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STOCHASTIC DYNAMIC LOT-SIZING MODEL WITH SHORTAGE AND DISTRIBUTION COSTS: APPLICATION IN A SINGLE-ITEM MANUFACTURING COMPANY

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ABSTRACT

In this work, a dynamic lot-size model with probabilistic normally distributed demand is presented. The model is an extension of the stochastic version of the Wagner-Whitin dynamic lot-size model. The Wagner-Whitin model provides an algorithm for minimizing the production and inventory cost of an item over N time periods. The cost function of the model presented in this paper comprises of fixed set-up cost, variable production cost, holding cost, shortage cost as well as distribution cost. Production of the item is considered to be instantaneous with fixed set-up cost and per unit production cost in any given period. The model presented was illustrated with data collected from a single-item production company, Boltsman Nig. Ltd. With a normally distributed demand for the item and with set-up, production, holding, shortage and distribution costs, an optimal lot-sizing plan that satisfies the demand over a 12-period time interval at minimum total cost was desired. The EXCEL software was used to analyze the data using the backward dynamic programming algorithm. The optimal production, inventory and distribution plan was obtained which satisfied expected demand while minimizing production, inventory and distribution costs in some periods. Also it minimized shortage cost in periods where the expected demands cannot be met. The optimal minimum cost policy of ₦43,622,570.00 was obtained.

Keywords: Dynamic programming, lot sizing, shortage cost, probabilistic demand, inventory cost.

INTRODUCTION

Production management involves planning and monitoring the processes related to quantity of product manufactured, distributed and stocked at any given time. This is the lot-sizing problem. Lot-sizing in production involves the determination of the size of production, and the timing of such decision to satisfy the requirements of demand over a defined future horizon. The main concern of this class of problems is to determine production or procurement lots for the given product over a finite (in the case of dynamic demand) or infinite (in the case of static demand) planning horizon so as to minimize the total cost while known demand is satisfied. The total relevant cost generally consists of set-up costs, inventory holding costs and production or procurement costs. The Wagner-Whitin (1958) model is considered a classical model with respect to the lot-sizing problem. It permits the determination of optimal lot sizes for single item when demand, inventory holding charges, and set-up cost vary over N periods of time. The solution provided by the Wagner-Whitin algorithm is considered as benchmark or standard against which other lot-sizing rules and heuristics are judged. Since the development of the dynamic lot-size algorithm by Wagner-Whitin in 1958 , many researchers have expanded the application of this dynamic lot-size model by either adding other relevant costs that are incurred in the production process or for cases with stochastic demand, lead time and shortages. Sharma and Ali (2017) reduced the lot-sizing problem with set-up, production, shortage and inventory costs to a lot sizing problem with set-up, production and inventory costs. Chand, Hsu and Deshpande (2007) considered the lot-sizing problem faced by a producer who supplies a single product to multiple customers characterized by backorder as well as shipping costs. Alfares and Turnadi (2016) proposed a general model for single item lot-sizing problem with multiple suppliers, quantity discount and backordering of shortages. Li, Hsu and Xiao (2003) in their paper studied two important variants of the dynamic economic lot-sizing problem. This include a model in which production in each time period is restricted to a multiple of a constant batch size where backlogging is allowed and all cost parameters are time varying. The first model was used to develop a second model in which the product acquisition cost structure, including a fixed charge for each acquisition, a variable unit production cost and a freight cost with truckload discount. Asken, Altinkemer and Chand (2003) introduce a profit maximization version of the well-known Wagner-Whitin model for the deterministic incapacitated single-item lot-sizing problem with lost sales. Cost and selling prices were assumed to be time variant. Gonzalez and Tullous (2004) in their paper extended the Wagner-Whitin N period's lot-sizing model by considering backorders. The problem was formulated as a transshipment network model and then reformulated as a transportation model in order to facilitate the process of getting the optimal solution by using commonly available tools. Lasserre, Bes and Roubellant (1985) investigated the stochastic dynamic lot size model with probabilistic state variable constraints. They solved new equivalent deterministic dynamic lot size problem with new parameters using the open loop approach. Sankar (2010) investigated an economic production lot-size (EPL) model in an imperfect production system in which the production facility may shift from an in-control state to an out-of-control state at any random time. Wakinaga and Sawaki (2008) considered a dynamic lot-size model for the case where single-item is produced and shipped by an overseas export company. They explored an optimal production scheduling with the constraint of production and production capacity so as to minimize the total cost over the planning horizon. Sidhoum and Absi (2008) addressed a multi-item capacitated lot-sizing problem with set-up times and shortage costs with demand that cannot be backlogged but has total and partial cost. Asken (2005) studied the loss of customer goodwill in incapacitated single-item lot sizing with a mixed integer programming model extending the well-known Wagner-Whitin model. Meanwhile Iwuji, Umezurike and Isaac (2019) extended the stochastic version of the Wagner-Whitin dynamic lot-size to include supply cost where products are

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supplied to multiple supply points. The demand was normally distributed and unfulfilled demand (i.e. shortage) in any given period is partially backlogged where the backlogging functions invloves the normal loss function. The cost structure included fixed set-up cost, variable production cost, inventory holding cost, backlogging cost and supply cost. The production and inventory levels had upper bounds while the backlogging cost was constant through all time periods. Using the EXCEL solver an optimal production-inventory schedule for the planning horizon that minimizes total expected costs was obtained using the forward dynamic programming algorithm. In this paper we present a dynamic lot-size model with probabilistic normally distributed demand with shortage and distribution cost. The unfulfilled demand in each period (i.e. shortage) is not backlogged bot incurs a shortage cost. The model is an extension of the stochastic version of Wagner-Whitin dynamic lot-size model. The cost function of the model comprises of production, holding, shortage and distribution costs.

MATERIALS AND METHODS

The Wagner-Whitin Dynamic Lot sizing model.

Suppose a manufacturer produces an item and anticipates a known deterministic demand for the items scheduled over N periods. Producing the item requires a setup cost while producing in excess of demand incurs an inventory cost. The manufacturer will intend to know the quantity of the item to produce in order to minimize total cost. The Wagner-Whitin (1958) dynamic lot-sizing algorithm presents a minimum cost policy for periods t through N as

$$f_t(I) = \text{Min}\,[i_{t-1}I + \delta(x_t)s_t + f_{t+1}(I + x_t - d_t] \quad (1)$$

Given that $I = I_0 + \sum_{j=1}^{t-1} x_j - \sum_{j=1}^{t-1} d_j \geq 0$ and $I + x_t \geq d_t$ (2)

$$\text{and } \delta(x_t) = \begin{matrix} 0 \text{ if } x_t = 0 \\ 1 \text{ if } x_t > 0 \end{matrix} \quad (3)$$

where $d_t =$ amount demanded

$I_0 =$ Inventory in period $\text{t} - 1$

$I =$ Inventory at beginning of period t

$i_t =$ interest charge per unit of inventory carried forward to period $\text{t} + 1$

$S_t =$ ordering (or Setup)cost

$x_t =$ amount ordered (or manufactured)

$t = 1,2, \dots, N$

Dynamic lot-sizing model with backlogged normally distributed demand

Iwuji *et al.* (2019) extended the stochastic version of the Wagner-Whitin lot-sizing model to accommodate supply cost to multiple supply points as well as backlogging cost. The backlogging cost involved a constant penalty cost for the partially backlogged demand which is a function of the normal loss function. The total available quantity for each period t is supplied to the supply points in fixed proportion in a prioritized order of higher demand centers before lower demand centers. The model intends to minimize the total cost in all time period which comprises of production cost, inventory cost, backlogging cost as well as supply cost in the planning horizon. The model was presented as follows;

$$Min\, T(C) = A + u_t p_t + \sum_{t=1}^{n} h_t \sigma_t \{z_t + I_N(z_t)\} + Y_t \sum_{t=1}^{n} \pi_t \sigma_t I_N(z_t) + \sum_{t=1}^{n} q_{it}\, c_{it} \quad (4)$$

Subject to $E(I_t) = \sigma_t z_t$, $E(b_t) = \sigma_t I_N(z_t)$, $z_t = \frac{Q_t - \mu_t}{\sigma_t}$ (5)

$$I_{t+1} = P_t + E(I_t) - \sum_{i=1}^{m} q_{it} = b_t \quad (6)$$

$$\sum_{i=1}^{m} q_{it} = Q_t = P_t + xI_t + yb_t \quad (7)$$

$$q_{it} = \alpha_{it} Q_t \quad (8)$$

where

A = fixed production set − up cost
u_t = unit production cost
π_t = backlogging cost
h_t = holding cost
σ_t = standard deviation of demand in period t
c_{it} = unit cost of supplying product to point i in period t
α_{it} = proportion of total quantity available supplied to point i in period t
$I_N(z_t)$ = normal loss function

Dynamic lot-sizing model with probabilistic demand with shortage and distribution costs.

The lot-sizing model presented in this work is an extension of the Wagner-Whitin (1958) dynamic lot-sizing model with shortage and distribution cost. It is also similar to the dynamic lot size model by Iwuji *et al.* (2019) except that the unfulfilled demand in each period (i.e. shortage) is not backlogged but rather incurs a shortage cost. It considers a T period planning horizon with normally distributed demand that is assumed to be distributed. The model minimizes the total cost which comprises of the production cost, inventory cost, shortage cost and distribution cost. Also production of the item is considered to be instantaneous with fixed set-up cost and per unit production cost in any given period together with bounds for the production and inventory levels. The dynamic lot-sizing model with probabilistic demand and with shortage and distribution is presented as follows;

Given the following notations,

T = number of periods in planning horizon
A_t = setup cost (fixed cost)of production in period t
h_t = unit holding cost in period t
c_t = unit production cost in period t
d_t = demand in period t
P_t = production quantity in period t
I_t = holding level at begginning of period t
I_{t+1} = end − of − period inventory for period t
S_t = unit shortage cost at period t
X_t = shortage quantity at period t
D_{it} = unit distribution cost from production factory to customer i, $1 \le i \le$ m
Y_{it} = units of product distributed in period t to satisfy demand of customer i, $1 \le i \le$ m
G_t = total units of product distributed in period t
Q_{it} = percentage of total demand in period t distributed to satisfy demand of customer i, $1 \le i \le$ m
R_{it} = unit cost to transport one unit of product to customer i in period t , $1 \le i \le$ m
w_t = upper bound on production in all periods.
u_t = lower bound on inventory in all periods.
v_t = upper bound on inventory in all periods.
d_{pt} = demand in period t
μ_t = mean demand in period t in

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previous years

σ_t = standard deviation of demand in period t in previous years

TC = Total cost

The dynamic programming model for the probabilistic lot-sizing problem minimum total cost policy is presented as follows;

Minimize TC = $[(A_t + c_tP_t) + E(h_tI_t) + E(X_tS_t) + E(Y_{it}D_{it}) + f_{t+1}(P_t + I_t + X_t - d_t)]$ (9)

Subject to the constraints

$P_t + I_t + X_t = d_t + I_{t+1}$ (10)

$P_t \leq w_t$ (11)

$u_t \leq I_t \leq v_t$ (12)

$\sum_{i=1}^{m} Y_{it} = G_t$ (13)

$G_t \leq w_t + I_t \leq d_t + I_{t+1}$ (14)

$\sum_{i=1}^{m} Q_{it} = G_t$ (15)

with $I_{t+1} \begin{cases} = 0, & \text{if } P_t + I_t \leq d_t \\ > 0, & \text{if } P_t + I_t > d_t \end{cases}$ (16)

$X_t \begin{cases} = 0, & \text{if there is no shortage in period } t \\ > 0, & \text{if there is shortage in period } t \end{cases}$ (17)

The demand in period t, d_t, is assumed to be normally distributed, i.e. $P(d_t) \sim N(\mu, \sigma)$.

The demand is converted to its standard normal variate, $N(0,1)$, using $z = \frac{d_t - \mu_t}{\sigma_t}$.

So then $P(d_t) \sim N(0,1)$ thereafter. Then expected demand in period t is obtained as follows;

$E(d_t) = d_{pt}P(z)$ (18)

In the lot-sizing model formulated above, the cost function (9) includes the set-up cost, the production cost, the inventory holding cost, the shortage cost and the distribution cost. Constraint (10) represents the end-of-period inventory equation. Constraints (12) and (13) indicates the production upper bound and the lower and upper bound of inventory respectively. Constraint (14) indicates that the sum of units of products distributed to each customer sums to total quantity distributed in the period. Constraint (15) indicates that the total quantity of product distributed in period t cannot exceed the sum of production upper bound and beginning of period inventory which in turn cannot exceed the sum of demand and end of period inventory. Constraints (17) indicate the scenarios when there is inventory at beginning of next or not. Also constraint (18) indicates the scenarios when we have or do not have shortages in a period.

Assumptions of the model.

The following assumptions are made with respect to the model

(i) There is a fixed set-up cost associated with production.

(ii) The unit production cost is known and constant throughout all periods.

(iii) The unit inventory cost is known and constant throughout all periods.

(iv) There is an upper and lower bound of inventory level throughout all periods.

(v) There is an upper bound production level throughout all periods.

(vi) Backlogging is not allowed.

(vii) Shortage cost is loss of supposed profit from sales and is constant throughout all periods.

(viii) Demand is probabilistic and is assumed to be normally distributed.

(ix) Unit distribution cost to each customer is constant throughout all periods.

Data Illustration

Data analysis.

The lot-sizing model with shortage and distribution cost formulated above is illustrated using data collected from a single-product manufacturing company, Boltzman Nigeria Ltd for 12 time period. The company produces and distributes ice cream to five distribution centers which includes Aba, Umuahia, Port-Harcourt, Uyo and Benin city. The expected demand of the current year for the product in the 12 periods, as presented in table 1, was obtained from previous demand in the last five years. Also table 2 contains the estimated set-up cost, unit production and inventory costs, upper and lower monthly production and inventory bounds, unit distribution cost to the different cities as well as percentage of total product in a month distributed to the different cities. The data was collected from the production and distribution departments of the factory.

Table 1. Expected demand of products in the different periods

	PERIODS											
	1	2	3	4	5	6	7	8	9	10	11	12
$E(d_t)$ (cartons)	8900	8100	9400	6700	5200	5200	5800	6100	5000	9700	6000	6900

Table 2. Unit supply costs, proportion of products to different cities and bounds for production and inventory.

Unit inventory cost in each period		₦120
Unit production cost per period		₦300
Set-up cost in each period		₦300,000
Inventory bound in each period		$0 \leq I_t \leq 300$
Production bound in each period		$0 < P_t \leq 7300$
Supply cost per carton of product to each supply point.	Aba	₦25
	Umuahia	₦86
	Port-Harcourt	₦87
	Akwa Ibom	₦200
	Edo	₦200
Proportion of total available product supplied to each supply point.	Aba	40%
	Umuahia	14%
	Port-Harcourt	23%
	Akwa Ibom	9%
	Edo	14%

RESULTS

The minimum cost lot-sizing policy of periods 1 through 12 using the backward dynamic programming algorithm is given in table 3. The optimal solution was obtained using the EXCEL software. From the analysis there were shortages in periods 1,2,3 and 10 incurring shortage costs in addition to the production and distribution costs. There were no shortage cost in the other periods. Also we can see inventory holding costs were incurred in periods 4 through 9 as

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well as periods 11 and 12 as there were end-of-period inventory in those periods. This actually incurred inventory costs in those periods in addition also to production and distribution costs. There were no inventory cost for the other periods.

Table 2. Optimal production and inventory policy

Period	I_t	P_t	X_t	G_t	I_{t+1}	COSTS (Naira)					
	(cartons of ice cream)					Fixed	Production	Inventory	Shortage	Distribution	Total
1	300	7300	1300	7500	0	300000	949000	99900	130000	783271.2	2162171.2
2	300	7300	500	7500	0	300000	949000	99900	50000	712864.8	2011764.8
3	300	7300	1800	7500	0	300000	949000	99900	180000	827275.2	2256175.2
4	300	6700	0	7000	300	300000	871000	99900	0	589653.6	1760553.6
5	300	5200	0	5500	300	300000	676000	99900	0	457641.6	1413541.6
6	300	5200	0	5500	300	300000	457641.6	99900	0	457641.6	1413541.6
7	300	5800	0	6100	300	300000	754000	99900	0	510446	1564346.4
8	300	6100	0	6400	300	300000	793000	99900	0	536848.8	1629748.8
9	300	5000	0	5300	300	300000	650000	99900	0	440040	1389940
10	300	7300	2100	7500	0	300000	949000	99900	210000	853677.6	2312577.6
11	300	6000	0	6300	300	300000	780000	99900	0	528048	1607948
12	300	6700	0	7000	300	300000	871000	99900	0	607255.2	1778155
TOTAL COST						3600000	9648642	1198800	570000	7304664	43,622,570

The fixed production cost, the variable production cost, the inventory cost, the shortage cost as well as the distribution costs through all periods were ₦3600000, ₦9648642, ₦1198800, ₦570000 and ₦7304664. The sum of these costs gives an optimal total minimum cost policy of ₦43,622,570.00 at the end of the planning horizon.

DISCUSSION.

The optimal production-inventory solution obtained in this work, in which unfulfilled demand is not backlogged but incurs shortage cost, had shortages in four periods which occurred mostly in the beginning periods of the planning horizon. The total shortage cost at the end of the planning horizon is ₦570,000 while the optimal minimum total cost policy is ₦43,622,570. In the work of Iwuji *et al.* (2019), in which the unfulfilled demand is partially backlogged, the shortage cost which was ₦105,000 which appears to be lower than that in this work. This of course might be as a result of backlogging of unfulfilled demand, though its optimum total cost was ₦53,936,535. So the optimal minimum cost policy in this work is seen to be lower than the optimal minimum cost policy for the case in which the unfulfilled demand is partially backlogged. Hence we can say that the optimal production-inventory policy incurs a lower minimum cost policy when the unfulfilled demand is not backlogged but incurs a constant shortage cost than for the case in which the unfulfilled demand is partially backlogged with the backlogging cost including the normal loss function. This of course is based on the data used in both works.

CONCLUSION

In this paper we present a dynamic lot-size model with probabilistic normally distributed demand. The model is an extension of the stochastic version of Wagner-Whitin dynamic lot-size with shortage and distribution cost. The classical Wagner-Whitin dynamic lot-sizing model considers just the procurement (manufacturing) and holding costs. Hence it is extended here to accommodate a shortage cost as well as a distribution cost. . The cost function of the model comprises of production, holding, shortage and distribution costs. Also production of the item is considered to be instantaneous with fixed set-up cost and per unit production cost in any given period. The proposed model is thereafter illustrated with data collected from a single-item company, Boltzman Nig. Ltd that produces and distributes ice-cream to five distribution centers. After analyzing the data using the EXCEL software an optimal minimum cost policy of ₦43,622,570 was obtained at the end of the planning horizon.

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