Monthly Wind Characteristics and Wind Energy in Rwanda

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Abstract

Evaluating wind power potential for a site is indispensable before making any decision for the installation of wind energy infrastructures and planning for relating projects. This paper presents a branch of a composite analysis whose objective was to investigate the potential of wind energy resource in Rwanda. Statistical methods were used to analyze long term time series of monthly daily wind speed measured on four meteorological stations in Rwanda for the period between 1981 and 1993. The Weibull distribution was used to model empirical distribution of measured long term monthly average wind speeds. The scale and shape parameters were estimated using four methods, the least square method, the likelihood method, the method of moments and the Chi-square method. It was observed that the method used for the estimates of the probability density of wind speed and giving the best overall fit to the distribution of the measured wind data varies from a location to another. However, the energy output calculated using wind speeds derived from the Chi-square method gave the best overall fit to the empirical distribution of the wind power density.

Keywords: *Wind speed- Least square method-Likelihood method- Moments Method-Chi-square method-Weibull distribution-Wind energy-Energy- Rwanda.*

1. Introduction

Recently, wind energy has been getting a lot of interest because of the focus on renewable energies. The effective use of wind energy requires having a detailed knowledge of its potential at a location. Determining of wind energy potential for selected site is made by investigating detailed knowledge of the wind characteristics, such as wind speed, direction, continuity, and availability [1-3].

In Rwanda, quite few studies have been conducted on wind energy resource and yet wind energy potential in Rwanda has not been totally exploited for power generation though potential wind power that Rwanda possesses in some parts may offer possible solutions to electricity generation, water pumping and windmill [4-5].

This paper presents a branch of a composite analysis whose objective is to investigate the potential of wind energy resource in Rwanda. The aim of this article is to model wind speed at four observatories in Rwanda. The

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observatories of Gisenyi, Kigali, Butare and Kamembe. These observatories have the highest long term wind speed data in Rwanda.

The Weibull distribution function was used to model empirical distribution of measured long term monthly average wind speeds and power density. The scale and shape parameters were estimated using four methods, the least square method, the likelihood method, the method of moments and the Chi-square method

2. Data

The National Meteorological Service is responsible for the Rwandan synoptic stations, and provides data records. Time-series of measured hourly daily wind speed and wind direction data were supplied by the National Meteorological Service. The data for the four stations, Gisenyi, Kigali, Butare and Kanombe were chosen for this study because they had the largest long term average wind speeds among the six synoptic stations in Rwanda.

Fig.1. Geographical locations of the four synoptic observatories.

Fig. 1 and **Table 1** present the geographical coordinates and elevations of the four stations of the study. All of these stations are located in the local airports with windmill type anemometers installed at 10m above ground level.

Table 1

Geographical coordinates and elevations of the four observatories of the study.

OBSERVATORY	φ	λ	h (m)
GISENYI	01°40' S	29°15' E	1,554 m
KIGALI	01°58' S	30°08' E	1,490 m
BUTARE	02°36' S	29°44' E	1,760 m
KAMEMBE	02°48' S	28°89' E	1,487 m

φ: latitude*, λ: longitude, h: elevation*

3. Methods for estimating the parameters of the Weibull distribution

3.1 The Weibull probability density function

A random variable *v*, here the wind speed, has a Weibull distribution if its probability density function is defined by [5]:

$$\begin{cases} f(v) = f(v;k,c) = \dfrac{k}{c}\left(\dfrac{v}{c}\right)^{k-1} e^{-(v/c)^k}, v > 0,\, k,c > 0 \\ f(v) = 0,\, v \le 0 \end{cases}, \tag{1}$$

where *k* is a so-called shape parameter (a dimensionless number) and *c* is scale parameter (m/s).

If $k = 2$, then we have a special case of the Weibull distribution called the Rayleigh distribution whose distribution density is:

$$f(v) = f(v;c) = \frac{2v}{c^2} e^{-(v/c)^2} . \tag{2}$$

With such a distribution, the expected value of a probability variable is:

$$\mu = \frac{c\sqrt{\pi}}{2} . \tag{3}$$

In this situation, the scale parameter *c* is in proportion to the average.

The Weibull cumulative distribution function expressing the probability that $0 \le v \le v$ is given by:

$$F(v;k,c) = \int_0^v \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-(v/c)^k} dv = 1 - e^{-(v/c)^k} \tag{4}$$

3.2 Estimation of the Weibull parameters k and c

Various methods have been developed for estimating the parameters of the Weibull probability distribution function. The most commonly used have been the Method of Moments [6, 8-9], the Maximum Likelihood Method [7], the Least Square Method [6, 9-10] and Chi-square Method [9]. However, the Maximum Likelihood Method has proved to be the most efficient [7-8] in determining the parameters of Weibull probability distribution function. In order to determine the best method to be used in the present study, the two mostly used methods in estimation of parameters, the Least Square and Maximum Likelihood have been compared.

3.2.1 The Least Square or Regression Method

The cumulative distribution function can be linearized by taking the natural logarithm of both sides of Eq. (4) twice as follows:

$$\ln\left[-\ln\{1-F(v)\}\right]=k\ln(v)-k\ln(c) \tag{5}$$

The parameters in the resulting equation are obtained using the least squares method. A plot of $\ln\left[-\ln\left[1-F(v)\right]\right]$ versus $\ln(v)$ presents a straight line. If a and b are the estimators of the intercept and slope from the regression equation, then the scale and shape parameter estimators are $\hat{k}=b$ and $\hat{c}=\exp(-a/\hat{k})$

3.2.2 Maximum Likelihood Method

Assume $(v_{1,}v_2,...,v_n)$ is a random sample with a probability density function, here the Weibull function, of the form given by Eq. (1). The likelihood function of the random sample $(v_{1,}v_2,...,v_n)$ denoted by $L(k,c,v_1,v_2,...,v_n)$ is the joint density of the variables involved, that is:

$$L(k,c,v_1,v_2,...,v_n)=\prod_{i=1}^{n}f(k,c,v_i) \tag{6}$$

Then, we have:

$$\ln L=\sum_{i=1}^{n}\ln\left[f(v_i)\right]=n\left[\ln k-k\ln c\right]+(k-1)\sum_{i=1}^{n}\ln(v_i)-c^{-k}\sum_{i=1}^{n}(v_i)^k \tag{7}$$

For n independent data $(v_{1,}v_2,...,v_n)$ of variable v, the maximum of the function Eq. (7) is determined by solving the following system of equation:

$$\begin{cases}\dfrac{\partial \ln L}{\partial k}=0\\[2mm]\dfrac{\partial \ln L}{\partial c}=0\end{cases} \tag{8}$$

Solutions of Eq.(8) must satisfy the following system of equations:

$$\begin{cases}\hat{c}=\left(\dfrac{1}{n}\displaystyle\sum_{i=1}^{n}v_i^{\hat{k}}\right)^{\frac{1}{\hat{k}}} & (a)\\[2mm]\dfrac{n}{\hat{k}}-n\ln(\hat{c})+\displaystyle\sum_{i=1}^{n}\ln(v_i)-\sum_{i=1}^{n}\left(\dfrac{v_i}{\hat{c}}\right)^{\hat{k}}\ln\left(\dfrac{v_i}{\hat{c}}\right)=0 & (b)\end{cases} \tag{9}$$

By eliminating $\hat{c}$ from the system of Eq. (9), we obtain the following equation which gives the value of $\hat{k}$ from which the value of $\hat{c}$ can be obtained by equation (9(a)):

$$\hat{k} = \left[\frac{\sum_{i=1}^{n} v_i^{\hat{k}} \ln(v_i)}{\sum_{i=1}^{n} v_i^{\hat{k}}} - \frac{1}{n} \sum_{i=1}^{n} \ln(v_i) \right]^{-1} \tag{10}$$

Equation (10) is solved with an iterative method starting with the value $\hat{k}_0$ given by [11]:

$$\hat{k}_0 = \left(\overline{v} \sqrt{\text{var}} \right)^{-1.086} , \tag{11}$$

where $\overline{v}$ and var are respectively the sample mean and variance of the series.

3.2.3 Moments' method

The first moment and second corrected moment of the two parameter Weibull distribution are obtained with the following relations [12]:

$$E(v) = c\,\Gamma(1 + \frac{1}{k}), \tag{12}$$

$$\sigma^2 = c^2 \left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right) \right]. \tag{13}$$

Therefore, given sample estimators of the mean speed $\overline{v}$, and its standard deviation s, the following can be solved iteratively for k.

$$\frac{\Gamma\left(1 + \frac{2}{\hat{k}}\right) - \Gamma^2\left(1 + \frac{1}{\hat{k}}\right)}{\Gamma^2\left(1 + \frac{1}{\hat{k}}\right)} = \frac{s^2}{\overline{v}^2} \tag{14}$$

The scale parameter c can then be estimated from:

$$\hat{c} = \frac{\overline{v}}{\Gamma(1 + \frac{1}{\hat{k}})} \tag{15}$$

3.2.4 Chi-square method

Let $\hat{F}(v_i)$ be the empirical distribution obtained from any wind speed data. Then, the parameters k and c are estimated such that:

$$\sum\left[\frac{\left(\hat{F}(v_i)-1+\exp\left(-\frac{v_i}{c}\right)^k\right)^2}{1-\exp\left(-\left(\frac{v_i}{c}\right)^k\right)}\right] \qquad (16)$$

is minimum.

3.3. Evaluation of power density distribution

The most important wind characteristic is the wind energy density. Assume A is a cross-section through which the wind of speed v flows perpendicularly. The available wind power is defined as the flow of kinetic energy which is obtained by the relation [6-7]:

$$P(v)=\left(\frac{1}{2}v^2\right)vA=\frac{1}{2}\rho v^3 A \qquad [\mathrm{W}] \qquad (17)$$

where ρ is the air density which depends on pressure (altitude), temperature and humidity. It is assumed to be constant since its variation does not affect significantly wind resource calculation [6-7]. The commonly used value is $\overline{\rho}=1.225\ kgm^{-3}$ corresponding to standard conditions (sea level, 15^0 C).

The power density distribution gives the distribution of wind energy at different wind speeds. It is obtained by multiplying the wind power density with the value corresponding to the probability density function $f(v)$ of each wind speed v as follows:

$$\frac{P(v)}{A}f(v)=\frac{1}{2}\overline{\rho}v^3 f(v) \qquad [\mathrm{Wm^{-3}s}] \qquad (18)$$

By integrating equation (17) for the period of study we obtain the mean wind power density:

$$\overline{P}=\frac{1}{2}\overline{\rho}\int_0^\infty v^3 f(v;k,c)dv=\frac{1}{2}\overline{\rho}\ \Gamma\left(1+\frac{3}{k}\right) \qquad [\mathrm{Wm^{-2}}] \qquad (19)$$

The wind speed v_{mec} at which the power density distribution is a maximum is called wind Speed of Maximum Energy Carrier. It corresponds to the mode of the power density distribution and is given by [13]:

$$v_{mec}=c(1+2/k)^{1/k} \qquad [\mathrm{m\ s^{-1}}] \qquad (20)$$

4. Computation

The four methods described above were used to estimate the scale and shape parameters. Long term distributions were estimated and compared to the long term empirical distribution of wind speed for the four observatories. The empirical distributions by month were moreover computed for estimating the shape and scale parameters by month.

A statistical programme was performed to fit the regression equation in the least square or regression method. From the regression parameters, the scale and shape parameters were estimated.
To compute the shape and scale parameters form the likelihood, Equations (7) was optimized relative to k and c and iterative method was used to solve Equations (9) and (10).
The shape and scale parameters from the moments' method were obtained by minimizing the following function with respect to k for a given sample average speed, $\overline{v}$ and its standard deviation, s .

$$G(k) = \left[\frac{\Gamma\left(1+\frac{2}{\hat{k}}\right)}{\Gamma^2\left(1+\frac{1}{\hat{k}}\right)} - \frac{s^2}{\overline{v}^2} - 1 \right]^2 \tag{21}$$

For the Chi square method, the scale and shape parameters were obtained by optimizing Equation (16). The estimates of the scale and shape parameters obtained from the least square or regression method were used as initial values for this optimization.
A simplex algorithm for function optimization was developed with a FORTRAN programme in computing the parameter values that optimize the functions in the likelihood, moments and Chi square methods.

4. Results and discussion

Long term empirical distribution and estimated Weibull distributions using the four methods for the determination of the scale and shape parameters, are presented in **Fig. 2**. Long term root mean square errors (RMSE), for the four methods used for the estimation of the Weibull distribution relative to the empirical distribution are also presented in **Table 2**. The discrepancies between distributions were very low for the four methods.

Table 2

Long term root mean square error (RMSE), for the four methods used for the estimation of the Weibull distribution relative to the empirical distribution.

STATION	METHOD			
	Likelihood	Least-Square	Moments	Chi square
GISENYI	0.037	0.035	0.033	0.032
KIGALI	0.021	0.048	0.037	0.044
BUTARE	0.109	0.136	0.126	0.134
KAMEMBE	0.023	0.021	0.021	0.022

Estimated Weibull shape and scale parameters for each observatory are presented in **Table 3** for each month. In general, the maximum likelihood method gave the highest values of the shape and scale parameters. For all observatories, the shape parameter k and scale parameter c did not vary considerably month by month. Meanwhile, for the observatory of Kigali and Butare, the highest values of the shape parameter *k* and scale parameter *c* were observed in June, July and August, months corresponding to the dry season.

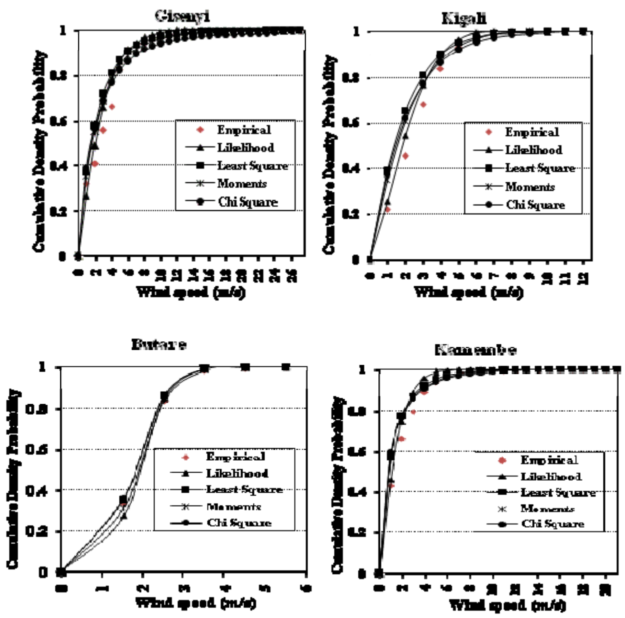

Fig. 2. Comparison of the long term Weibull Cumulative distribution function obtained by the use of the four methods of determination of the Weibull parameters to the cumulative empirical distributions.

Comparisons of monthly mean power energies obtained by the four methods to the empirical monthly mean power energies are presented in **Fig. 3** for the four observatories. Also, the annual averages of monthly mean power energy density and root mean square errors (RMSE) of monthly estimated values relative to monthly empirical values are presented in **Table 4**. It was observed that the estimate from the Chi square method gave the best fit of power energy density. **Fig. 4** gives the Monthly mean power energies determined by the Weibull wind distribution using the Chi-square method for the shape parameter k and scale parameter c (m/s).

Table 3
Weibull shape parameter k and scale parameter c (m/s) using the four methods.

Likelihood Method

STATION	GISENYI		KIGALI		BUTARE		KAMEMBE	
Months	k	c	k	c	k	c	k	c
January	1.25	2.69	1.51	2.17	2.19	2.31	1.17	1.33
February	1.31	2.72	1.48	2.11	2.29	2.31	1.26	1.37
March	1.18	2.92	1.48	2.27	2.36	2.54	1.23	1.52
April	1.22	2.87	1.52	2.23	2.56	2.43	1.19	1.56
May	1.10	3.19	1.56	2.33	2.33	2.28	1.24	1.81
June	1.17	3.11	1.64	2.56	2.56	2.33	1.23	1.95
July	1.14	3.14	1.66	2.62	2.61	2.50	1.20	2.38
August	1.01	3.30	1.66	2.86	2.55	2.56	1.23	2.09
September	1.14	3.15	1.49	2.64	2.64	2.63	1.18	1.70
October	1.16	3.15	1.36	2.60	2.43	2.61	1.17	1.43
November	1.14	2.75	1.36	2.49	2.33	2.64	1.24	1.43
December	1.16	2.47	1.43	2.51	2.24	2.42	1.26	1.32
Average	1.17	2.96	1.51	2.45	2.42	2.46	1.22	1.66

Least Square Method

STATION	GISENYI		KIGALI		BUTARE		KAMEMBE	
Months	k	c	k	c	k	c	k	c
January	0.92	2.16	1.17	1.85	1.07	1.80	0.73	0.99
February	0.97	2.14	1.13	1.82	1.37	1.83	0.86	1.10
March	0.94	2.24	1.10	1.86	1.31	1.92	0.80	1.09
April	0.92	2.25	1.10	1.87	1.50	1.90	0.78	1.10
May	0.89	2.32	1.09	1.88	1.09	1.81	0.82	1.19
June	0.97	2.39	1.11	1.96	1.27	1.89	0.90	1.34
July	0.92	2.43	1.18	2.00	1.21	1.89	0.82	1.35
August	0.88	2.48	1.24	2.05	1.05	1.79	0.83	1.40
September	0.87	2.48	1.17	2.09	1.65	1.98	0.91	1.47
October	0.87	2.49	1.04	2.05	1.21	1.94	0.82	1.36
November	0.92	2.50	1.00	2.04	1.17	1.96	0.84	1.34
December	0.89	2.43	1.03	2.08	1.23	1.97	0.83	1.30
Average	0.91	2.36	1.11	1.96	1.26	1.89	0.83	1.25

Moments Method

STATION	GISENYI		KIGALI		BUTARE		KAMEMBE	
Months	k	c	k	c	k	c	k	c
January	0.95	2.32	1.11	1.90	1.10	2.06	0.68	0.89
February	1.01	2.37	1.08	1.82	1.17	2.08	0.73	0.94
March	0.92	2.55	1.11	2.00	1.26	2.33	0.75	1.09
April	0.95	2.51	1.13	1.96	1.38	2.27	0.75	1.14
May	0.88	2.82	1.18	2.08	1.20	2.07	0.82	1.41
June	0.94	2.78	1.31	2.36	1.35	2.15	0.84	1.56
July	0.90	2.77	1.34	2.43	1.48	2.38	0.89	2.01
August	0.79	2.88	1.36	2.67	1.46	2.45	0.87	1.71
September	0.88	2.75	1.19	2.41	1.53	2.53	0.77	1.29
October	0.92	2.79	1.05	2.32	1.34	2.45	0.70	0.99
November	0.88	2.37	1.03	2.20	1.27	2.46	0.74	1.00
December	0.87	2.10	1.11	2.26	1.16	2.18	0.72	0.89
Average	0.91	2.58	1.17	2.20	1.31	2.28	0.77	1.24

Chi Square Method

STATION	GISENYI		KIGALI		BUTARE		KAMEMBE	
Months	k	c	k	c	k	c	k	c
January	0.78	2.26	1.02	1.94	1.18	1.84	0.68	0.95
February	0.79	2.26	1.02	1.90	1.20	1.84	0.70	0.97
March	0.78	2.33	1.02	1.95	1.24	1.92	0.71	1.03
April	0.77	2.35	1.03	1.96	1.30	1.94	0.70	1.06
May	0.76	2.41	1.04	2.00	1.31	1.92	0.71	1.13
June	0.76	2.44	1.06	2.06	1.34	1.91	0.73	1.21
July	0.77	2.49	1.08	2.12	1.37	1.93	0.73	1.32
August	0.76	2.53	1.09	2.19	1.39	1.96	0.74	1.37
September	0.76	2.54	1.09	2.22	1.40	1.98	0.74	1.36
October	0.76	2.56	1.09	2.24	1.40	1.99	0.73	1.33
November	0.76	2.52	1.09	2.24	1.39	2.01	0.73	1.30
December	0.76	2.48	1.09	2.24	1.16	2.18	0.72	1.26
Average	0.77	2.43	1.06	2.09	1.31	1.95	0.72	1.19

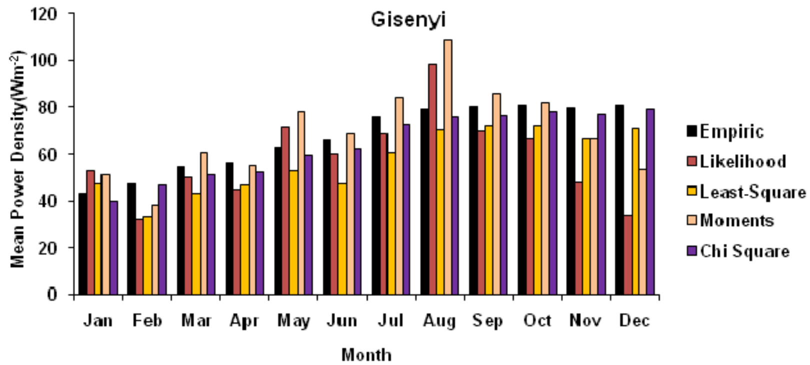

Fig. 3 a. Comparison of monthly mean power energies obtained by the four methods to the empirical monthly mean power energies for the observatory of Gisenyi.

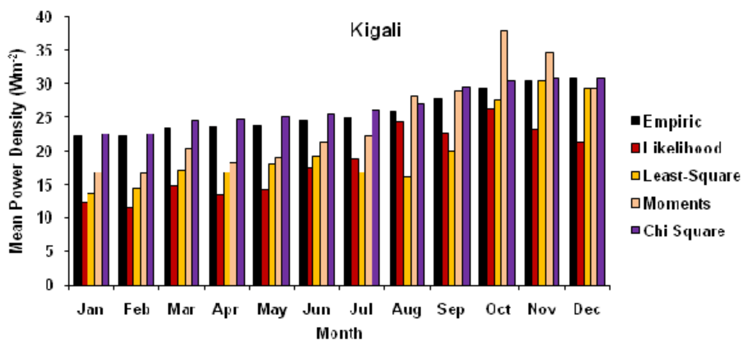

Fig. 3 b. Comparison of monthly mean power energies obtained by the four methods to the empirical monthly mean power energies for the observatory of Kigali

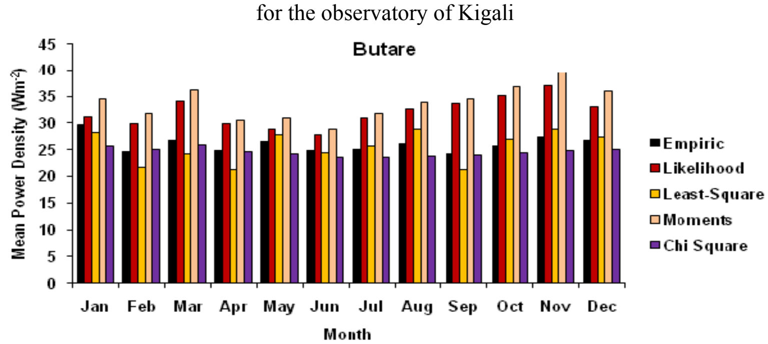

Fig. 3 c. Comparison of monthly mean power energies obtained by the four methods to the empirical monthly mean power energies for the observatory of Butare.

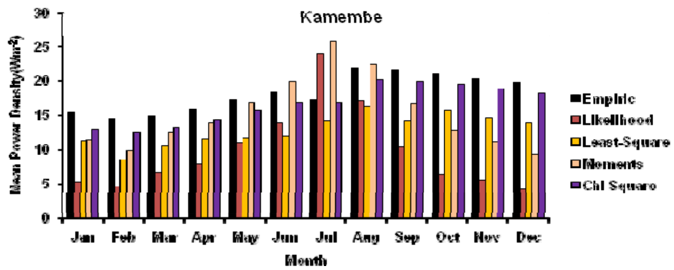

Fig. 3 d. Comparison of monthly mean power energies obtained by the four methods to the empirical monthly mean power energies for the observatory of Kamembe.

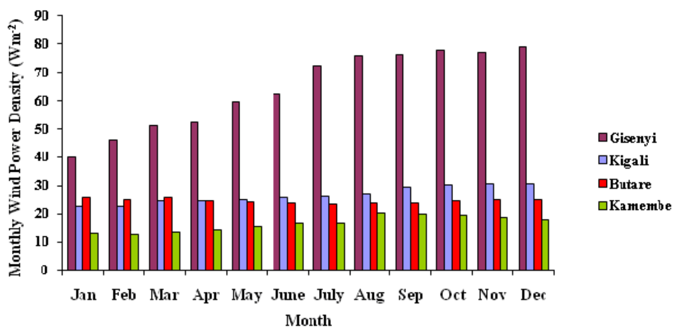

Fig. 4. Monthly mean power energies determined by the Weibull wind distribution using the Chi-square method for the shape parameter k and scale parameter c (m/s).

Table 4

Annual averages of monthly mean power energy and root mean square errors (RMSE) of monthly estimated values relative to monthly empirical values (W/m^2).

STATION		METHOD							
	EMPIRIC	LIKELIHOOD		LEAST SQUARE		MOMENTS		CHI SQUARE	
	Average	Average	RMSE	Average	RMSE	Average	RMSE	Average	RMSE
GISENYI	67.20	57.92	19.58	56.77	11.73	69.38	14.08	64.20	3.11
KIGALI	25.73	18.39	7.86	19.94	6.52	24.48	4.42	26.62	1.02
BUTARE	26.04	31.98	9.34	25.55	3.06	33.76	7.09	24.54	0.55
KAMEMBE	18.14	9.69	10.25	12.82	5.43	15.17	5.81	16.55	1.66

5. Conclusion

Wind data from four observatories in Rwanda were modeled using the two parameter Weibull distribution function. The parameters have been estimated using the least square or regression method, the maximum likelihood method, the moments' method and the chi-square method. It was found that in general the maximum likelihood method gave better estimates for the Weibull shape and scale parameters. Meanwhile the Chi square method gave the best fit of power energy density.

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Appendix

Wind characteristics for the four observatories.

GISENYI OBSERVATORY												
Number of Observation	N											
	J	F	M	A	M	J	J	A	S	O	N	D
	6626	5710	7077	6868	6714	6869	7475	7433	6388	7380	7208	7403
Empirical Mean Wind Speed (m/s)	$< v >_{emp} = \sum_{i=1}^{N} v_i f_i / \sum_{i=1}^{N} f_i$											
	J	F	M	A	M	J	J	A	S	O	N	D
	2.9	2.9	3.0	3.0	3.1	3.1	3.2	3.3	3.3	3.3	3.3	3.2
Empirical Standard Deviation of Wind speed (m/s)	$\sigma_{emp} = [\frac{1}{N-1} \sum_{i=1}^{N} (v_i - < v >_{emp})^2 f_i]^{1/2}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	2.5	2.4	2.6	2.6	2.8	2.8	2.9	3.1	3.1	3.1	3.1	3.0
Weibull Mean Wind Speed (m/s)	$< v >_w = c\Gamma(1 + 1/k)$											
	J	F	M	A	M	J	J	A	S	O	N	D
	2.5	2.5	2.7	2.6	3.1	2.9	3.0	3.3	3.0	3.0	2.6	2.3
Weibull Standard Deviation of Wind speeds (m/s)	$\sigma_W = c[\Gamma(1 + 2/k) - \Gamma^2(1 + 1/k]^{1/2}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	2.0	1.9	2.3	2.2	2.8	2.5	2.6	3.3	2.7	2.6	2.3	2.0
Wind speed of maximum energy carrier (m/s)	$v_{mec} = c(1 + 2/k)^{1/k}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	5.8	5.5	6.8	6.4	8.2	7.3	7.6	9.8	7.7	7.5	6.7	5.9
Empirical Power Density (W/m^2)	$\overline{P}_{emp} = \sum_{i=1}^{N} 1/2 \rho v_i^3 f_i / \sum_{i=1}^{N} f_i$											
	J	F	M	A	M	J	J	A	S	O	N	D
	42.9	47.3	54.2	56.3	62.6	66.5	75.6	79.0	80.2	81.0	79.9	80.9
Weibull Power Density (W/m^2)	$\overline{P}_W = 1/2 \rho\Gamma(1 + 3/k)$											
	J	F	M	A	M	J	J	A	S	O	N	D
	39.9	46.3	51.2	52.3	59.6	62.5	72.6	76.0	76.2	78.0	76.9	78.9

KIGALI OBSERVATORY

Number of Observation	N											
	J	F	M	A	M	J	J	A	S	O	N	D
	6626	5710	7077	6868	6714	6869	7475	7433	6388	7380	7208	7403
Empirical Mean Wind Speed (m/s)	$<v>_{emp} = \sum_{i=1}^{N} v_i f_i / \sum_{i=1}^{N} f_i$											
	J	F	M	A	M	J	J	A	S	O	N	D
	2.3	2.3	2.3	2.3	2.4	2.4	2.5	2.5	2.6	2.6	2.6	2.6
Empirical Standard Deviation of Wind speed (m/s)	$\sigma_{emp} = [\frac{1}{N-1} \sum_{i=1}^{N} (v_i - <v>_{emp})^2 f_i]^{1/2}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	1.6	1.6	1.7	1.7	0.9	1.7	1.7	1.7	1.7	1.8	1.8	1.8
Weibull Mean Wind Speed (m/s)	$<v>_w = c\Gamma(1+1/k)$											
	J	F	M	A	M	J	J	A	S	O	N	D
	2.0	1.9	2.1	2.0	2.1	2.3	2.3	2.6	2.4	2.4	2.3	2.3
Weibull Standard Deviation of Wind speeds (m/s)	$\sigma_W = c[\Gamma(1+2/k) - \Gamma^2(1+1/k]^{1/2}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	1.3	1.3	1.4	1.4	1.4	1.4	1.5	1.6	1.6	1.8	1.7	1.6
Wind speed of maximum energy carrier (m/s)	$v_{mec} = c(1+2/k)^{1/k}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	3.8	3.7	4.1	3.9	3.9	4.2	4.2	4.6	4.7	5.1	4.9	4.6
Empirical Power Density (W/m^2)	$\overline{P}_{emp} = \sum_{i=1}^{N} 1/2\rho v_i^3 f_i / \sum_{i=1}^{N} f_i$											
	J	F	M	A	M	J	J	A	S	O	N	D
	22.3	22.1	23.4	23.6	23.7	24.4	24.9	26.0	27.8	29.4	30.4	30.9
Weibull Power Density (W/m^2)	$\overline{P}_W = 1/2\rho\Gamma(1+3/k)$											
	J	F	M	A	M	J	J	A	S	O	N	D
	22.6	22.5	24.4	24.6	25.0	25.6	26.1	27.0	29.6	30.5	30.8	30.7

BUTARE OBSERVATORY

Number of Observation	N											
	J	F	M	A	M	J	J	A	S	O	N	D
	6626	5710	7077	6868	6714	6869	7475	7433	6388	7380	7208	7403
Empirical Mean Wind Speed (m/s)	$< v >_{emp} = \sum_{i=1}^{N} v_i f_i / \sum_{i=1}^{N} f_i$											
	J	F	M	A	M	J	J	A	S	O	N	D
	2.7	2.6	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.8	2.8	2.8
Empirical Standard Deviation of Wind speed (m/s)	$\sigma_{emp} = [\frac{1}{N-1}\sum_{i=1}^{N}(v_i - < v >_{emp})^2 f_i]^{1/2}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	1.7	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
Weibull Mean Wind Speed (m/s)	$< v >_w = c\Gamma(1 + 1/k)$											
	J	F	M	A	M	J	J	A	S	O	N	D
	2.0	2.0	2.2	2.2	2.0	2.1	2.2	2.3	2.3	2.3	2.3	2.1
Weibull Standard Deviation of Wind speeds (m/s)	$\sigma_W = c[\Gamma(1 + 2/k) - \Gamma^2(1 + 1/k]^{1/2}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	1.0	0.9	1.0	0.9	0.9	0.9	0.9	1.0	1.0	1.0	1.1	1.0
Wind speed of maximum energy carrier (m/s)	$v_{mec} = c(1 + 2/k)^{1/k}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	3.1	3.0	3.3	3.0	3.0	2.9	3.1	3.2	3.3	3.3	3.4	3.2
Empirical Power Density (W/m^2)	$\overline{P}_{emp} = \sum_{i=1}^{N} 1/2\rho v_i^3 f_i / \sum_{i=1}^{N} f_i$											
	J	F	M	A	M	J	J	A	S	O	N	D
	29.7	24.6	26.8	24.8	26.4	24.8	25.1	26.2	24.2	25.8	27.3	26.8
Weibull Power Density (W/m^2)	$\overline{P}_W = 1/2\rho\Gamma(1 + 3/k)$											
	J	F	M	A	M	J	J	A	S	O	N	D
	25.6	25.0	25.9	24.7	24.3	23.6	23.6	23.8	23.9	24.4	24.8	25.0

KAMEMBE OBSERVATORY

Number of Observation	N											
	J	F	M	A	M	J	J	A	S	O	N	D
	6626	5710	7077	6868	6714	6869	7475	7433	6388	7380	7208	7403
Empirical Mean Wind Speed (m/s)	$< v >_{emp} = \sum_{i=1}^{N} v_i f_i / \sum_{i=1}^{N} f_i$											
	J	F	M	A	M	J	J	A	S	O	N	D
	2.7	2.6	1.7	1.7	1.8	1.7	2.0	2.0	2.0	2.0	2.0	1.9
Empirical Standard Deviation of Wind speed (m/s)	$\sigma_{emp} = [\frac{1}{N-1} \sum_{i=1}^{N} (v_i - < v >_{emp})^2 f_i]^{1/2}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	1.7	1.6	1.7	1.7	1.7	1.7	2.0	2.0	2.0	2.0	1.9	1.9
Weibull Mean Wind Speed (m/s)	$< v >_w = c\Gamma(1+1/k)$											
	J	F	M	A	M	J	J	A	S	O	N	D
	1.3	1.3	1.4	1.5	1.7	1.8	2.2	2.0	1.6	1.4	1.3	1.2
Weibull Standard Deviation of Wind speeds (m/s)	$\sigma_W = c[\Gamma(1+2/k) - \Gamma^2(1+1/k]^{1/2}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	1.1	1.0	1.2	1.2	1.4	1.5	1.9	1.6	1.4	1.2	1.1	1.0
Wind speed of maximum energy carrier (m/s)	$v_{mec} = c(1+2/k)^{1/k}$											
	J	F	M	A	M	J	J	A	S	O	N	D
	3.1	2.9	3.3	3.6	3.9	4.3	5.4	4.6	3.9	3.4	3.1	2.8
Empirical Power Density (W/m^2)	$\overline{P}_{emp} = \sum_{i=1}^{N} 1/2 \rho v_i^3 f_i / \sum_{i=1}^{N} f_i$											
	J	F	M	A	M	J	J	A	S	O	N	D
	15.4	14.5	14.9	16.0	17.2	18.4	17.2	21.7	21.5	20.9	20.3	19.7
Weibull Power Density (W/m^2)	$\overline{P}_W = 1/2 \rho \Gamma(1+3/k)$											
	J	F	M	A	M	J	J	A	S	O	N	D
	12.9	12.4	13.2	14.3	15.6	16.8	16.8	20.1	19.9	19.4	18.8	18.1