An Optimal Design Model for New Water Distribution Networks in Kigali City

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Abstract
This paper is concerned with the problem of optimizing the distribution of water in Kigali City at a minimum cost. The mathematical formulation is a Linear Programming Problem (LPP) which involves the design of a new network of water distribution considering the cost in the form of unit price of pipes, the hydraulic gradient and the loss of pressure. The objective function minimizes the cost of the network which is computed as the sum of the initial cost of the individual pipes. The model is solved using the Simplex algorithm which is implemented by the Linear Interactive and Discrete Optimizer (LINDO) using data from a sample network in Kigali. The optimal solutions show that the cost is reduced compared to the cost of the sampled existing networks of Kigali city.

Keywords: *Linear Programming models, water distribution network, hydraulic gradient, pressure loss, minimize cost, Kigali City*

1. Introduction

Kigali is the largest city in Rwanda and its population is increasing at a drastic rate and thus widening surface area. This has resulted in water rationing because water production has not matched with the ever increasing needs. The cost of distributing clean water is high and the aged distribution piping system badly needs rehabilitation. A water distribution network consists of pipes, reservoirs, pumps, valves, and other hydraulic components and its purpose is to provide reliable service to the customers under various demand conditions. The least cost design of water distribution networks is an optimization problem, which has been solved using linear programming, nonlinear programming, dynamic programming and heuristic based optimization methods (Kessler and Shamir, 1989; Eiger et al. 1994; Dandy et al 1996). This paper presents an attempt to achieve the optimal solution with the minimum design cost for the Kigali water distribution network with available pipe sizes, using the gradients calculated from the dual variables.

2. Methodology

2.1. The Linear Programming Problem

Definition1: A *mathematical programming problem* (MPP) is an optimization problem of finding the values of the unknown variables $x_1, x_2, \ldots x_n$ that

$$\text{Maximize (or minimize)} \quad f(x_1, x_2, \ldots x_n) \tag{1}$$

$$\text{Subject to } g_i(x_1, x_2, \ldots x_n)\ (\leq, =, \geq) b_i \qquad i = 1,2,\ldots,m \tag{2}$$

Where the b_i are real constants and the functions f and g_i are real-valued. The function $f(x_1, x_2, \ldots x_n)$ is called the *objective function* of the MPP (equations (1) and (2)) while the functions $g_i(x_1, x_2, \ldots x_n)$ are called the *constraints* of the MPP.

In vector notations, (1) and (2) can be written as:

$$\text{Maximize (or minimize)} \qquad f(X^T) \tag{3}$$

$$\text{Subject to } g_i(X^T)\ (\leq, =, \geq) b_i \ \ i = 1,2,\ldots,m \tag{4}$$

Where$X^T = (x_1, x_2, \ldots x_n)$ is the solution vector.

A *linear programming problem* (LPP) is a mathematical programming problem having a linear objective function and linear constraints, expressed as,

$$\text{Minimize or maximize } \ z = f(x) = \sum_j c_j x_j \tag{5}$$

$$\text{Subject to} \qquad \begin{cases} a_{ij}\, x_j\ (\leq, =, \geq) b_i \\ \qquad x_j \ \geq 0 \end{cases} \quad i = 1,2,\ldots,m; j = 1,2,\ldots,n \tag{6}$$

Equations (5) and (6) form the LPP model. Equation (5) is the linear objective function in the decision variablesx_j that the decision maker wants to maximize (revenue or profit) or minimize (cost). The decision-variables x_j are the unknown to be determined by the solution of the model. Equation (6) are the constraints on the decision variables with coefficients a_{ij} while $b_i$are the equality or inequality right hand side of the linear combination. The constraints represent the physical limitations of the system with the constraint that the decision variables x_j are nonnegative. It is assumed that the known constants a_{ij}, b_i and c_j are real. If all the constraints are inequalities and the unknowns x_j are restricted to nonnegative values, then the form is called *canonical*. The *canonical form* of an LPP is

$$\text{Max or min } z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n \tag{7}$$

$$\text{Subject to} \begin{cases} a_{11}x_1 + \ \dots + a_{1n}x_n \le b_1 \\ \vdots \\ a_{m1}x_1 + \ \dots + a_{mn}x_n \le b_m \end{cases} \quad (8)$$

Where $x_i \geq 0, \quad i = 1,2, \dots, n.$ (9)

If all $b_i \geq 0$, then the form is called a *feasible canonical form*. The Simplex method will be used to solve the LPP. As such, the canonical form must be converted into the standard form:

$$\text{Max } z = c_1x_1 + c_2x_2 + \ \dots + c_nx_n \quad (10)$$

$$\text{Subject to} \begin{cases} a_{i1}x_1 + \ \dots + a_{in}x_n \le b_i, \ \ i = 1,2, \dots, m \\ \vdots \\ x_j \geq 0, \ \ j = 1,2, \dots, n \end{cases} \quad (11)$$

It is assumed that the b_i are nonnegative while the number of variables may or may not be the same as before. The LPP can easily be changed into the canonical form or into the standard form (see for example Dantzig, 1963).

For convenience the standard form of the LPP is expressed in matrix notation as:

$$\text{Max or Min } c^T x \quad (9)$$

$$\text{Subject to} \begin{cases} Ax = b \\ x \geq 0 \end{cases} \quad (10)$$

Where $x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^{n\times 1}, \ b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix} \in \mathbb{R}^{m\times 1}$ and

$c = \begin{pmatrix} c_{11} & \cdots & c_{1m} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nm} \end{pmatrix} \in \mathbb{R}^{n\times m}$ and $rank(A) = m.$

2.2. Existence of an optimal solution of a LP problem

It is important to know whether the LPP has an optimal solution or not.

Definition2:

a) If x satisfies $Ax = b, \ x \geq 0$, then x is a *feasible solution*. The set of all feasible solutions is called the *feasible region*.
b) A feasible solution to an LPP is said to be an *optimal solution* if it maximizes the objective function of the LPP, i.e., an optimal

solution $x*$ is a feasible solution such that $c^T x^* = \min\{c^T x: Ax = b, x \geq 0$.

c) A linear program is unbounded if $\forall\, \lambda \in \mathbb{R}, \exists$ a feasible x^* s.t. $c^T x^* \leq \lambda$.

Definition3: Given a system of m x n linear equations $Ax = b$, let B be any nonsingular m x m sub-matrix made up of columns of A. Then, if all $n - m$ components of x not associated with columns of B are set equal to zero, the solution to the resulting set of equations is said to be a basic solution to $Ax = b$ with respect to the basis B. A feasible solution to an LPP is said to be a *basic feasible solution* (BFS) if it is a basic solution with respect to the linear system $Ax = b$. If a BFS is non-degenerate, then we call it a *non-degenerate basic feasible solution*.

Theorem 1 (Dantzig, 1963)**:** *If there is a feasible solution then there is a basic feasible solution.*

Assuming that there is a feasible solution x with p positive variables where $p \leq n$, then the feasible solution can be written as $x^T = (x_1, x_2, \dots, x_p, 0, 0, \dots, 0)$ so we have $\sum_{j=1}^{p} x_j a_j = b$. For simplicity we can write $x^T = (x_1, x_2, \dots, x_p, 0, 0, \dots, 0)$ as $x^T = (x_1, x_2, \dots, x_p)$

Theorem 2 (Border, 2003): Let the LPP be

$$\max\ z = c^T x$$

$$\text{Subject to} \begin{cases} Ax = b \\ x \geq 0 \end{cases}$$

Assuming $b \geq 0$ and $rank(A) = m$. Let a_j be columns of A i.e. $col\, A = [a_1, a_2, \cdots a_n]$. Let B be a $m \times m$ non-singular matrix whose columns are linearly independent columns of A and denote $B = [b_1, b_2, \dots, b_m]$,where b_i, is the i^{th} column of A. For any choice of basic matrix B there corresponds a basic solution $Ax = b$ given by the m-vector $x_B = [x_{B1}, x_{B2}, \cdots x_{Bm}]$ where $x_B = B^{-1} b$. Since A is mxn and $rank(A) = m$, the column space of A is m-dimensional. Thus the columns of B form a basis for the column space of A. Let $a_j = \sum_{i=1}^{m} y_{ij} b_i, \forall j = 1,2, \dots, n$. Put $\begin{bmatrix} y_{1j} \\ \vdots \\ y_{mj} \end{bmatrix}, \forall j = 1,2, \dots, n$ then $a_j = B y_j$; hence $y_j = B^{-1} a_j$.

Theorem 3 (Border, 2003)**:** Let x_B be a BFS to an LPP with corresponding basic matrix B and objective value z. Let $z_j = c^T y_j$. If

a) there exists a column a_j in A but not in B such that the condition $c_j - z_j > 0$ holds and if
b) at least one $y_{ij} > 0$, then it is possible to obtain a new BFS by replacing one column in B by a_j and the new value of the objective function $\hat{z}$ is larger than or equal to z.

Proof** (Border, 2003)**:

We first note that given any feasible solution x, then by the assumption that $z_j - c_j \geq 0, \forall j = 1, 2 \cdots n$ and we have

$$z = \sum_{j=1}^{n} c_j x_j \leq \sum_{j=1}^{n} z_j x_j = \sum_{j=1}^{n} (c^T y_j) x_j = \sum_{j=1}^{n} \left(\sum_{i=1}^{m} c_i y_{ij} \right) x_j = \sum_{i=1}^{m} \left(\sum_{j=1}^{n} y_{ij} x_j \right) c_i \tag{11}$$

Thus

$$z \leq \sum_{i=1}^{m} \left(\sum_{j=1}^{n} y_{ij} x_j \right) c_i \tag{12}$$

It is claimed that

$$\tilde{x}_i \equiv \sum_{j=1}^{n} y_{ij} x_j = x_i, \ \ i = 1,2, \cdots m \tag{13}$$

Since x is a feasible solution, $Ax = b$. Thus

$$b = \sum_{j=1}^{n} x_j a_j = \sum_{j=1}^{n} x_j (B y_j) \sum_{j=1}^{n} \left(\sum_{i=1}^{m} y_{ij} b_i \right) x_j \ \sum_{i=1}^{m} \left(\sum_{j=1}^{n} y_{ij} x_j \right) b_i = \sum_{i=1}^{m} \tilde{x}_i b_i = B\tilde{x} \tag{14}$$

Since B is non-singular and $Bx = b$, it follows that $\widetilde{x} = x$. Thus

$$z \leq \sum_{i=1}^{m} c_i x_i = z_0 \tag{15}$$

$\forall x_i$ in the feasible region.

2.3. Literature Review

Alperovits and Shamir (1977) presented a linear programming gradient (LPG) in optimizing water distribution network. Segmental length of pipe with differential diameter was used as decision making variable. The LPG method was later further improved by Kessler and Shamir (1989) who presented two stages LPG method. In the first stage, parts of the variables are kept constant while other variables are solved by LP. For a given set of flows, the corresponding sets of heads are determined by LP. In the second step, search is conducted based on the gradient of the objective function. Flows are modified according to gradient of the objective functions. Fujiwara and Khang (1990) proposed a nonlinear programming gradient (NLPG) approach that considered the flow distribution and pumping head as the decision variables and used a gradient approach to arrive at their optimal values. However, most NLP methods assume that the pipe diameters are continuous variables and hence, cannot guarantee optimality when the continuous diameters are rounded off to discrete commercially available diameter values. Werey (2000) used a dynamic programming approach to schedule pipe. However, both these approaches don't allow the designer to consider the hydraulic performance of the network.

The surplus head at a node refers to the excess of the available head at a node over the desirable head. It is assumed that the node with the minimum surplus head is the most critical in terms of the potential to fail. Hence, designs based on minimum surplus head try to maximize this critical residual head. However, this approach only considers the most critical node and doesn't consider the performance at other nodes during the periods in which the critical node fails. Further, the use of minimum surplus head as a reliability measure implies that partial flows are not considered in arriving at reliability (Nirmal Jayaram, 2006).

This research uses a LPP model which involves the design of a new network of water distribution considering the cost in the form of unit price of pipes, the hydraulic gradient and the loss of pressure.

3. The LP Model for Water Distribution Networks in Kigali

3.1. Model data

The objective of cost minimization can be obtained by employing scientific optimization techniques in order to reduce the life cycle cost of the project. One of the biggest components of cost associated with any water distribution network is the initial cost. However, a new water distribution network would have to be optimally designed to handle forecast demands at a desired level of service, throughout its service life. The cost of a water distribution network depends upon proper selection of the geometry of the network. The decision variables in the optimal design problem are the pipe diameters, which are discrete in nature. The optimal solutions to the design problem should be a set of commercially available diameters that minimize the cost of the network, while maximizing its reliability.

Table 1 gives the prices of different pipes in Rwanda Francs obtained in Kigali. The price varies based on pipe length and pipe diameter.

Table 1: Prices of pipes for different sizes of diameters in Rwanda francs (RWF)

121166.7	242333.4	363500.1	484666.8	605833.5	727000.2
189567	379134	568701	758268	947835	1137402
273533.3	547066.6	820599.9	1094133.2	137666.5	1641199.8
381458.3	762916.6	114474.9	1525833.2	1907291.5	2288749.8
670734.5	1341469	2012203.5	2682938	3353672.5	4024407
907762.3	1815524.6	2723286.9	3631049.2	4538811.5	5446573.8
1752810	3505620	5258430	7011240	8764050	10516860
2536230	5072460	7608690	10144920	12681150	15217380
6913705	13824710	20741115	27654820	34568525	41482230
8236383.3	16472766.6	2470949.9	32945533.2	41181916	49418299.8

Source: Data collected from the Energy, Water and Sanitation Authority (EWSA), Rwanda

Table 2 gives data on relationship between hydraulic gradient with pressure loss when water is distributed at a constant velocity of water of 25m/s, commonly known as head loss. The diameters of the pipes have a large effect on the internal head losses, which in turn determine the adequacy of supply at the output nodes at desirable pressures. The hydraulic gradient depends on the discharge, the pipe diameter and the Hazen William's coefficient.

Table 2: Hydraulic gradient with loss of pressure

Pressure in Pa	Hydraulic gradients in mm									
40	2.7779	1.5724	1.1856	1.2417	1.0447	1.1448	0.5695	0.4649	0.346	0.3074
70	3.3198	2.02	1.5951	1.4482	1.4423	1.3591	0.6383	0.534	0.3987	0.3545
200	8.5792	5.3193	3.9872	3.6979	2.9021	3.1358	1.6091	1.3174	0.9850	0.8750
400	19.0515	11.8749	8.9646	9.1475	6.1325	7.1195	4.0273	3.026	2.4904	2.2125
500	29.1065	17.0994	12.8108	11.1097	9.2605	9.259	5.657	4.279	3.2439	2.8154
600	41.2638	24.0239	17.9925	15.5906	12.9706	12.9639	7.9213	5.991	4.5419	4.0351

Source: Data from EWSA

3.2. Model Formulation

The objective function to minimize the cost of distribution network of clean water is:

$$\text{Min } \sum_{i=1}^{m} c_{ij}\, x_{ij}, \; j = 1,2,\dots,n \tag{16}$$

Subject to the constraints
$\sum_{i=1}^{m} a_{ij} x_{ij} \geq b_i \; j = 1,2,\dots,n$
(17)
$x_{ij} \geq 0, \; i = 1,2,\dots,m; j = 1,2,\dots,n$ (18)

Equations 16, 17 and 18 form the LPP model. x_{ij} are sizes of pipe diameters at a given section. The constraints represent the condition that the total pressure losses in a hydraulic path between a pump station or tank and every critical node (i.e. the end of the pipe network or the extreme elevation inside the network) should be less than or equal to the hydraulic gradient of the diameter with size j. These constraints are based on the minimum network pressure requirements needed for the operation of the system. Given the minimization requirement for the investment costs, the objective function is the sum of the products of the individual pipeline unit prices and their required size of diameters. By incorporating multi-demand conditions in the model we have a system of constraints for every demand pattern. When pumps are also included in the model, the main input parameter is its pump curve. The right sides of the constraints vary according to the pump's operating conditions.

Table 3: The unknown pipe diameters x_{ij} and the unit prices c_{ij} of pipeline with diameter j used in the sum $\sum_{i=1}^{m} c_{ij} x_{i,} \; j = 1,2,\dots,n$ (Equation (16))

x_i	c_{i1}	c_{i2}	c_{i3}	c_{i4}	c_{i5}	c_{i6}
x_1	121,166.7	242,333.4	363,500.1	484,666.8	605,833.5	727,000.2
x_2	189,567	379,134	568,701	758,268	947,835	1,137,402
x_3	273,533.3	547,066.6	820,599.9	1,094,133.2	137,666.5	1,641,199.8
x_4	381,458.3	762,916.6	114,474.9	1,525,833.2	1,907,291.5	2,288,749.8
x_5	670,734.5	1,341,469	2,012,203.5	2,682,938	3,353,672.5	4,024,407
x_6	907,762.3	1,815,524.6	2,723,286.9	3,631,049.2	4,538,811.5	5,446,573.8
x_7	1,752,810	3,505,620	5,258,430	7,011,240	8,764,050	10,516,860
x_8	2,536,230	5,072,460	7,608,690	10,144,920	12,681,150	15,217,380
x_9	6,913,705	13,824,710	20,741,115	27,654,820	34,568,525	41,482,230
x_{10}	8,236,383.3	16,472,766.6	2,470,949.9	32,945,533.2	41,181,916	49,418,299.8

These constraints are formulated using data given in Table 4 and Table 5 below.

Table 4: Hydraulic gradients a_{ij} used in the constraints $\sum_{i=1}^{m} a_{ij}\, x_i \;\geq b_i\; j = 1,2, \ldots, n$

a_{1j}	2.7779	1.5724	1.1856	1.2417	1.0447	1.1448	0.5695	0.4649	0.346	0.3074
a_{2j}	3.3198	2.02	1.5951	1.4482	1.4423	1.3591	0.6383	0.534	0.3987	0.3545
a_{3j}	8.5792	5.3193	3.9872	3.6979	2.9021	3.1358	1.6091	1.3174	0.985	0.875
a_{4j}	19.0515	11.8749	8.9646	9.1475	6.1325	7.1195	4.0273	3.026	2.4904	2.2125
a_{5j}	29.1065	17.0994	12.8108	11.1097	9.2605	9.259	5.657	4.279	3.2439	2.8154
a_{6j}	41.2638	24.0239	17.9925	15.5906	12.9706	12.9639	7.9213	5.991	4.5419	4.0351

Table 5: Pressure loss b_i used in the constraints $\sum_{i=1}^{m} a_{ij}\, x_i \;\geq b_i\; j =$ *1,2, ..., n*

b_1	b_2	b_3	b_4	b_5	b_6
40	70	200	400	500	600

The objective function is thus formulated as follows:

Minimize

$$(121166.7+ 242333.4 + 363500.1 + 484666.8 + 605833.5 + 727000.2\,)x_1 + (189567 + 379134 + 568701 + 758268 + 947835 + 1137402\,)x_2 + (273533.3+547066.6 + 820599.9 + 1094133.2 + 1367666.5 + 1641199.8)x_3 + (381458.3 + 762916.6 + 1144374.9 + 1525833.2 + 1907291.5 + 2288749.8)x_4 + (670734.5 + 1341469 + 2012203.5 + 2682938 + 3353672.5 + 4024407)x_5 + (907762.3+1815524.6 + 2723286.9 + 3631049.2 + 4538811.5 + 5446573.8\;)x_6 + (1752810+3505620 + 5258430 + 7011240 + 8764050 + 10516860\,)x_7 + (2536230+5072460 + 7608690 + 10144920 + 12681150 + 15217380)\; x_8 + (6913705 + 13827410 + 20741115 + 34568525 + 27654820 + 41482230)x_9 + \big(8236383.3 + 16472766.6 + 32945533.2 + 32945533.2x_{10,4} + 41181916.5 + 49418299)x_{10} \tag{19}$$

Subject to:

1. $$2.7779x_1 + 1.5724x_2 + 1.1856x_3 + 1.2417x_4 + 1.0447x_5 + 1.448x_6 + 0.5695x_7 + 0.4649x_8 + 0.346x_9 + 0.3074x_{10} \geq 40 \tag{20}$$

2. $$3.3198x_1 + 2.02x_2 + 1.5951x_3 + 1.4482x_4 + 1.1423x_5 + 1.3591x_6 + 0.6383x_7 + 0.534x_8 + 0.3987x_9 + 0.3545x_{10} \geq 70 \tag{21}$$

3. $$8.5792x_1 + 5.3193x_2 + 3.9872x_3 + 3.6979x_4 + 2.9021x_5 + 3.1358x_6 + 1.6091x_7 + 1.3174x_8 + 1.9850x_9 + 0.8750x_{10} \geq 200 \tag{22}$$

4. $$19.0515x_1 + 11.8749x_2 + 8.9646x_3 + 9.1475x_4 + 6.1325x_5 + 7.1195x_6 + 4.0273x_7 + 3.026x_8 + 2.4904x_9 + 2.2125x_{10} \geq 400 \tag{23}$$

5. $29.1065x_1 + 17.0994x_2 + 12.8108x_3 + 11.1097x_4 + 9.2605x_5 + 9.259x_6$
$+5.657x_7 + 4.2790x_8 + 3.2439x_9 + 2.8154x_{10} \geq 500$ (24)

6. $41.2638x_1 + 24.0239x_2 + 17.9925x_3 + 15.5906x_4 + 12.9706x_5 +$
$12.9639x_6 + 7.9213x_7 + 5.9910x_8 + 4.5419x_9 + 4.0351x_{10} \geq 600$ (25)

To ensure the model has an objective function we apply it to the Theorem.

Choose

$B = [b_1 \ \ b_2 \ \ b_3 \ \ b_4 \ \ b_5 \ \ b_6]$ be the first 6 columns of A (26)

$c^{\mathrm{T}} = [121166.7 \quad 189567 \quad 273533.3 \quad 381458.3 \quad 670734.5 \quad 907762.3]$ (27)

$b = [40 \ \ 70 \ \ 200 \ \ 400 \ \ 500 \ \ 600]^{\mathrm{T}}$ (28)

Then the corresponding solutions are

$x_B{}^{\mathrm{T}} = [604.0332 \quad 2440.757 \quad 707.761 \quad 9764.48 \quad 21204.33 \quad 12316.26]$ (29)

These are feasible solutions since $x_B > 0$.

$y_1 = [1\ 0\ 0\ 0\ 0\ 0\ ]^T;\ y_2 = [0\ 1\ 0\ 0\ 0\ 0\ ]^T;\ y_3 = [0\ 0\ 1\ 0\ 0\ 0\ ]^T;\ y_4 = [0\ 0\ 0\ 1\ 0\ 0\ ]^T;$
$y_5 = [0\ 0\ 0\ 0\ 1\ 0\ ]^T;\ y_6 = [0\ 0\ 0\ 0\ 0\ 1\ ]^T.$ (30)

$z_1 = 121170$, $z_2 = 189570$, $z_3 = 273530$, $z_4 = 381460$, $z_5 = 670700$
and $z_6 = 907760$. (31)

Since

$z_1 - c_1 = \ 120{,}565.9668 \ \geq 0, z_2 - c_2 = 187{,}129.243 \geq 0, z_3 - c_3 = 272{,}822.239 \geq 0, z_4 - c_4 = 371{,}695.52 \geq 0, z_5 - c_5 = 649{,}495.67 \ \geq 0$ and $z_6 - c_6 = 895{,}443.74 \ \geq 0.$ (32)

Thus x_i are optimal solutions. Hence the data verify the theorem.

4. Results

Data used in the LPP model are obtained from an existing water distribution network in Kigali City. This has been illustrated using a sample network configuration with pipes of different lengths and diameters. The diameters of pipes considered are: 1.97inches, 2.56inches, 3.14inches, 3.94inches, 4.92inches, 5.51inches, 7.87inches, 9.84inches, 11.81inches, 15.75inches and the lengths are: 100m, 200m, 300m, 400m, 500m, 600m. The model was solved using the Simplex method which was implemented on the Linear Interactive and Discrete Optimizer (LINDO) software package. The data were collected from the Energy, Water and Sanitation Authority (EWSA). The optimal solution has been found after 6 iterations and it requires RWF 38,761,640 for a new water distribution network, using pipes of 14.399366inches, 21.085608inches, 20.995722inches, 17.178293inches, 14.540589 inches of 100m of length and 0.022857 inches of 600m of length.

5. Discussions

The obtained results show reduction in the cost compared to the actual cost of the given data. For illustration purposes Figure 1 shows the cost of the existing sample pipelines in Kigali city and the reduced cost obtained from the model computations for a pipe of 2.56inches of diameter for different lengths of the pipes.

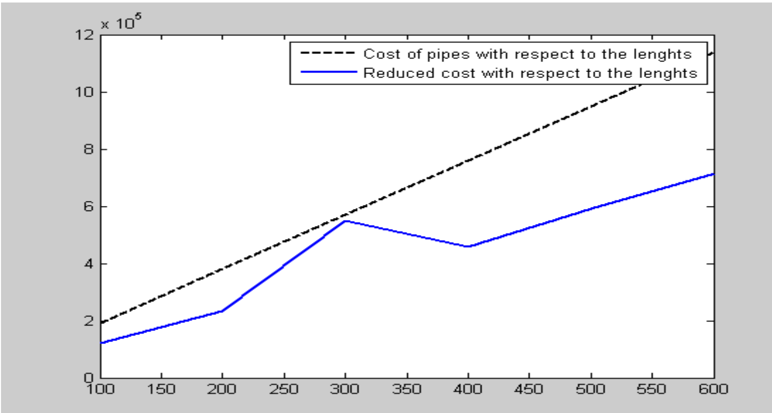

Figure 1: Actual cost and reduced cost for a pipe diameter of 2.56 inches and various lengths

6. Conclusion

This paper presents a linear programming method to solve an optimal water distribution network to new locations in Kigali City. The main aim is to achieve the optimal solution with the minimum design cost to the new locations and, at the same time, satisfy the demand with available pipe sizes, using the gradients calculated from the dual variables. The results obtained show a reduction in cost compared to the actual cost of the sampled pipelines in Kigali city. It is recommended that optimization techniques be used before designing and constructing new water distribution networks.

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