employed for beneficial reservoir level control. A suitable choice of irrigation reservoir release strategy is, however, not obvious for a number of reasons. The objectives that should be pursued by such a strategy are not generally agreed upon. Irrigation demands should obviously be met, while the risk of water shortage and/or the risk of flood damage may be minimised, or evaporation losses may be minimised. Unpredictable weather patterns furthermore cause reservoir annual inflows to vary substantially, thus making planning and water allocation exceedingly difficult. The determination of irrigation demand is also a non-trivial problem, which is influenced by the climate as well as the distribution of crop types under irrigation and various agricultural policies. Finally, the decision makers responsible for the selection of a release strategy may differ vastly in their attitude toward risk, which plays a critical role in the selection of a consensus strategy.

The problem considered in this paper is the design and implementation of a user-friendly, computerised *decision support system* (DSS) which may aid the operators of an open-air irrigation reservoir in deciding upon a suitable water release strategy. This DSS provides a means for the effective comparison of different water release strategies through quantitative performance metrics and is also capable of suggesting a strategy which best achieves user-specified levels of these performance metrics. The DSS put forward in this paper relies on a mathematical modelling framework, previously suggested by van der Walt and van Vuuren [24]. This framework accommodates trade-off decisions between the fulfilment of demand and future strategy repeatability. These trade-offs are accomplished by balancing the conflicting objectives of maximising the fulfilment of irrigation demand (and hence reservoir releases), whilst simultaneously minimising the risk of water shortage, within certain specified legal and environmental constraints.

The paper is organised as follows. A concise review of the literature related to the notion of reservoir release strategy formulation is presented in §2, after which the mathematical modelling framework for reservoir releases of van der Walt and van Vuuren [24] is briefly reviewed in §3. A novel computerised DSS, based on the framework of §3, is described in §4 and then applied in §5 to a special case study involving Keerom Dam, a large open-air irrigation reservoir in the South African Western Cape. The working of the DSS is, however, generic and its potential use at other irrigation reservoirs may hold significant benefit in view of its successful application in the case of Keerom Dam. The paper finally

closes in §6 with a brief appraisal of the contribution and some ideas with respect to possible follow-up future work.

2 Literature review

In the first section of this literature review, we describe a number of methods available for meteorological data prediction, after which the focus turns in §2.2 to approaches available in the literature for calculating irrigation demand. The review closes in §2.3 with a description of the methods available for evaporation estimation and modelling as well as an overview in §2.4 of DSSs previously established for open-air reservoir control.

2.1 Meteorological prediction and generation models

Precipitation in the catchment area of a reservoir and the resulting inflows into the reservoir are the most volatile and unpredictable variables in the reservoir sluice control problem under consideration. While extensive research has been devoted to weather prediction, long-term weather patterns remain largely unpredictable [9]. As a result, weather generators are often used to simulate volatile weather conditions exhibiting seemingly random behaviour when the effects of weather patterns have to be studied. Most weather generators have been developed for the study of precipitation, because of the far reaching effects of rainfall on many environmental processes [27].

According to Wilks and Wilby [27], most weather generators are based on the assumption that the precipitation volumes on consecutive wet days are independent. The Monte Carlo simulation method, which is often used in such weather generators, consists of using a pseudo-random number generator in conjunction with a precipitation distribution curve, fitted to historical data, in order to generate random precipitation amounts.

It has, however, been observed that wet and dry days do not occur randomly and independently, but that they rather tend to cluster together in wet or dry spells [27]. Dry spells cause a discontinuity in the precipitation distribution and thus cannot be modelled adequately using only the procedure described above. The occurrence of a wet or dry spell may be simulated effectively as a Markov chain. Once the state of a day has been simulated (wet or dry), the precipitation amount associated with a wet day may be simulated using the Monte Carlo method.

If the flows of a river are to be modelled using the Monte Carlo method, it is unnecessary to embed a Markov chain in the simulation, however, since there are no corresponding discontinuities in the flow distributions. The effects of discontinuous precipitation are instead absorbed by the gradual water flows in the catchment area and are also lessened by other sources of water, such as water springs or smaller rivers.

Successful river flow predictions have been achieved by standard time series modelling, as described by Box and Jenkins [2]. Artificial neural networks have also been employed for this purpose, achieving greater accuracies than time series modelling in some cases [8].

The choice between a generation or prediction approach to reservoir inflows may have a significant impact on the usefulness of the DSS put forward in this paper. Time series and

artificial neural network prediction models rely on the assumption that the immediate future behaviour of the system depends more sensitively on recent historical behaviour than on non-recent historical behaviour. While this may prove true for short-term predictions (a small number of days for general weather prediction), it remains inaccurate over annual predictions [9]. Using prediction models in the DSS put forward in this paper may therefore result in overly optimistic strategy suggestions.

2.2 Irrigation demand calculation

The calculation of crop irrigation demand is an important constituent part of the proposed solution to the problem under consideration in this paper. It is essential to be able to forecast irrigation demand, at least annually, for planning purposes. Several factors may influence the irrigation demand on a farm, depending on the calculation method used. Irrigation demand may be estimated from an economic point of view, taking into account water cost, crop yields and cost-benefit trade-offs. This approach to deriving irrigation demand may be adopted in conjunction with statistical estimation methods or programming models, as described by Bontemps and Couture [1]. The use of statistical estimation methods for deriving irrigation water demand results in inelastic water demand profiles. The reason for this may be a lack of data on crop-level water use [1]. When using programming models, however, inelastic demand profiles result only below a certain threshold water price.

In the DSS proposed here, a fixed (inelastic) demand profile is assumed as input to serve as a fixed reference point for performance measures in the model. This profile serves the dual purpose of being the primary input in the selection of an initial water release strategy (which is improved iteratively by the DSS), and as a measure of how well demand is met by comparing the actual reservoir outflows to this demand profile. Therefore, the demand profile employed in this study should ideally reflect crop needs, irrespective of water price. It is envisaged that the model developed here may ultimately be used in an economic benefit study, instead of incorporating the cost-benefit analysis in the demand calculation.

According to Willis and Whittlesey [26], a farmer's risk appetite determines the degree to which his preferred irrigation policy exceeds the net irrigation requirements of his crops. There is a cost associated with this over-allocation of water, referred to as a *self-protection cost*. An analysis of each farmer's utility function and preferred self-protection cost is beyond the scope of this paper. Since it is expected that several farmers may benefit from the reservoir for which a sluice release strategy is sought, using a method for determining each farm's irrigation requirements, which is standardised and independent of subjective preferences, may be beneficial. Such an approach excludes individual risk preference when determining the annual water demand profile for a reservoir; it rather depends only on crop water requirements. This is referred to as the *net irrigation requirement*.

A globally accepted standard for the calculation of fixed crop water requirements in irrigation studies was published by the Food and Agriculture Organisation of the United Nations in 1974 [15]. This standard was updated and improved in 1990 by a panel consisting of members of the International Commission for Irrigation and Drainage as well as the World Meteorological Organisation. The resulting current standard is referred to as the FAO Penman-Monteith method [15].

CROPWAT 8.0 for Windows [19] is a DSS for the calculation of net irrigation requirements, based on soil, climate and crop data. All calculations performed in CROPWAT follow the FAO Penman-Monteith method. The equivalent South African standard is the DSS SAPWAT [23].

2.3 Evaporation estimation and modelling

Evaporation from a water surface is the net rate of water transported from the surface into the atmosphere [18]. Estimations of evaporation losses are required by all water balance models for water reservoir systems. The modelling approach adopted in this paper to predict future evaporation losses is discussed here.

Evaporation losses are usually assumed to be proportional to the average exposed water surface area [17]. Van Vuuren and Gründlingh [25] modelled evaporation loss during period i in a set $\mathcal{T} = \{0, \dots, n-1\}$ of calculation periods spanning a hydrological year as

$$E_i = e_i \frac{A_i + A_t}{2}, \quad (1)$$

where $t \equiv i - 1(\text{mod } n)$ and A_i denotes the exposed water surface area at the end of calculation period $i \in \mathcal{T}$. Here e_i is the evaporation rate associated with calculation period $i \in \mathcal{T}$, as determined from historical evaporation rates.

As their aim was to employ (1) in a linear programming model, van Vuuren and Gründlingh [25] assumed a linear relationship between reservoir storage and exposed water surface area. The relationship between the exposed water surface area and reservoir storage, as defined by the local topography, is, however, usually non-linear. Sun *et al.* [17] developed a piecewise linear model for approximating evaporation losses as a function of reservoir storage. The latter approach is also adopted in this paper.

2.4 Decision support systems

There exists no universally agreed-upon definition for the notion of a DSS. A range of definitions have been proposed, with the one extreme focussing on the notion of *decision support* and the other on the notion of a *system* [10]. Shim *et al.* [14] state that “DSSs are computer technology solutions that can be used to support complex decision making and problem solving.” The development of DSSs draws from two main areas of research, namely theoretical studies and organisational decision making [14].

DSSs may be partitioned into the categories of collaborative support systems, optimisation-based decision support models and active decision support [14]. *Collaborative support systems* aid groups of agents engaged in cooperative work by sharing information across organisational, space or time boundaries with the aim of facilitating effective consensus decision making. *Optimisation-based decision support models* generally consist of three sequential stages. First, a system is abstractly modelled in such a manner that solution objectives can be expressed in a quantitative manner, after which the model is solved according to some algorithmic approach. Finally, the solution or set of solutions is analysed in terms of their possible effects on the system [14]. The term *active decision support* refers to a DSS’s ability to adapt its output actively when new inputs are entered, as

used by van Vuuren and Gründlingh [25], thus contrasting DSSs used mainly for strategic planning purposes to those which can be used for operational decision making. The DSS proposed in this paper falls in the latter category.

The farmers who benefit from crop irrigation reservoirs are often in agreement that the release of more water (up to maximum sluice capacity, thus not including floods) is more beneficial than the release of less water [5]. It may therefore be assumed that the benefit function for normal operation of an irrigation reservoir is a strictly increasing function of release volume. This assumption renders the problem of determining a suitable release strategy fairly simple: release the maximum amount of water, keeping in mind the risk of not being able to achieve repeatability of the strategy over successive hydrological years. The focus of a reservoir release DSS should therefore be on quantifying the risk related to a given release strategy, rather than merely searching for an optimal strategy or set of strategies.

Two DSSs have previously been developed for implementation at Keerom Dam. The first is ORMADSS, developed by van Vuuren and Gründlingh [25] in 1998. This DSS relies on a linear programming model for determining an optimal release strategy during average years with the objective of minimising evaporation losses. The DSS receives the actual reservoir level and other user-defined parameters as inputs and attempts to steer the actual reservoir level to the optimal reservoir level for average years.

ORMADSS was validated using data from the 1990/91 hydrological year, by comparing the total water yield and evaporation losses corresponding to the historical release strategy (based on the benefiting farmers' intuition) to those of the release strategies suggested by ORMADSS. The overly conservative nature of the farmers became apparent during the validation process, with the suggested strategies outperforming the historically implemented strategy for 0%, 20%, and 40% reservoir reserve levels. The DSS was taken in use in 2000, but after extremely dry years in 2002 and 2003, where the reservoir level fell to below 25% of its capacity, confidence in the DSS decreased and it fell out of use.

Strauss [16] then attempted in 2014 to improve upon this DSS by adapting the way in which risk is accommodated. In addition to a minimum reserve for the operating level of the reservoir, Strauss implemented risk-related constraints in the form of release quantity upper and lower bounds in the underlying mathematical model. This adaptation does not, however, quantify risk explicitly, and therefore does not provide greater security in terms of the risk of non-repeatability of the suggested strategy. Since no quantitative measure of risk related to a given strategy is available, users cannot ascertain the extent to which they are exposing themselves to the possibility of future water shortages if they choose to follow the suggested strategy. It may follow that a decision maker who was reluctant to continue using ORMADSS will view this adaptation in the same regard. The latter DSS has not, in fact, been taken in use.

Strauss [16] criticised the simplistic manner in which van Vuuren and Gründlingh [25] accommodated risk by only including a minimum reserve for the operating level of the reservoir, while not directly allowing for the possibility of unmet demand. Strauss nevertheless implemented a very similar model, together with the additional risk-related constraints in the form of release quantity upper and lower bounds. It is important to note, however, that risk was also not quantified explicitly in the model of Strauss [16].

Having access to a quantitative representation of risk is a crucial element lacking in the two DSSs reviewed above. Furthermore, models which provide a single optimal solution may be insufficient, since reservoir operation commonly involves trade-off decision options. In response, multi-objective models have often been implemented in the context of reservoir management.

3 Mathematical modelling framework

In this section, the 2015 mathematical modelling framework for reservoir releases of van der Walt and van Vuuren [24] is briefly reviewed. This framework attempts to balance the conflicting objectives of water demand fulfilment and future water shortage risk, taking into account certain user preferences. The framework facilitates a comparison of different release strategies in terms of a trade-off between various quantitative performance metrics.

3.1 Modelling assumptions

In order to develop a mathematical modelling framework for irrigation reservoir operation, van der Walt and van Vuuren [24] made a number of important modelling assumptions. They discretised the scheduling horizon over which a release strategy is to be determined into a number of time intervals, called *calculation periods*, which are typically days or weeks. Their model is therefore discrete in nature. The minimum possible duration over which a constant water release rate can be implemented, as determined by the frequency with which sluice adjustments are allowed, is called the *decision period* length. Decision periods are typically weeks, fortnights or months, but their length is in any case a multiple of the calculation period length. Water demand during a specific decision period was also assumed to be constant, which is an acceptable assumption for short decision periods.

Van der Walt and van Vuuren [24] furthermore assumed that the evaporation rate experienced at the reservoir during a given calculation period is directly proportional to the average exposed water surface area of the reservoir and thus a function of the average reservoir volume during that period, according to (1). The coefficient of proportionality e_i in (1) was taken to depend on the historical meteorological conditions of the time interval in question. The South African Department of Water Affairs and Forestry [6] maintains a database of all reservoirs exceeding a certain minimum storage capacity, which includes historical daily evaporation losses for these reservoirs. These loss rates may be used to estimate the coefficient e_i in (1) for all $i \in \mathcal{T}$. For new reservoirs, the evaporation rates of older reservoirs in the vicinity may be used as an initial estimate.

The repeatability of any reservoir release strategy is taken to depend on an estimate of the most likely reservoir volume at the end of the hydrological year as a result of this strategy. In particular, van der Walt and van Vuuren [24] took the reservoir volume during the transition between two consecutive hydrological years as a measure of future demand fulfilment security. A hydrological year in South Africa runs from October 1st to September 30th.

Van der Walt and van Vuuren [24] finally adopted a standard conservation law in the form of the assumption that the change in water volume during a given calculation period

equals the net influx (including all the reservoir’s water sources, precipitation onto the reservoir and in its catchment area, as well as seepage losses), less evaporation losses and all reservoir outflows, including controlled sluice outflow and overflow.

3.2 Modelling framework for reservoir releases

The conceptual mathematical modelling framework of van der Walt and van Vuuren [24] is illustrated graphically in Figure 1. In this framework, the model inputs have been partitioned into historical data, as well as various user-inputs and reservoir-related parameters. Historical data refer to past inflows, used as an indication of possible future inflows, and past evaporation losses which may be used to estimate the coefficient of proportionality of evaporation during any given calculation period.

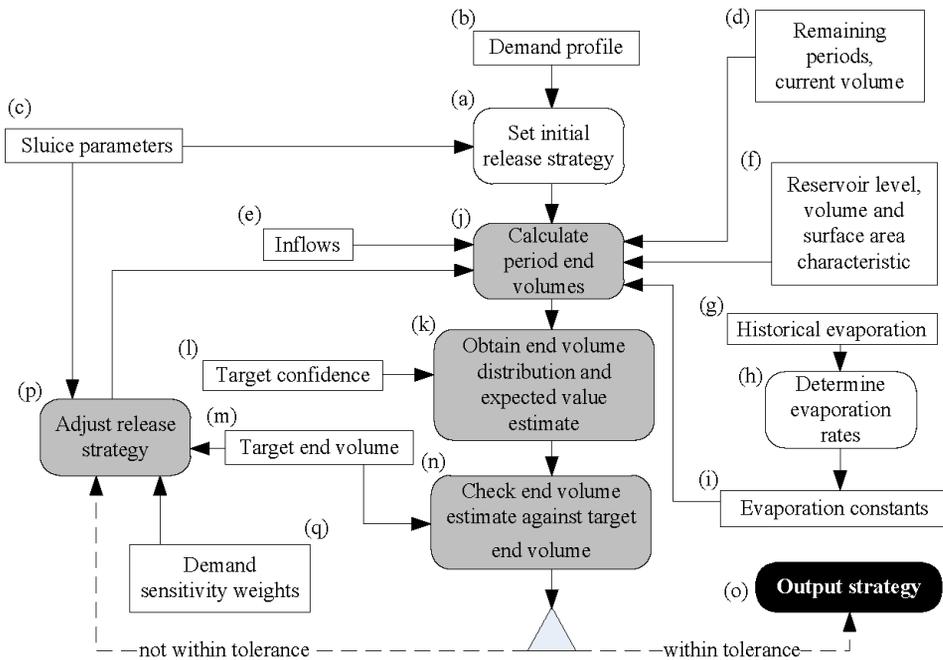


Figure 1: Modelling framework for the problem of deciding on irrigation reservoir release strategies [24].

The required user inputs are the decision period length (typically biweekly or monthly), the number of remaining calculation periods in the current hydrological year, the current reservoir volume and some target end-of-hydrological-year volume. The decision period demand profile, which may be computed using standard irrigation decision support software, such as CROPWAT [19] or SAPWAT [23], as well as demand-importance weights, are also considered user inputs. The water demand and demand-importance weights are recorded over decision period intervals. For application in calculation periods these values are adjusted *pro-rata*.

Reservoir-related parameters include the sluice release capacity per calculation period, the minimum allowed release volume per calculation period according to legal requirements,

the reservoir's storage capacity and its *shape characteristic*, which relates the water level, stored water volume and exposed water surface area of the reservoir.

First, an initial release strategy in Figure 1(a) may be determined using the demand profile and reservoir sluice parameters in Figure 1(b)–(c), or the user may simply input his preferred strategy, to be analysed. Starting at the current reservoir volume in Figure 1(d), this initial strategy is then used, in conjunction with expected inflows in Figure 1(e) and an estimation of evaporation losses in Figure 1(g)–(i), to compute the calculation period end volumes in Figure 1(j) for the remainder of the current hydrological year. More specifically, the expected volume fluctuations of the reservoir, from the current date to the end of the hydrological year, are calculated for each expected inflow input. For example, if twenty years' inflow data are available, twenty sets of possible reservoir volumes may be obtained for the remainder of the hydrological year, in intervals not shorter than the inflow data time intervals.

In the estimation of evaporation losses used in this procedure, the historically observed evaporation rates in Figure 1(g) may be employed to obtain an expected evaporation rate for each calculation period in Figure 1(h). These rates are then taken as the coefficients of proportionality relating the evaporation loss to the exposed reservoir water surface area as in (1).

Since the expected reservoir volume at the end of the hydrological year is of interest as a metric of future demand fulfilment security, the hydrological year end-volume distribution in Figure 1(k) is obtained, using the final volumes of each reservoir volume data set. The minimum reservoir level at the end of the hydrological year associated with a user-specified confidence in Figure 1(l) is then estimated from this distribution.

The expected end-of-hydrological-year volume is compared to some user-specified target end volume in Figure 1(m), as illustrated in Figure 1(n). If the expected end-of-hydrological-year volume falls within a user-specified tolerance interval centred on the target volume, the current strategy is returned as output, as illustrated in Figure 1(o).

If, however, the expected year-end volume fails to fall within the tolerance interval, the current release strategy is adjusted, as illustrated in Figure 1(p), taking into account, amongst other things, the size of the deviation of the expected year-end volume from the target end volume, user-specified weights in Figure 1(q) which represent each demand period's sensitivity to demand fulfilment and the reservoir sluice parameters. The adjusted strategy then serves as input for the calculation of new sets of reservoir volume fluctuations in Figure 1(j). This procedure is repeated until the estimated year-end volume falls within the user specified tolerance interval.

3.3 Modelling reservoir inflows

In order to model the fluctuations in reservoir volume during the hydrological year, expected reservoir inflow data are required. There are three possible sources for these data [11]. One option is direct utilisation of the historical inflow data of the reservoir. The other options involve simulation by random sampling and are distinguished by the distributions utilised during the simulation process. Either empirical distributions of the inflows may

be used for each simulation period, or some theoretical distribution may be fitted to the inflow data.

The simulation period length may be chosen equal to the calculation period length. Visualising a two-dimensional cumulative distribution plot, historical net inflow for a given period may be placed in bins on a horizontal axis, with each bin containing the number of historical inflows less than the bin's upper limit. The vertical axis then denotes the number of inflow data points. The vertical axis may be normalised in order to represent proportions of total inflows.

As mentioned, the South African Department of Water Affairs and Forestry's database includes past daily inflows for all large reservoirs in South Africa, typically resulting in ample historical inflow data. This allows for the determination of accurate reservoir inflow distributions. In other words, it is usually not necessary to fit a theoretical distribution to the data — the empirically obtained distributions may instead be utilised directly. For a newly built reservoir, the prediction of future inflows in the absence of historical data would constitute a challenging, separate research project.

Utilising the inflow distributions described above, inflows may be emulated by Monte Carlo simulation. Let I_i be the net reservoir inflow during simulation period $i \in \mathcal{T}$ and let U be a uniform random variable on the interval $[0, 1]$. If I_i has the cumulative distribution function F_i , then $F_i^{-1}(U)$ has the same distribution as I_i . An instance of I_i may therefore be simulated according to the inverse transform method [12] by generating a uniform $[0, 1]$ variate u and recording the value $F_i^{-1}(u)$. Once this has been done for each simulation period, the inflows of a single hydrological year have been simulated. A large number of hypothetical parallel years may thus be simulated.

In the simulation approach described above, it is assumed that inflows during adjacent simulation periods are independent. The validity of this assumption may be tested by comparing selected statistical properties of inflows thus simulated to those of the historically observed inflows. A simulation approach with the incorporation of memory, such as an artificial neural network or a modified Markov chain, would not rely on the aforementioned assumption. Such approaches would, however, include a degree of prediction, as mentioned in §2.1, which may result in overly optimistic planning. In fact, given the extremely volatile nature of weather patterns, any attempt at synthetically increasing the information inherent in historical inflow data over long planning horizons (such as one year, for example) may result in an unsubstantiated increase in knowledge related to the behaviour of inflows.

3.4 Estimating period end-volume distributions

Let V_i denote the reservoir volume at the end of calculation period i , let x_i be the water volume released during calculation period i , and let E_i be the volume of water lost due to evaporation during calculation period i , where $i \in \mathcal{T}$, for some set $\mathcal{T} = \{0, \dots, n - 1\}$ of calculation periods spanning a hydrological year. Furthermore, let e_i denote the evaporation rate per unit of average exposed water surface area during calculation period $i \in \mathcal{T}$, denoted by A_i . Then it follows that

$$V_i = V_t + I_i - x_i - E_i \quad (2)$$

for all $i \in \mathcal{T}$ and $t \equiv i - 1 \pmod{n}$, into which expression (1) may be substituted. Furthermore, the exposed surface area of the reservoir in (1) is related to the stored water volume according to some reservoir shape characteristic f in the sense that

$$A_i = f(V_i) \quad (3)$$

for all $i \in \mathcal{T}$. A preliminary release strategy is determined according to the water demand profile and the sluice release parameters. Let D_i denote the water demand during calculation period $i \in \mathcal{T}$, and let x_{\min} and x_{\max} denote respectively the minimum and maximum possible release volumes during any calculation period. Then the water volume released during calculation period i is assumed to be

$$x_i = \begin{cases} x_{\max} & \text{if } D_i \geq x_{\max} \\ D_i & \text{if } D_i \in (x_{\min}, x_{\max}) \\ x_{\min} & \text{if } D_i \leq x_{\min} \end{cases} \quad (4)$$

for all $i \in \mathcal{T}$. Using the current reservoir volume, expected inflows during the remaining calculation periods of the hydrological year and the preliminary release strategy described above, a cumulative distribution function may be obtained for the reservoir volume at the end of the hydrological year, as a result of the release in (4). This distribution, denoted by F_V , may be analysed by standard statistical methods of inference, to obtain an estimate V_c^* of the expected reservoir end volume for a given probability c in the sense that

$$V_c^* = F_V^{-1}(1 - c). \quad (5)$$

The estimate V_c^* may be compared to a target end-volume specified by the decision maker. If the estimate falls outside a certain tolerance band centred around the target end-of-hydrological year volume (also specified by the decision maker), the release strategy may be adjusted with the aim of centring the expected end volume on the target value. The user-specified tolerance within which the target end volume should be met is denoted by $\alpha \in (0, 1]$.

In the model, various factors are taken into account for this adjustment: the number of calculation periods remaining in the current hydrological year, denoted by m , the end volume estimate V_c^* , the target end volume, denoted by Φ , the minimum and maximum sluice release parameters, and user-specified weight factors which represent each demand period's sensitivity to adjustments in release volume during that period, denoted by $w_i \in [0, 1]$, where a lower value represents a less adjustable period.

Let μ_w denote the mean of the user-defined weight factors. The adjustment process proposed by van der Walt and van Vuuren [25] for determining a preliminary release strategy is iterative in nature. Each iteration of this process may be accomplished in two stages. First the adjusted volume

$$x'_i = x_i + \frac{w_i(V_c^* - \Phi)}{m\mu_w} \quad (6)$$

is computed, after which the capped corresponding release quantity

$$x''_i = \begin{cases} x_{\max} & \text{if } x'_i \geq x_{\max} \\ x'_i & \text{if } x'_i \in (x_{\min}, x_{\max}) \\ x_{\min} & \text{if } x'_i \leq x_{\min} \end{cases} \quad (7)$$

is determined for each remaining calculation period $i \in \mathcal{T}$ after the current calculation period. During each iteration of the adjustment procedure in (7), the end-volume distribution is recalculated and a new end-volume estimate closer to the target value obtained, until the estimate falls within the interval $[(1 - \alpha)V_c^*, (1 + \alpha)V_c^*]$. Once the minimum end-volume is thus centred on the target value, the particular incarnation of the release strategy (7) during that iteration is suggested to the decision maker.

3.5 Quantifying the risk of water shortage

As mentioned, the repeatability of a release strategy is taken to depend on the reservoir volume during the transition between two successive hydrological years as a result of applying the strategy. The probability of water shortage associated with a reservoir volume during this transition may be determined by equating the starting and target end-volumes in the model, and solving the model for a given probability level. The number of times that the reservoir volume drops below a user-specified threshold volume and the length of time the volume remains below this threshold, for all historically observed inflows, may be taken as an estimate of the probability of water shortage associated with a given release strategy, transition volume and probability level. This probability estimate may be adopted as a performance metric when comparing release strategies.

During any period in an actual year, the probability of reaching the end of the hydrological year with at least a certain reservoir volume may be obtained from the end-volume probability distribution resulting from the current release strategy. Either the probability of obtaining a certain minimum end volume, or the minimum end volume expected to be obtained for a fixed probability, may thus be adopted as a second performance metric when comparing release strategies.

In the case of a particularly dry year, when the notion of risk requires special attention, trade-off decisions between the fulfilment of the current hydrological year's demand and future repeatability associated with the release strategy may be required. Future repeatability here refers to a level of confidence in the ability to fulfil the irrigation demands of subsequent years. Due to particularly low reservoir storage levels during a dry year, the decision maker may prefer to aim for a lower target end volume, since the proposed strategy which centres the end-volume distribution on the specified target value for repeatability to within the acceptable tolerance interval fails to meet the current year's demand adequately. The fulfilment of the current hydrological year's demand will thereby be improved, but at the cost of a decrease in the security of future years' water supply. The improvement in demand fulfilment, as well as the decrease in security, may finally be quantified and compared in terms of benefit and cost trade-offs.

4 Decision support system

The design and implementation of a novel DSS concept demonstrator, based on the mathematical modelling framework of §3, is described in this section. Section 4.1 is devoted to a description of the working of the concept demonstrator graphical user interface, while the method of implementation of this concept demonstrator is described in §4.2.

4.1 Working of the concept demonstrator

In order to validate the modelling framework of §3, a concept demonstrator of the framework was implemented in Python 2.7 on a personal computer. This DSS is referred to as WRDSS — an acronym for *Water Reservoir Decision Support System*. The user may link WRDSS to a specific irrigation reservoir by providing its historical inflow profile I_0, \dots, I_{n-1} , the historical evaporation rates e_0, \dots, e_{n-1} experienced at the reservoir, the irrigation demand profile D_0, \dots, D_{n-1} and the reservoir shape characteristic (3) in the form of Excel files. A screenshot of the main user interface of this concept demonstrator is shown in Figure 2.

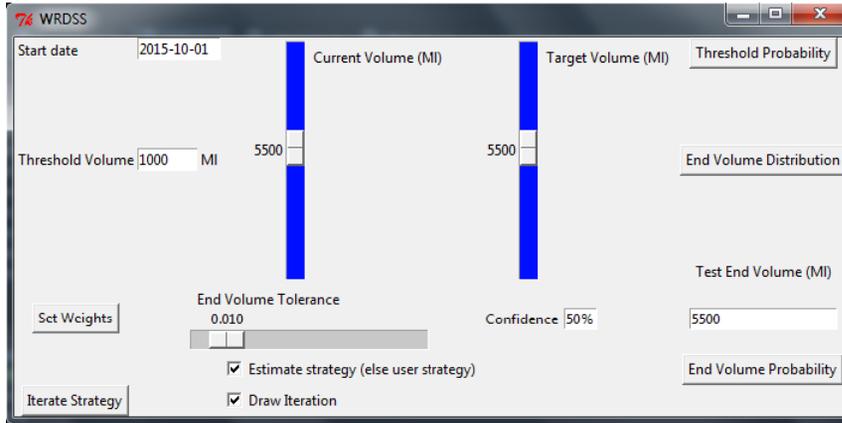


Figure 2: The main user interface of the WRDSS concept demonstrator.

The start date of the scheduling horizon and a lower threshold volume for the reservoir may be entered into the text boxes labelled accordingly in the left-hand top corner. The current reservoir volume, as well as the target end-of-hydrological-year volume, may be specified using the blue vertical scale widgets, while the end-volume tolerance may be specified using the horizontal scale widget. The user may enter the probability (as a percentage) with which at least the end-volume target should be obtained in the text box labelled confidence. The difference in meaning between the notions of probability and confidence, as defined in probability theory, is noted. In this context, confidence is defined as the probability of obtaining at least the target end volume. This slight abuse of terminology is intentional as it is expected to coincide with the intuition of the end-users of the DSS, who are expected to be farmers.

The user may specify whether the model should estimate an initial strategy using the demand input data or whether the model should employ a user-specified initial strategy, by clicking the corresponding check box. The *Set Weights* button may be used to access a separate window, shown in Figure 3, in which the demand-importance weight of each month may be set, for either monthly or biweekly decision period lengths. The *Draw Iteration* check box may be used to specify whether the final iteration plot, of which an example is shown in Figure 4(a), should be displayed. This iteration plot does not provide additional output information to the user. It merely depicts the model's volume fluctuation estimates, which may be used for explanatory purposes when introducing a new user to WRDSS.

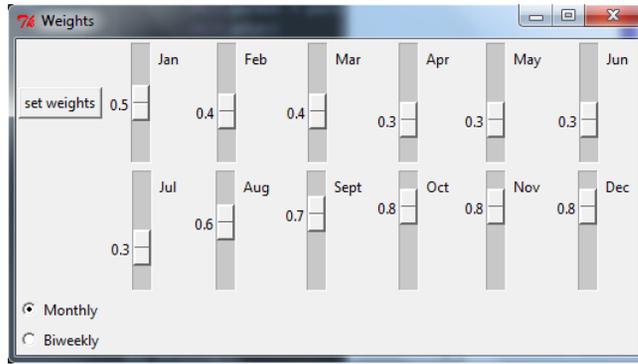
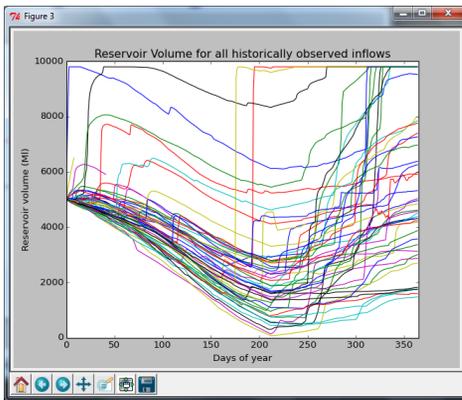
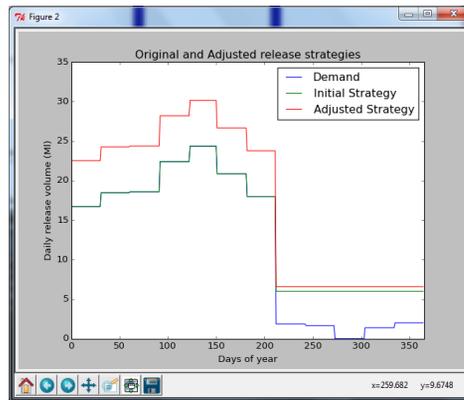


Figure 3: The user interface for specifying the demand-importance weights in the WRDSS concept demonstrator.



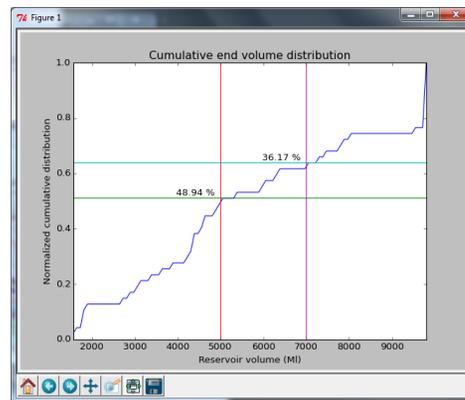
(a) End-of-year volume fluctuations



(b) Release strategy plot

	Empty	Threshold	Overflow
Initial strategy:	0.00 %	16.23 %	5.06 %
Adjusted strategy:	0.00 %	33.72 %	4.07 %

(c) End-volume cumulative distribution



(d) Estimated strategy probabilities

Figure 4: Output produced by the WRDSS concept demonstrator.

Once the above-mentioned parameters have been specified, the user may initiate the model iteration process by clicking the *Iterate Strategy* button, which results in a suggested output strategy, as shown in Figure 4(b). The buttons in the right-hand column of the main user interface may be used to analyse the model output. Clicking the *Threshold*

Probability button displays the probability of the reservoir volume dropping below the specified threshold volume, for the initial and iteratively adjusted strategies. An example of such output is shown in Figure 4(c). The *End Volume Distribution* button opens a plot of the end-volume cumulative distribution for the latest iteration, as shown in Figure 4(d). The probability of obtaining at least a given end volume may be obtained by entering a volume into the text box labelled *Test End Volume (Mℓ)* and clicking the *End Volume Probability* button. The probabilities of obtaining 5 000 Mℓ or 7 000 Mℓ are, for example, shown in Figure 4(d).

4.2 Implementation of the concept demonstrator

The unified modelling language [22] class structure of the concept demonstrator implementation of WRDSS is shown in Figure 5. In the figure, each class is represented as a rectangle, listing its attributes followed by its operations, with the exception of the **GUI** class (its attributes have been summarised for the sake of brevity). The attributes of a given class are the variables which exist in an instance of the class, whilst operations are the methods which may be performed on these attributes. If one class utilises another at some point in time, the first class is said to depend on the latter. Class dependency is indicated by dashed arrows in the figure.

The **GUI** class displays the graphical user interface which creates an instance of the **Weights** class. This class is used when the user specifies the sensitivity of each demand period. By default, constant weight values are stored for each month of the hydrological year, but the *convert_to_biweekly* operation is called if the user checks the corresponding radio button, after which the weights are stored in constant biweekly periods by calculating the weighted average importance values on a biweekly basis. The **Dates** class is used to load the current date and manage the start date of the model. The **GUI** class depends on the **Strategy** class for model execution once the user parameters have been specified. The **Strategy** class performs the iterative procedure of the mathematical modelling framework, as described in §3.4, and depends on several other classes for its operation. First, the *set_initial_release* operation of the **Demand** class is used to specify the initial release strategy. The **EndVolumeDistribution** class depends on the **Volumes** and **Inflows** classes. It is used to determine the end-volume distribution resulting from a given release strategy by means of the *end_volume_distribution* operation.

The **Inflows** class loads the historical inflow data upon initialisation, which are then passed to the **Volumes** class by the **EndVolumeDistribution** class during model execution. The functionality adopted to simulate inflows is present in this class and may be adapted or reviewed in future work. It is, however, not used during model execution in the context of this paper, as motivated in the case study to follow. The operation *get_end_vol_estimate* implements (5) to obtain an estimate of the expected end volume associated with a given user-specified probability according to the aforementioned end-volume distribution. The *draw_volumes* operation may be used to plot the end-volume distribution resulting from a given strategy, while the *shortage_probability* operation obtains the probability of the reservoir volume dropping below a certain user-specified threshold volume, as explained in §3.5. The *adjust_strategy* operation of the **AnaliseStrategy** class implements (7) and

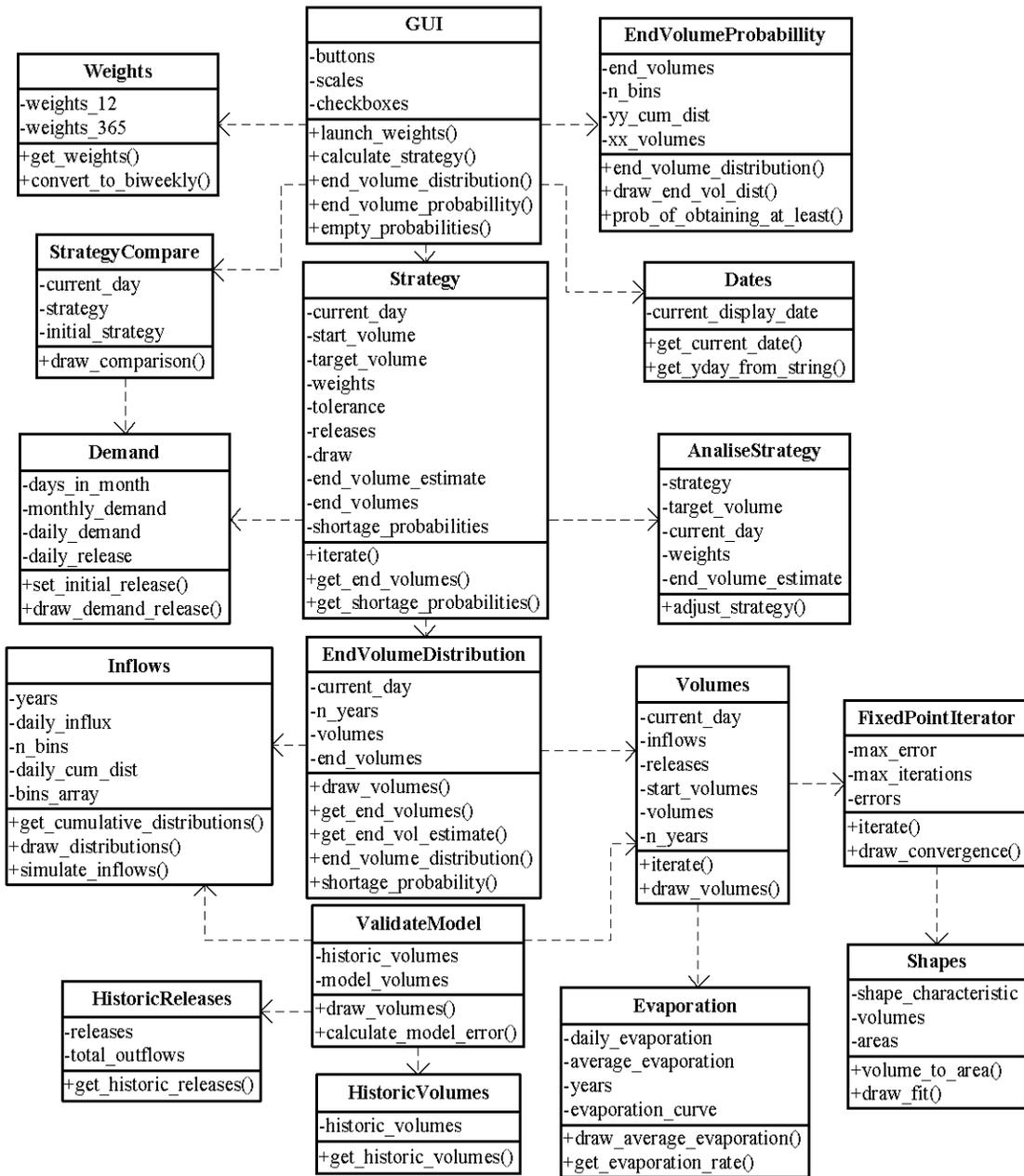


Figure 5: The unified modelling language [22] class structure of the concept demonstrator implementation of WRDSS.

is called by the *iterate* operation in the **Strategy** class to perform an adjustment of a water release strategy after each iteration.

The **Volumes** class depends on the **FixedPointIterator** and **Evaporation** classes in the calculation of period volumes, which is performed by the *iterate* operation of this class. The **FixedPointIterator** class, in turn, depends on the **Shapes** class to compute the exposed water surface area for a given reservoir volume using the *volume_to_area* operation. This surface area is employed in the estimation of evaporation losses. The *iterate* operation in the **FixedPointIterator** class implements the method of fixed point iteration to solve (2) for V_i during each replication of each calculation period. The **Evaporation** class calculates the historical daily average evaporation rate and fits a polynomial function to these averages upon initialisation. The *get_evaporation_rate* operation of this class returns the evaporation rate estimation function value for a given day, according to the polynomial function fitted. This value is passed to an instance of the **FixedPointIterator** class, by the *iterate* operation of the **Volumes** class.

Finally, the **EndVolumeProbability** and **StrategyCompare** classes facilitate analysis of the system output. The **StrategyCompare** class may be used to draw a comparison between water demand, the initial release strategy and the adjusted release strategy by means of the *draw_comparison* operation. The **EndVolumeProbability** class is used to plot the end-volume distribution, using the *draw_end_vol_dist* operation, and to estimate the probability of ending the hydrological year with at least a certain reservoir storage level, using the *prob_of_ending_at_least* operation.

Not all the attributes, operations or even classes shown in Figure 5 are employed directly in the execution of the modelling approach — some were implemented to analyse the efficiency and precision of the DSS during the development process. Thus, if adjustments or updates are made to the DSS, these may perhaps be tested and analysed using the existing functionality. An example of such an operation is the *draw_convergence* operation of the **FixedPointIterator** class, which plots the estimated error for each iteration. This is useful when analysing the efficiency of the fixed point iteration procedure, but is of little value to the end user of the DSS. Other examples include the *draw_fit* operation of the **Shapes** class, used to visualise the effect of fitting a piecewise linear function to the reservoir characteristic, and the *draw_average_evaporation* operation of the **Evaporation** class, which plots the function fitted to the historical average evaporation rates and may be used to analyse its ability to represent the historical trend adequately.

The **ValidateModel**, **HistoricReleases** and **HistoricVolumes** classes are not employed in the DSS execution, but may be used to estimate modelling errors via the *calculate_model_error* operation. Using the *draw_volumes* operation of the **ValidateModel** class, for example, the model accuracy may be depicted visually by plotting historically observed reservoir volumes against the model's volume estimations for the corresponding historical input data.

5 Keerom Dam: A case study

Keerom Dam is a typical example of an open-air reservoir with the primary purpose of water supply for irrigation. It is the second largest privately owned open-air reservoir in South Africa and is situated in the Nuy agricultural irrigation district, north-east of Worcester, in the Western Cape. The reservoir's wall height from dam crest to river bed level is 38 *metres* (m) and, when at its maximum storage capacity of 9 600 *mega litres* (Mℓ), the water surface area is 92 *hectares* (ha) [7].



Figure 6: Keerom Dam, the irrigation reservoir of the Nuy agricultural district.

Nineteen farmers benefit from its water supply, of which six serve on the reservoir board of management. This board determines the release strategy for the reservoir on an annual basis. The DSS of §4 is applied in this section to a special case study involving Keerom Dam in order to demonstrate the workability and usefulness of the system in a real-world context.

5.1 Background

A measuring station situated on the dam wall is visible in Figure 6. This measuring station records the reservoir water level on a daily basis, while a second measuring station situated downstream from the reservoir's sluice measures the water release rate on a daily basis. Both of these measuring stations transmit their data via satellite to the Department of Water Affairs and Forestry for incorporation into their national database.

The reservoir volume is determined from the measured water level using the reservoir shape characteristic, shown in Figure 7, which relates the reservoir volume and surface area. Using sonar, this shape characteristic was determined by the consulting and engineering company Tritan Inc. [7].

The measured outflow, reservoir volume and evaporation rates obtained from the database of the Department of Water Affairs and Forestry [6] for the hydrological years spanning 1 October 1955 to 31 September 2013 were used to calculate the actual daily reservoir inflow.

5.2 Simulation of inflows

The method described in §3.3 for the simulation of inflows was applied using historical inflow data for Keerom Dam. The mean annual inflow obtained from a large number (a thousand years in this instance) of Monte Carlo simulations of daily inflows was found

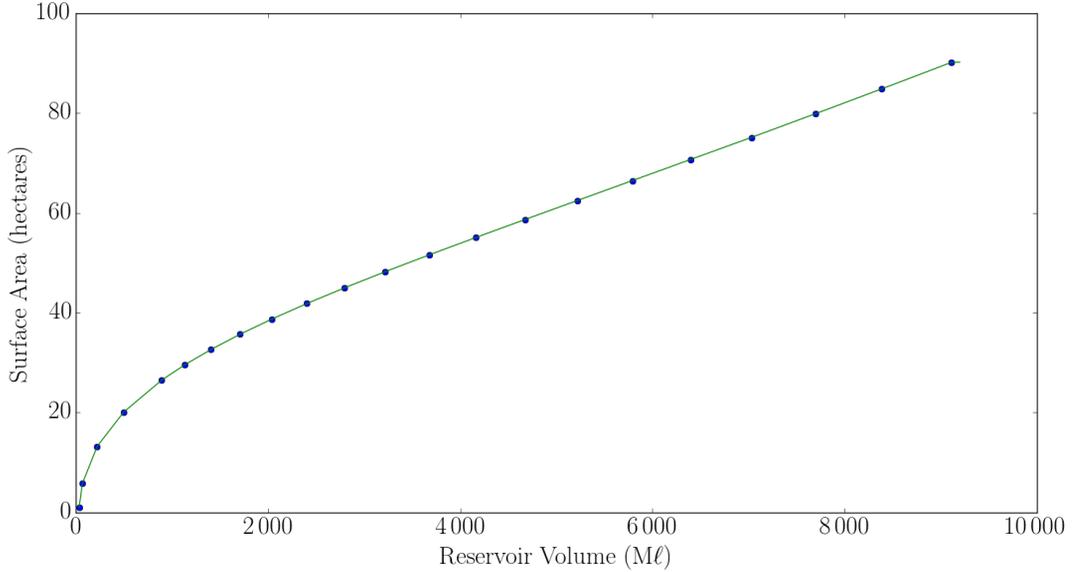


Figure 7: The shape characteristic f in (3) for Keerom Dam.

to fall within 5% of the historically observed average, yet the standard deviation of the total annual simulated inflow was approximately a sixth of that associated with the actual historical inflow, as shown in Table 1.

	Mean	Standard deviation
Historical inflows	16 481.95	47 215.24
1000 years' simulated inflows	15 758.24	7 878.92

Table 1: Historical and simulated inflows for Keerom Dam.

The reason for this discrepancy is the assumption of independence between adjacent simulation periods inherent in this modelling approach, as mentioned in §3.3. In reality, large inflows tend to decrease gradually over a period of a couple of days or weeks, whilst in the Monte Carlo simulation it may happen that a large inflow, lasting only a single day, is simulated. This means that over longer periods (such as years, for example), the variation in the total inflow obtained during the simulations may be substantially less than that historically observed.

Since one of the underlying assumptions on which this simulation approach relies causes a substantial decrease in variation over several simulation periods, the approach may lead to overly optimistic planning, resulting in strategies corresponding to inadequate reservoir reserve levels for absorbing realistic inflow variation. For this reason it was decided to abandon the simulation approach as a means of modelling reservoir inflows mathematically. Considering other sources of inflow input data for the model, as mentioned in §3.3, it was decided that historical inflows would instead be utilised directly in this case study.

5.3 Volume calculation accuracy

The method described in §3.3 for determining the reservoir volume was validated by applying (2) to historical data of Keerom Dam and comparing the model’s predicted volumes to the historically observed reservoir volumes. The historical evaporation losses and Keerom Dam’s volume-area characteristic were employed in this process. The historical evaporation losses were used to obtain a mean evaporation rate, measured in millimetres per day, for each day of the year. A polynomial function was then fitted to these rates, using least squares regression. The degree of the polynomial was incrementally increased until the best visual fit was acquired. The corresponding least squares regression errors are shown in Table 2.

Degree	3	4	5	6	7
R^2 error	48.38%	48.21%	18.59%	18.03%	12.77%

Table 2: The degree of the polynomial function fitted to the mean historical daily evaporation rate and the corresponding least squares regression error.

The seventh-degree polynomial representation of the mean daily evaporation rates in Figure 8 was eventually obtained as a result of this process. As may be seen in the figure, there are no irregular function oscillations on the interval $[0, 364]$ and no rank warning was issued by the computation package, NUMPY [13], used to achieve the fit.

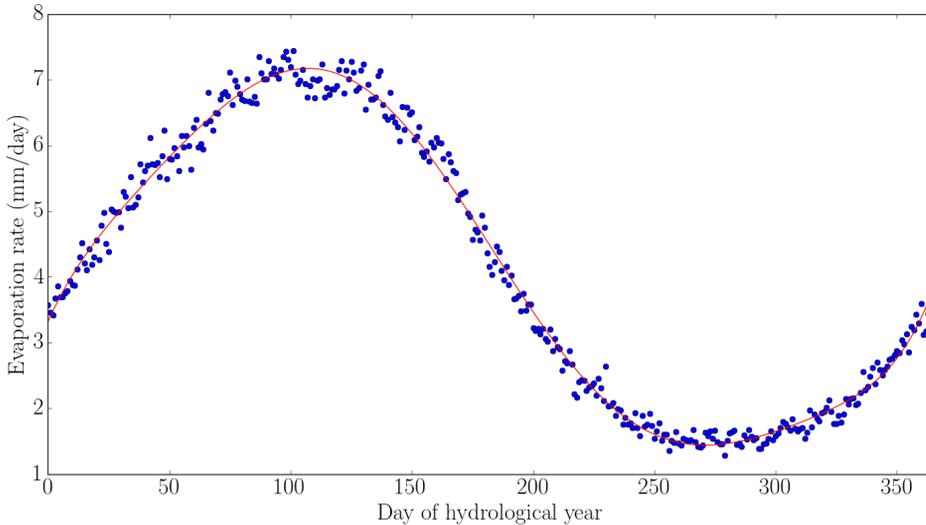


Figure 8: Seventh-degree polynomial fitted to the Keerom Dam daily evaporation rate.

The reservoir shape characteristic in Figure 7 and the mean daily evaporation rate polynomial representation in Figure 8 were used in the estimation of evaporation water losses E_0, \dots, E_{364} , as defined §3.4.

The release strategy was set equal to the historically implemented release strategy, and the expected reservoir volume was calculated using the historical inflows. A calculation

period length of one day was chosen, since this is the resolution of the input data, leading to the calculation period index set $\mathcal{T} = \{0, \dots, 364\}$. Let p denote the polynomial function fitted to the average daily evaporation rate, as shown in Figure 8. Then

$$e_i = p(i) \quad (8)$$

for all $i \in \mathcal{T}$. By substituting (1), (3) and (7) into (2) it follows that

$$V_i \equiv V_t + I_i - x_i'' - p(i) \frac{f(V_i) + f(V_t)}{2} \quad (9)$$

for all $i \in \mathcal{T}$ and $t \equiv i - 1 \pmod{n}$. Since (9) is difficult to solve analytically, a numerical method was used instead. The method of *fixed point iteration* [3] was selected for this purpose, since (9) is already in the correct format for this method and function differentiation is not required in fixed point iteration. A maximum estimated error of 0.1% was allowed and all volume estimates in (9) converged to within this tolerance within four iterations.

The *daily volume error* is defined as the percentage deviation of the model's volume prediction from the historically observed volume. The method used for volume calculation was found to be sufficiently accurate for its intended application, because an average daily volume error of 0.52% and corresponding standard deviation of 0.55% was thus achieved over the twenty-year validation period 1993–2013.

5.4 DSS input data

The irrigation water demand profile for the Nuy irrigation district, calculated using CROP-WAT [19] and shown in Table 3, was loaded into the DSS. These demand values were calculated by Strauss [16] and were validated by the authors.

The demand-importance weights were specified as indicated in Figure 3. For the months from August to December, water demand is low (less than the minimum allowed release). By fixing the importance weights of these months at small values, it is therefore ensured that excess water is released in greater proportions during months of higher demand. In the case of a water shortage, less water cannot be released during the months of August to December, since releases during these months already equal the minimum allowed release. The assigned weights would, therefore, also expedite the model iteration process in the case of a dry year.

5.5 Transition volume analysis

After loading the required data inputs into the DSS, a transition volume analysis was performed. The starting and target volumes were equated, a release strategy was employed according to (9) and the model was solved for a range of transition volumes. The probabilities of the reservoir volume dropping below certain threshold volumes were estimated according to the method described in §3.5. The results of this analysis are shown in Table 4.

From the results in Table 4, transition volumes of larger than 4 000 Mℓ seem adequate if irrigation demand fulfilment is the only requirement. For a starting volume of 4 000 Mℓ, the

	Table grapes	Wine grapes	Orchards	Olives	Vegetables	Cereals	Lucerne	Total
Oct	0.82	154.63	13.34	80.48	8.11	31.89	229.00	518.27
Nov	2.54	158.59	13.68	103.18	8.32	32.71	234.87	553.89
Dec	3.93	163.64	17.37	106.47	8.58	33.75	242.34	576.08
Jan	5.39	220.58	17.08	131.56	9.64	37.92	272.23	694.40
Feb	5.18	194.27	18.05	126.40	10.19	40.07	287.71	681.87
Mar	4.14	193.91	10.29	100.93	10.17	40.00	287.18	646.62
Apr	2.25	168.68	8.96	65.85	8.85	34.79	249.82	539.20
May	0.78	0.00	0.00	56.84	0.00	0.00	0.00	57.62
Jun	0.00	0.00	0.00	48.97	0.00	0.00	0.00	48.97
Jul	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Aug	0.00	0.00	4.66	38.11	0.00	0.00	0.00	42.77
Sep	0.71	0.00	7.12	52.34	0.00	0.00	0.00	60.17
Total	25.74	1 254.30	110.55	911.13	63.86	251.13	1 803.15	4 419.86

Table 3: Irrigation demand of the Nuy agricultural district in Mℓ [16].

Transition/year-end volume (Mℓ)	Probability of reservoir level dropping below threshold					
	Empty	≤ 1 000	≤ 2 000	≤ 3 000	≤ 4 000	Overflow
1 000	13.81 %	55.97 %	75.49 %	83.61 %	88.08 %	2.71 %
2 000	5.09 %	27.06 %	61.53 %	78.91 %	85.63 %	3.42 %
3 000	0.89 %	11.74 %	30.72 %	64.05 %	80.29 %	4.43 %
4 000	0.00 %	2.83 %	14.43 %	32.66 %	65.24 %	4.96 %
5 000	0.00 %	0.00 %	3.59 %	16.23 %	34.45 %	5.33 %
6 000	0.00 %	0.00 %	0.00 %	4.10 %	17.34 %	6.70 %
7 000	0.00 %	0.00 %	0.00 %	0.00 %	4.59 %	8.12 %
8 000	0.00 %	0.00 %	0.00 %	0.00 %	0.05 %	9.44 %
9 000	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	12.45 %

Table 4: Transition volume analysis for Keerom Dam. Volumes are given in Mℓ.

risk of water shortage is negligibly small over a period of one hydrological year. Since the expected reservoir volume during the hydrological year, for a starting volume of 8 000 Mℓ, is approximately 4 000 Mℓ, it follows that the risk of water shortage is negligibly small over a period of at least two hydrological years, for a starting volume of 4 000 Mℓ.

At lower reservoir levels, evaporation losses are smaller since the average exposed water surface area of the reservoir is smaller. Thus, the probability of reaching a given target end volume, for the same release strategy, is greater for lower transition target volumes. Optimality in the context of maximising the total annual reservoir outflow by minimising evaporation losses, as pursued by van Vuuren and Gründlingh [25], and subsequently by Strauss [16], therefore corresponds to managing the reservoir at the lowest possible level. Such “optimal” management is, however, only beneficial in systems with very limited variation in input variable behaviour. Optimal management strategies in pursuit of little or no redundancy within systems exposed to substantial volatility result in high exposure to risk [20]. In the case of Keerom Dam, there is extreme variation in the total annual inflow volume, with the standard deviation approximately three times the mean, as may be seen in Table 1.

It is therefore not necessarily better to choose the smallest possible transition volume in pursuit of minimising evaporation losses. On the contrary, the risk of not being able to satisfy demand becomes negligible for higher transition volumes, although the WRDSS user will have to accept a slightly lower level of confidence in reaching the target end volume. For lower transition volumes, the risk of water shortage increases, which represents a very undesirable situation for farmers who depend on the reservoir water supply.

The precipitation norms in the Nuy district typically cause Keerom Dam to reach its largest storage volume during the transition between hydrological years. It may, therefore, in general (and specifically also in the context of Keerom Dam) be best to select the largest possible transition volume which still allows releases of acceptable magnitude for the purpose of meeting irrigation demand.

Aiming for a transition volume in the vicinity of 8000 Mℓ seems to be a prudent choice in the context of Keerom Dam. With such a choice, there should be no occurrences of water shortage, if the last 58 hydrological years' data are used as an indication of possible likely futures. Even for the driest years observed as of yet, the end volume should not drop to catastrophically low levels. This recommendation is analysed and substantiated in hindsight in the following section by considering a set of historically observed hydrological years.

5.6 Release strategy suggestion

For an analysis of WRDSS's capability of suggesting good release strategies, the concept demonstrator of §4 is applied in hindsight in this section to the 2003/2004 hydrological year observed at Keerom Dam — the year of volatile meteorological conditions after which the previous DSS (ORMADSS) fell out of favour. The expected volume fluctuations resulting from the suggested strategies are then compared to the actual historical volume fluctuations.

Suppose a year-end target volume of 8000 Mℓ is initially selected and that suggested strategies are obtained from WRDSS at 50% and 75% confidence levels, where the latter would represent a more risk-averse user. Keerom Dam's volume on 1 October 2003 was 9645.23 Mℓ. The water demand, the actual historical release strategy and the two strategies suggested by WRDSS are shown in Figure 9(a) for a target end volume of 8000 Mℓ. The volume fluctuations for the 2003/2004 hydrological year corresponding to these three strategies are shown in Figure 9(b).

It may be noted that the 75%-strategy outperforms the actual historical strategy in terms of maintaining reservoir storage levels, whilst the strategy suggested at a 50% confidence level fares slightly poorer than the historically adopted strategy. Suppose that six months into the year WRDSS were once again to be consulted. The resulting strategy suggestions and corresponding volume fluctuations are shown in Figures 10(a) and 10(b), respectively. It may be noted that even the strategy suggested by WRDSS at a 50% confidence level now outperforms the historically adopted strategy by obtaining a slightly higher end volume.

Suppose WRDSS were finally to be consulted with three months of the 2003/2004 hydrological year remaining. At this point the DSS outputs a failure to converge message, indicating that the strategy cannot be adjusted enough to obtain the target end volume at

the selected confidence levels. Strategies managing the end volume as close as possible to the target are nevertheless suggested as output. These strategies and the corresponding volume fluctuations are shown in Figures 11(a) and 11(b), respectively.

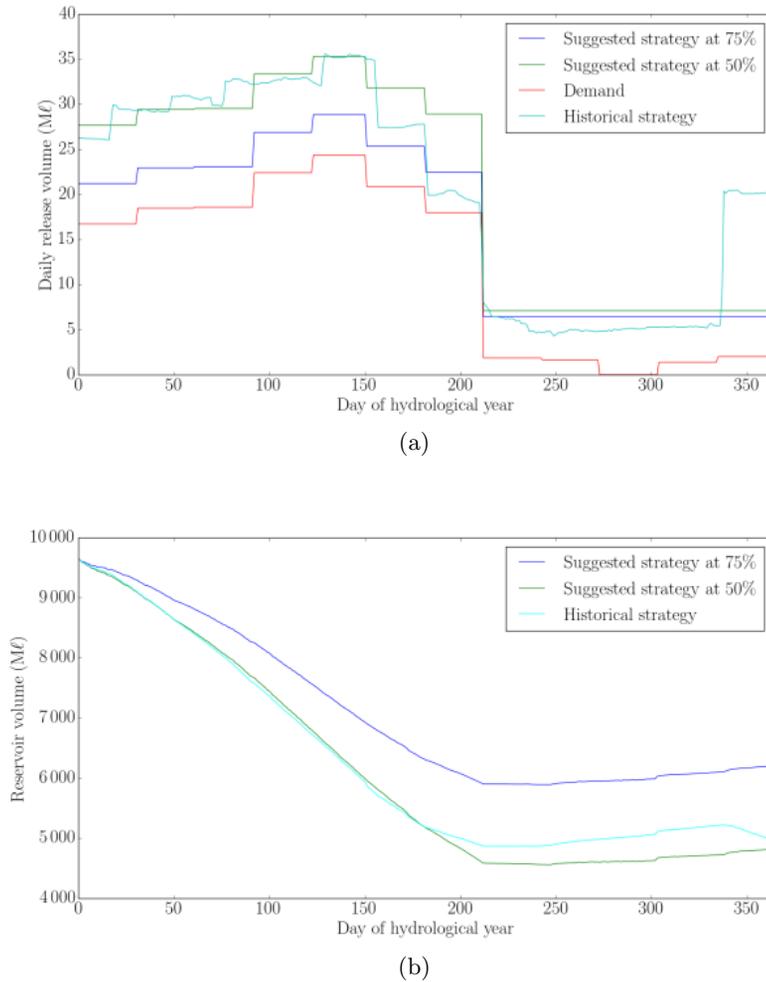
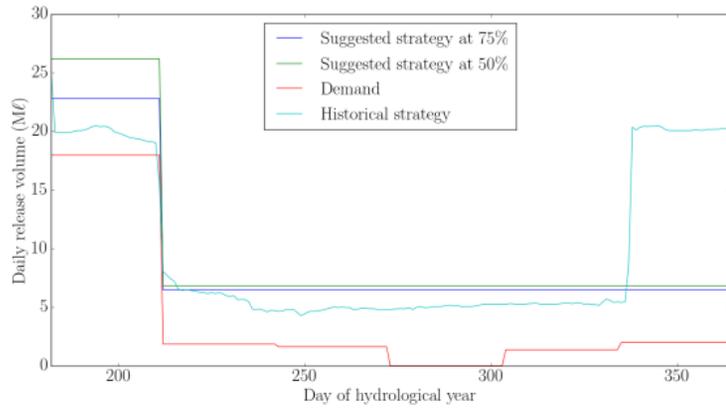
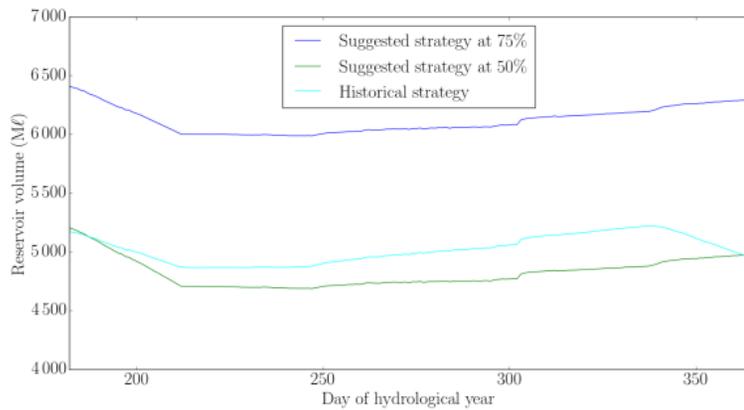


Figure 9: Release strategies suggested by WRDSS at the start of the 2003/2004 hydrological year and corresponding reservoir volume fluctuations for the actual starting volume of the year and a target end volume of 8 000 Mℓ.



(a)



(b)

Figure 10: Release strategies suggested by WRDSS six months into the 2003/2004 hydrological year and corresponding reservoir volume fluctuations for the actual starting volume of the year and a target end volume of 8 000 Mℓ.

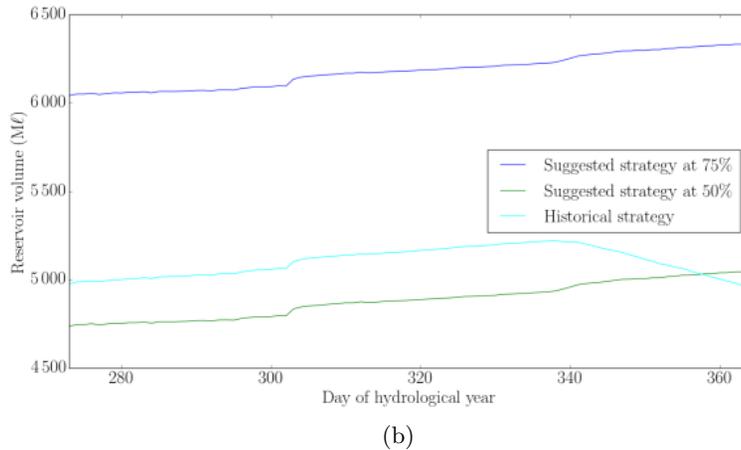
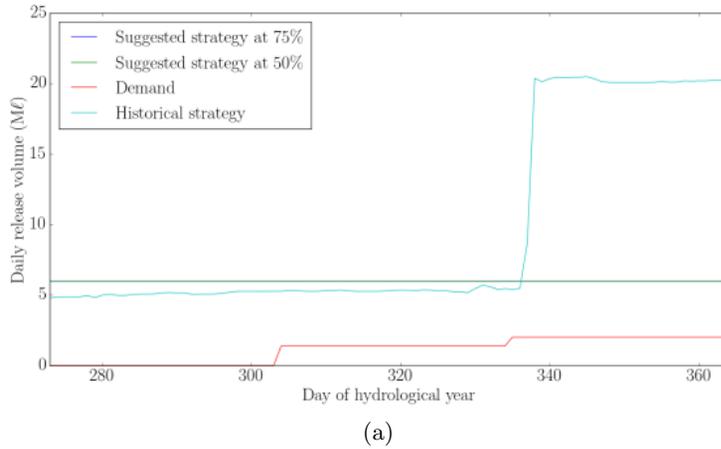


Figure 11: Release strategies suggested by WRDSS nine months into the 2003/2004 hydrological year and corresponding reservoir volume fluctuations for the actual starting volume of the year and a target end volume of 8 000 Mℓ.

The true and expected end volumes for the 2003/2004 hydrological year are listed in Table 5. Both strategies suggested by WRDSS may be seen to outperform the historically adopted strategy in hindsight, by achieving higher reservoir storage levels at the end of a particularly dry year.

Strategy	End volume
75% confidence	6 335.73 Mℓ
50% confidence	5 047.73 Mℓ
Historical	4 945.34 Mℓ

Table 5: End volumes for the 2003/2004 hydrological year at Keerom Dam for a 8 000 Mℓ target transition volume.

5.7 Response of Keerom Dam board of management

In October 2015, the concept demonstrator of WRDSS was presented to the chairperson of the Keerom Dam board of management as well as ten farmers who benefit from the reservoir's water supply. The DSS was positively received and the farmers were excited by the possibility of quantifying the future water shortage risk related to a chosen strategy [5]. The ability to gauge whether operational decisions are overly conservative on the one hand or heedless on the other, aroused enthusiasm amongst the farmers present. The concept demonstrator of WRDSS was subsequently installed on the personal computers of several of these farmers, and the system was taken in use by the board of management on 15 January 2016.

6 Conclusion

A novel DSS concept demonstrator for open-air irrigation reservoir control, called WRDSS, was proposed in this paper. The system is based on the mathematical modelling framework of van der Walt and van Vuuren [24]. This framework allows for the quantification of water shortage risk, the degree of strategy repeatability and the extent to which demand fulfilment is achieved. This information may enable operators to compare strategy choices objectively, thereby aiding them in selecting a consensus reservoir release strategy.

The release strategy suggestions of WRDSS depend on several user-specified parameters, including demand importance weight factors denoting a given period's demand flexibility, a year-end target volume and a confidence level by which this target end volume is to be obtained. This increases the model's versatility, as it incorporates the user's attitude toward risk to some extent, instead of simply suggesting a strategy based on reservoir dynamics.

The computerised concept demonstrator of WRDSS was applied to a real case study involving Keerom Dam in a bid to validate the DSS, by comparing its strategy suggestions to various historically employed strategies and the reservoir volume fluctuations resulting from these strategies. It was found that WRDSS's strategy suggestions would have fared better in hindsight in terms of preserving reservoir storage levels than the historically employed strategies, especially during dry hydrological years, thereby diminishing the farmers' exposure to water shortage risk. The concept demonstrator was received with positive enthusiasm by the members of the Keerom Dam board of management who took it in use for strategy suggestion. The objective manner in which strategies can be compared, as facilitated by the quantitative performance metrics calculated by WRDSS, was noted as a significant benefit by members of the board.

In WRDSS, expected volumes are estimated using possible volume fluctuations based on historically observed inflows. Previous models employed at Keerom Dam estimated volumes using inflow averages instead. The reservoir volume is, however, a non-linear function of inflows and release volumes, since evaporation losses are proportional to the reservoir's exposed water surface area, which is a non-linear function of the reservoir storage volume, and since the reservoir volume may only increase up to its maximum storage capacity. More specifically, for a given fixed release strategy, the limits of possible

hydrological year-end volumes, based on historically observed inflow data, are concave functions of the total annual inflow, up to the reservoir storage capacity. According to Jensen's well-known inequality [4], the expected value of a concave function of a random variable is not greater than the concave function evaluated at the expected value of the random variable. In the context of this paper, reservoir inflow is a random variable and the expected reservoir volume is a concave function of this variable. The model on which the WRDSS is based is therefore expected to produce more conservative, and more realistic, volume estimations than previous models, because estimations are performed on the function values, instead of on the variable values themselves. The standard approach of utilising historical cumulative inflow distributions for discrete simulation periods in a Monte Carlo simulation setting was analysed in the context of Keerom Dam and found to be an insufficient representation of inflow behaviour. It was determined that historical inflows, which represent existing knowledge on inflow behaviour, should instead be utilised directly in the model.

7 Future work

Various ideas for future work, which may be pursued as extensions to the work documented in this paper, are mentioned in this final section. A cost-benefit analysis may be performed in order to gain a concrete understanding of the financial implications of experiencing water shortage. The influence of the shortage magnitude, duration and time of occurrence may thus be investigated. Such information may perhaps be utilised in the selection of demand-importance weights to be used in WRDSS. An analysis of inflow variation may also be performed in respect of a number of large irrigation reservoirs so as to gain an understanding of a reservoir's role either as buffer in terms of limiting outflows or as a source of security in hedging farmers against water shortage risk. In this paper, it was observed in the case of Keerom Dam that the volatility of annual inflow volumes is so extreme that mean inflow values are of little use. Since the effects of small annual inflows differ vastly from the effects of large annual inflows, considering only standard deviation is not adequate.

As WRDSS was developed with the intention of being a generic DSS for the selection of water release strategies at open-air irrigation reservoirs, it may be applied to case studies other than that of Keerom Dam in order to explore its flexibility in terms of suggesting good water release strategies.

A study may further be performed to determine beneficial release strategies at large water reservoirs, in the presence of a network of smaller secondary reservoirs located downstream. In the case of Keerom Dam, for example, all of the farmers who benefit from water from the reservoir also have smaller dams on their farms in which they can store water released from Keerom Dam. These secondary reservoirs may differ in size and may each have a unique demand profile. Methods for the aggregation of these demand profiles into a demand profile for the larger, upstream reservoir may also be investigated.

The functionality of WRDSS may finally be extended to consider strategy formulation over scheduling horizons exceeding one year. The effects of a given strategy on storage levels when considering $N \in \{1, 2, 3, \dots\}$ consecutive hydrological years' inflows may be analysed

to determine an explicit, non-conditional N -year water shortage risk corresponding to a given transition volume.

References

- [1] BONTEMPS C & COUTURE S, 2002, *Irrigation water demand for the decision maker*, Environment and Development Economics, **7(4)**, pp. 643–657.
- [2] BOX GE & JENKINS GM, 1976, *Time series analysis: Forecasting and control*, 2nd Edition, Holden-Day, San Francisco (CA).
- [3] BURDEN RL & FAIRES JD, 1989, *Numerical analysis*, PWS Kent Publishing Co., Boston (MA).
- [4] CHANDLER D, 1987, *Introduction to modern statistical mechanics*, Oxford University Press, Oxford.
- [5] CONRADIE A, 2015, Chairperson of the *Keerom Dam Board of Management*, [Personal Communication].
- [6] DEPARTMENT OF WATER AFFAIRS AND FORESTRY, 2015, *Hydrology database*, [Online], [Cited August 17th, 2015], Available from <http://www.dwa.gov.za/Hydrology/Verified/HyDataSets.aspx?Station=H4H020&SiteDesc=RIV>
- [7] HUMAN O & HAGEN DJ, 2014, *Raising of Keerom Dam*, (Unpublished) Technical Report 9523/107752, Aurecon South Africa (Pty) Ltd., Cape Town.
- [8] JAIN S, DAS A & SRIVASTAVA D, 1999, *Application of ANNs for reservoir inflow prediction and operation*, Journal of Water Resources Planning and Management, **125(5)**, pp. 263–271.
- [9] JIANPING H, YUHONG Y, SHAOWU W & JIFEN C, 1993, *An analogue-dynamical long-range numerical weather prediction system incorporating historical evolution*, Quarterly Journal of the Royal Meteorological Society, **119(511)**, pp. 547–565.
- [10] KEEN PG, 1987, *Decision support systems: The next decade*, Decision Support Systems, **3(3)**, pp. 253–265.
- [11] KELTON WD & LAW AM, 2000, *Simulation modeling and analysis*, McGraw-Hill, Boston (MA).
- [12] RIZZO ML, 2008, *Statistical computing with R*, Chapman & Hall/CRC, London.
- [13] SCIPY.ORG, 2014, [Online], [Cited August 24th, 2015], Available from <http://docs.scipy.org/doc/numpy/reference/generated/numpy.polynomial.polynomial.polyfit.html>
- [14] SHIM JP, WARKENTIN M, COURTNEY JF, POWER DJ, SHARDA R & CARLSSON C, 2002, *Past, present, and future of decision support technology*, Decision Support Systems, **33(2)**, pp. 111–126.
- [15] SMITH M, ALLEN R & PEREIRA L, 1998, *Revised FAO methodology for crop-water requirements*, (Unpublished) Technical Report ISSN 1011-4289, Joint FAO/IAEA Division of Nuclear Techniques in Food and Agriculture, Vienna.
- [16] STRAUSS JC, 2014, *A decision support system for the release strategy of an open-air irrigation reservoir*, Final-year Project, Department of Industrial Engineering, Stellenbosch University, Stellenbosch.
- [17] SUN YH, YANG S, YEH WW & LOUIE PW, 1996, *Modeling reservoir evaporation losses by generalized networks*, Journal of Water Resources Planning and Management, **122(3)**, pp. 222–226.
- [18] SUDHEER K, GOSAIN A, MOHANA RANGAN D & SAHEB S, 2002, *Modelling evaporation using an artificial neural network algorithm*, Hydrological Processes, **16(16)**, pp. 3189–3202.
- [19] SWENNENHUIS J, 2006, *CROPWAT version 8.0*, [Online], [Cited April 30th, 2015], Available from http://www.fao.org/nr/water/infores_databases_cropwat.html

- [20] TALEB NN, 2012, *Antifragile: Things that gain from disorder*, Random House Incorporated, New York (NY).
- [21] TIBANE E & VERMEULEN A, 2014, *South Africa Yearbook 2013/14*, [Online], [Cited March 9th, 2015], Available from <http://www.gcis.gov.za/content/resourcecentre/sa-info/yearbook2013-14>
- [22] RUMBAUGH J, JACOBSON I & BOOCH G, 2004, *Unified modeling language reference manual*, 2nd Edition, Pearson Higher Education, Boston (MA).
- [23] VAN HEERDEN P, CROSBY C, GROVÉ B, BENADÉ N, THERON E, SCHULZE R & TEWOLDE M, 2009, *Integrating and updating of SAPWAT and PLANWAT to create a powerful and userfriendly irrigation planning tool*, (Unpublished) Technical Report TT 391/08, Water Research Commission, Gezina.
- [24] VAN DER WALT JC & VAN VUUREN JH, 2015, *Decision support for the selection of water release strategies at open-air irrigation reservoirs*, Proceedings of the 44th Annual Conference of the Operations Research Society of South Africa, Hartbeespoort, pp.33–43.
- [25] VAN VUUREN JH & GRÜNDLINGH WR, 2001, *An active decision support system for optimality in open air reservoir release strategies*, International Transactions in Operational Research, **8(4)**, pp. 439–464.
- [26] WILLIS DB & WHITTLESEY NK, 1998, *The effect of stochastic irrigation demands and surface water supplies on on-farm water management*, Journal of Agricultural and Resource Economics, **23**, pp. 206–224.
- [27] WILKS DS & WILBY RL, 1999, *The weather generation game: A review of stochastic weather models*, Progress in Physical Geography, **23(3)**, pp. 329–357.
- [28] WINTER TC, 1981, *Uncertainties in estimating the water balance of lakes*, JAWRA Journal of the American Water Resources Association, **17(1)**, pp. 82–115.