OPTIMAL LOCATION OF DISTRIBUTED GENERATION FOR LOSS MINIMIZATION IN DISTRIBUTION SYSTEM

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ABSTRACT

Accurate loss minimization is the critical component for efficient electrical power flow. The introduction of distributed generation (DG) in networks is bound to have significant effect in network losses. Such dispersed generators can reduce distribution system loss by appropriate allocation. With the integration of DG into an existing utility, Line Loss reduction can be achieved. In order to demonstrate system loss minimization, distributed generator units (DGs) of trans-Amadi gas turbine and Omoku gas turbine generators were selected and sparsely located into the low-voltage buses (11kv) of the existing distribution substations near electrical consumers in the sampled Port Harcourt transmission/distribution network fed by Afam power station. A proposed line loss reduction index (LLRI) was implemented on the ten-bus test system using MATLAB 7.5 software. The sample system used to test the programme produced a reduction in line losses of 19MW. The minimum values of LLRI is 0.8323 and 0.8326 which correspond to the optimal DG location scenario in terms of line loss reduction

INTRODUCTION

Distributed Generation (DG) is the use of small, decentralized, off-grid or modular, grid connected generators spotted throughout a power system network, providing the electricity locally to load customers. The key element of this new environment is to build and operate several DG units near load centers instead of expanding the central station power plants located far away from customers to meet increasing load demand. Integrating distributed generation (DG) into an existing utility results in several benefits such as increased overall system efficiency, reduced line losses, improved system profile and transmission and distribution capacity relief to both utilities and the customers.

Among the many benefits of distributed generation is a reduced line loss. The loss can be significant because large voltage drop and high distribution losses along the lines experienced by high density load areas and rural electrification networks results to poor network performance and poor quality of power supply to customers. The factor of the optimal location is one of the important issues in the implementation of distributed generation for loss minimization in distribution system.

DG can be powered by both conventional and renewable energy sources. Technologies that utilize conventional energy sources includes turbines. micro turbines. internal gas combustion engines (IC). There are many new technologies that utilize renewable energy resources such as biomass system, PVs, solarelectric wind electric thermal system, conversion systems (WECS), and geothermal systems. The rating of DG can range from a few kilowatts up to 100MW). While smaller units (a few kilo-watts to a few megawatts) are typically installed in distribution networks.

Considerable research and investigation has proceeded in the area of optimal DG unit placement for loss minimization in distribution system. Joos et al (2002) have demonstrated the potential of DG with power electronic interface to provide ancillary services such as reactive power, voltage sag compensation and harmonic filtering. It has proved the ability of DG to compensate voltage sag resulting from faults in the power system. However the method did not analyze the amount of power loss reduction due to DG installation. Chiradeja et al (1999) have evaluated a probabilistic approach based on convolution technique to quantify the benefit of voltage profile improvement involving wind turbine generation. Hagazy et al (2003) have presented a Monte Carlo-based method for the adequacy assessment of distributed generation systems. Macken et al (2004) have presented solutions to prevent sensitive equations from disruptive operation by making use of DG in the pressure of voltage dips. Pregelj et al (2004) have demonstrated a combination of clustering techniques and convex hull algorithm for analysis of large sets in renewable distributed generation. Wang et al (2004) have presented non-iterative analytical approaches to determine the optimal location for placing DG in both radial and network systems to minimize power losses. All the above methods are mathematically modeled and hence are found to be complex in its approach.

Quezada et al (2006) have presented an approach to computer annual energy losses when different penetration and concentration levels of DG are connected to a distributed network. The method also identifies that when DG units are more dispersed along the network feeders the expected higher losses can be reduced up to a particular DG capacity beyond which loss increases. This idea is used in the proposed method for optimizing the DG capacity corresponding to minimum power loss. Chiradeja (2005) has quantified the benefit of reduced line loss in a radial distribution feeder

with concentrated load. Ramkumar et al (2002) have proposed an approach to enumerate the various power quality indices in terms of profile, line-loss reduction and voltage environment impact reduction. Chiradeja et al (2004) has proposed a general approach and a set of indices to assess some of the technical benefits in a quantitative manner. Several soft computing techniques have proposed to analyze the potential benefits of DG connected to a power system. Celli et al (2001) have proposed a genetic algorithm based software procedure to establish optimum DG allocation on an existing distribution network considering constraints like feeder capacity limits, feeder voltage profile and three-phase short circuit. Greatbanks (2003) has formulated a methodology for locating the most appropriate site and deciding the size of DG. Optimum sitting is done by sensitivity analysis of power flow equations. Optimum sizing is formed as a security constrained optimization problem and solved by genetic algorithm. The soft computing techniques for optimization are mainly based on GA. Though the GA methods have been employed successfully to solve complex optimization problems, recent research has identified some deficiencies in GA performance.

However, due to the complexity of the problems, optimization method may fail to find global solution. This paper considers the impact of the introduction of DG on line loss of the entire distribution network and employ's the line loss reduction index (LLRI) discussed in 6 to demonstrate the system loss minimization

MATHEMATICAL MODEL AND SYSTEM MODELING

In order to evaluate and quantify the benefit of DG, suitable mathematical model is employed along with distribution system models to arrive at index of benefit.

Among the many benefits, one major one is considered: line loss reduction

Line loss reduction index (LLRI)

The line loss reduction index (LLRI) method is employed in this work to show how the integration of distributed generators (DGs) unit into the existing power network has reduced electrical line losses. The index is computed using MATLAB 7.5 software package.

The line loss reduction index (LLRI) is defined as:

$$LLRI = \frac{LL_{W/DG}}{LL_{WO/DG}}$$

Where $LL_{W/DG}$ is the total line losses in the distribution system with the employment of DG and $LL_{WO/DG}$ is the total line loss in the distribution system without DG.

$$LL_{W/DG} = 3\sum_{i=1}^{M} I_i^2 RD_i$$

Where, Ii is the line current in distribution line i with the employment of DG, R is the line resistance (ohms), D_i is the distribution line length (km), and M is the number of lines in the system.

Similarly, $LL_{WO/DG}$ is expressed as

$$LL_{W/DG} = 3\sum_{i=1}^{M} I_i^2 RD_i$$

Where, Ii is the line current in distribution line i without DG. Based on this definition, the following attributes are:

LLRI < 1 DG has reduced electrical line losses,

LLRI = 1 DG has no impact on system line losses,

LLRI > 1 DG has caused more electrical line losses.

This index is used to identify the best location to install DG to maximize the line loss reduction. The minimum value of LLRI corresponds to the best DG location scenario in terms of line loss reduction.

System Modeling and Line Loss Analysis

Two simple radial systems are considered:

(i) System without DG

(ii) System with the inclusion of DG Nominal voltage at the low voltage bus of the injection substation is 11kv. It is assumed that loads are uniformly distributed along the feeder at load buses. The total length of the line is assumed to be L(km). Schematics of the two cases are shown in figures 2.1 and 2.2.

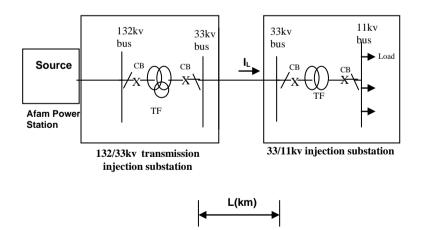


Figure 2.1: A simple radial distribution system without DG.

For the system with DG, the location of DG is at the low-voltage buses (11kv) of the existing substations near electrical consumers, DG will reduce loadings on substations power transformer during peak hours, thereby extending the useful life of this equipment, deferring planned substation upgrades and minimize line losses.

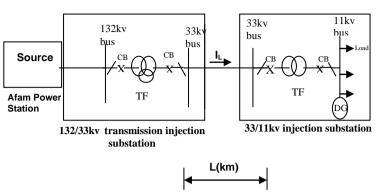


Figure 2.2: Schematic of a radial system with the inclusion of DG.

The following assumptions are made in the work:

- 1. The DG units is assumed to operate at a power factor of 0.89 lagging in all cases.
- 2. The DG considered here are synchronous gas turbine generators which will supply real power when located at the respective 11kv substation buses.
- 3. The sample network operates at a power factor of 0.8 lagging without DG.
- 4. With the integration of DG unit(s), the configuration of the network changes and the power factor increases. The new power factor is assumed to be 0.89 lagging. Line loss is inversely proportional to the power factor, so also to the supply voltage. Hence, the power loss in the line goes on increasing as the power factor of the load goes on decreasing.

The operating power factor of DG is also important in determining the benefit of distributed generation

Line Loss Analysis

Electrical line loss occurs when current flows through transmission and distribution systems. The magnitude of the loss depends on amount current flow and the line resistance. Therefore, line loss can be decreased by reducing either line current or resistance or both. If DG is used to provide energy locally to the load, line loss can be reduced because of the decrease in current flow in some part of the network.

- Line loss analysis without DG

Schematic of the system for this analysis is shown in figure 2.2. line loss on the distribution feeders is equal to the product of line current squared times the line resistance. Therefore, line loss equation for a three phase system is defined as :

 $Loss_{k} = 3I_{L}^{2} R$ $= 3\left(\frac{P}{\sqrt{3} V \cos\theta}\right)^{2} R,$

Where: P is the power in (MW) injected, V_L is the line voltage (kv), $\cos\theta$ is the(power factor and R is the line resistance in (ohms), $loss_k$ is the real power line loss in section k.

Line loss analysis with DG

Since DG is integrated into the 11kv bus of the existing substations, the line from DG location to the substation bus is assumed very short, voltage drop along the line is neglected. Schematic of this system is shown in figure 2.2. The real power supplied by the DG is synchronize with the real power injected into the bus by the source.

Therefore, the line loss with the integration of DG involves a combinational sum of DG unit power supplied and the injected power from the source.

Line Loss Reduction (LR)

Loss reduction (LR) is given by the difference in the line loss with and without DG. Hence.

 $LR \qquad = LL_w/_{DG} - LL_{wo}/_{DG}$

Where:

 $LL_{w}\!/_{DG}~$ is the line loss with DG and $LL_{w0}\!/_{DG}$ is the line loss without DG

The positive sign of LR indicates that the system loss reduces with the integration of DG. In contrast, the negative sign of LR implies that DG causes the higher system loss.

Percentage of line loss reduction is simply defined as the ratio of loss reduction to the loss without DG, expressed in percentage

$$\% LR = \frac{LR}{LL_{wo}/_{DG}} \quad x \quad 100$$

The ratio presents the benefits of DG in normalized form.

Relevance and Quantification of line Loss Reduction Index (LLRI) of the System

The line loss reduction index method will assist to justify and assess the technical benefits of distributed generation (DG) such as voltage profile improvement, line loss reduction etc. in the electric power system network.

Technical benefits of introducing DG can accure in one of two broad categories:

- (i) Improvement of a certain attribute such as voltage profile, reliability, power quality etc.
- (ii) Reduction of an attribute such as line losses, congestion etc.

By comparing and taking the ratio of a measure of an attribute with and without DG with the loads served being the same, an index can be derived for each of the attributes. If the introduction of DG is beneficial, indices corresponding to the attributes in category i) will be greater than unity and indices corresponding to the attributes in category ii) will be less than unity.

Designating the indices as II_i and RI_j for the different attributes in categories (i) and (ii), respectively, an overall composition BI can be formulated as

$$\mathbf{BI} = \sum_{i} \mathbf{BW}_{i}\mathbf{II}_{j} + \sum_{j} \mathbf{BW}_{j} \frac{1}{\mathbf{RI}_{j}}$$

Where:

 $\ensuremath{\mathrm{II}}_i$ is improvement index for ith attribute,

RI_j is reduction index for ith attribute,

BI is distributed generation benefit index,

BWi and BWj are the benefit weighting factors and

$$\sum_{i} \mathbf{BW}_{i} + \sum_{j} \mathbf{BW}_{j} = 1$$

The use of weighting factors will enable the emphasis of certain critical attributes depending on the location of the DG units, type of loads served by the distribution system and the region involved. With this formulation, the planner can select the locations and ratings of DG that will result in the highest value for BI to maximize the benefit.

A set of indices is proposed to quantify some of the technical benefits of DG. They are line loss reduction index and voltage profile improvement index (VPII) by Joss et al (2000) cited in the literature review. One of the justifications for introducing DG is to improve the voltage profile of the system and maintain the voltage at customer terminals to within an acceptable range. By introducing DG in the system, voltage profile can be improve because DG can provide a portion of the real and reactive power to the load, thus helping to decrease current along a section of the distribution line, which in turn, will result in a boost in the voltage magnitude at the customer.

The proposed VPII quantifies the improvement in the VP in a simple manner with the inclusion of DG. It is defined as the ratio of the voltage profile index of the system with DG to the voltage profile index of the system without DG (base case system) and is expressed

as VPII =
$$\frac{VP_{w/DG}}{VP_{WO/DG}}$$

Based on this definition, the following attributes are used:

VpII, <1, DG has not beneficial

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VPII = 1, DG has no impact on the system voltage profile,

VPIII > 1 DG has improved the voltage profile of the system.

Where, VP $_{W/DG}$, vp $_{WO/DG}$ are the measure of the voltage profile of the system with DG and without DG respectively. The general expression for VP is given as,

$$\mathbf{VP} = \sum_{i=1}^{N} ViL_iK_i$$

With

$$\sum_{i=1}^{N} K_i = 1$$

Where, V_i , is the voltage magnitude at bus i, L_i is the load at bus i, K_i is the weighting factor for bus i, and N is the total number of buses in the distribution system. The weighting factors are chosen based on the importance and criticality of different loads. As defined, the expression for VP provides an opportunity to quantify and aggregate the importance, amounts, and the voltage levels at which loads are being supplied at the various load buses in the system. This expression should be used only after making sure that the voltage at all the load buses are within allowable minimum and maximum limits, typically between 0.95 and 1.05 p.u. The weighting factors are chosen based on the importance and criticality of the different loads. No overarching rules can be formulated at the present time. Starting with a set of equal weighting factors, modifications can be made and, based on an analysis of the results, the set that will lead to the most acceptable voltage profile on a system-wide basis can be selected. It should be noted that if all the load busses are equally weighted, the value of K_i, is given as

$$K_1 = K_2 = K_3 = \dots = K_N = -\frac{1}{N}$$

In this case all the load buses are given equal importance. In reality, DG can be installed almost anywhere in the system. Therefore, VPII can be used to select the best location for DG. In general, the highest value of VPII implies the best location for installing DG in terms of improving voltage profile. The voltage profile expression recognizes the influences of the amount and importance of load at each bus. It allows the possibility of a low-load bus with important load to have a strong impact. In general, weighting factors are assigned based on the importance /criticality of load at each bus.

SIMULATION AND DG IMPLEMENTATION

In this work, line loss reduction index method was implemented on the ten-bus test system by selecting distributed generator units (DG-1) of 10MW of Trans-Amadi gas turbine and 20MW of Omoku gas turbine (DG-2) and locating these units on the low-voltage buses (11kv) of the existing distribution substations near electrical consumers as shown in the sample network (fig 4.1). The objective of the placement technique is to minimize the total real power line losses in the system. The operation of the distributed generators is considered to be at steady state and therefore the DG injects active power. By locality DG units on the low-voltage bus of an existing distribution substation, DG will reduce loadings on substation power transformers during peak hours, thereby extending the useful life of this equipment, deferring planned substation upgrade, reduce transmission and distribution line losses. In order to minimize the system losses, distributed generators were located at these respective buses. The input data to the sample test program was obtained from the daily hourly load readings record for the electric power injected in each distribution substations for 2008, compiled for Port Harcourt distribution zone from their respective dispatch centre log books. The average electric power injected (MW) with and without DG were obtained from the yearly summation and recorded in table 1 and 2. The lengths of the distribution lines and resistances are listed in table 3.

Cable resistance

a. Pre-calculated Resistance per km length of Bare All Aluminum multistranded conductor (AAC-type) 150mm² – 0.18250hms/km

 $100 \text{mm}^2 - 0.2704 \text{ohms/km}$

b. Pre-calculated resistance per km length of Bare Aluminium multistranded Conductor Steel Reinforced (ACSR-type)

> $150 \text{mm}^2 - 0.1828 \text{ohms/km}$ $100 \text{mm}^2 - 0.2733 \text{ohms/km}$

Source: Nigeria Wire and Cable Company Limited

TABLE 1: Load Data for the Sample System Network Under Study Without DG

| Sub-Stations | Load(MW) |
|--------------|----------|
| Woji | 58 |
| C.O.E | 54 |
| Oyigbo | 67 |
| Refinery 2 | 65 |
| Agip | 76 |
| Trans-Amadi | 66 |
| Old GRA | 63 |
| Bolokiri | 60 |
| Marine Base | 64 |
| UST | 56 |

TABLE 2:Load Data for the Sample SystemNetwork Under study With DGs.

| Sub-Stations | Load (MW) |
|--------------|-----------|
| | |
| Woji | 60 |
| C.O.E | 55 |
| Oyigbo | 68 |
| Refinery 2 | 70 |
| Agip | 79 |
| Trans-Amadi | 67 |
| Old GRA | 65 |
| Bolokiri | 62 |
| Marine Base | 66 |
| UST | 58 |

TABLE 3: DISTRIBUTION LINES(DL)LENGTH DATA

| DL | Length (KM) | Size of ACSR | Resistance in ohms |
|----|----------------|--------------------|--------------------|
| 1 | 15 | 150mm ² | 0.82125 |
| 2 | 7 | 100mm^2 | 5.7393 |

| 3. | 10 | 150mm ² | 5.475 |
|----|-----|--------------------|---------|
| 4 | 21 | 100mm ² | 17.2179 |
| 5 | 12 | 100mm ² | 9.8388 |
| 6 | 6.5 | 150mm ² | 3.55875 |
| 7 | 4 | 100mm ² | 3.2796 |
| 8 | 20 | 150mm ² | 10.95 |
| 9 | 18 | 150mm ² | 9.855 |
| 10 | 5 | 100mm^2 | 4.0995 |

STIMULATION RESULT

The results for loss minimization on the ten-bus test system using MATLAB 7.5 software package are presented in tabular and graphical form.

In table 5.1a the total line loss for the base case without DG is found to be 153MW and the value of the total line loss considerably reduces to 134MW with the inclusion of DG. The reduction in line loss is evident after connecting DG. It indicates the reduction in line losses with the installation of DGs at the respective buses. Based on the definition of the line loss reduction index (LLRI), the following attributes are:

LLRI < 1, DG has reduced electrical line losses.

LLRI = 1, DG has no impact on system line losses.

LLRI > 1, DG has caused more electrical line losses

Table 5.1a clearly indicates that LLRI <1, in all the buses where DG is integrated. Confirming the fact that the installation of DG into an existing utility can result in reduced electrical line losses.

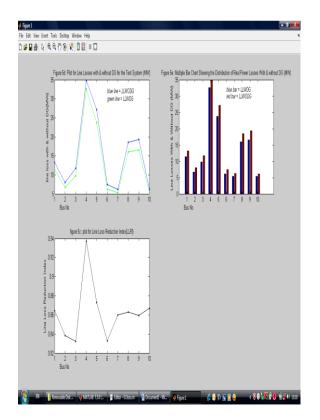
Fig. 5c is a plot showing the distribution of LLRI. The minimum value of LLRI are found to be in bus 3 and bus 6 respectively which corresponds to the optimal or best DG location scenario in terms of line loss reduction.

The ratio of loss reduction (LR) to the line loss without DG express in percentage as shown in table 5.1a presents the benefit of DG in normalized form. The positive sign of LR indicates that the system loss reduces with the integration of DG. Fig. 5d and 5e show the distribution of real power losses in MW with DG and without DG in graphical form. With all these indications, it is clearly proved and shown that by injecting DG into the distribution system, we can reduce the distribution line losses.

Table5.1a:**RESULTOFCOMPUTER**SIMULATION

| 'Bus I | No' " | 'LL _w /DG' ' MW ' | 'LL _{wo} /DG' 'MW ' | 'LR' 'MW ' | [LLRI] | '%' LR' `% ' |
|--------|----------|---------------------------------|---------------------------------|---------------|-----------|-----------------|
| [| 1] | [11.4255] | [13.2138] | [1.7883] | [0.8647] | [13.5338] |
| [| 2] | [6.7094] | [8.0047] | [1.2953] | [0.8382] | [16.1819] |
| [| 3] | [9.7836] | [11.7552] | [1.9716] | [0.8323] | [16.7722] |
| [| 4] | [32.6043] | [34.7941] | [2.1898] | [0.9371] | [6.2936] |
| [| 5] | [23.7297] | [27.1810] | [3.4513] | [0.8730] | [12.6974] |
| [| 6] | [6.1738] | [7.4146] | [1.2408] | [0.8326] | [16.7352] |
| [| 7] | [5.3548] | [6.2258] | [0.8710] | [0.8601] | [13.9907] |
| [| 8] | [15.9694] | [18.5100] | [2.5407] | [0.8627] | [13.7258] |
| [| 9] | [16.5898] | [19.3069] | [2.7171] | [0.8593] | [14.0733] |
| [| 10] | [5.3295] | [6.1490] | [0.8195] | [0.8667] | [13.3278] |
| ' | ·' | '' | ' | ' | ······' ' | ' ' |
| ' | | | | | | |

Total' [133.6697] [152.5552] [18.8855] " [137.3317] 'Average' [13.3670] [15.2555] [1.8886] [0.8627] [13.7332]



CONCLUSION

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In this work, we have presented a sample transmission/distribution network in Port Harcourt with multiple distributed generators (DGs) fed by Afam power station. Line loss reduction index analysis is used to demonstrate for loss minimization using MATLAB 7.5 software. An application has been developed to implement the use of distributed generators (DGs) units to reduce transmission/distributed line losses.

The placement or integration of DG into an existing utility system and its effects on loss minimization are demonstrated and implemented on the 10-bus test system.

The conclusion from the implementation are:

- With the introduction of DG, electrical line losses are reduced. This factor is analyzed, quantified and presented for varying locations of the distributed generators (DGs) in the low voltage buses (11kv) of an existing distribution substation near electrical consumers.
- The line loss reduction index approach clearly proved that by integrating DG into the distribution system, we can reduce the distribution line losses.

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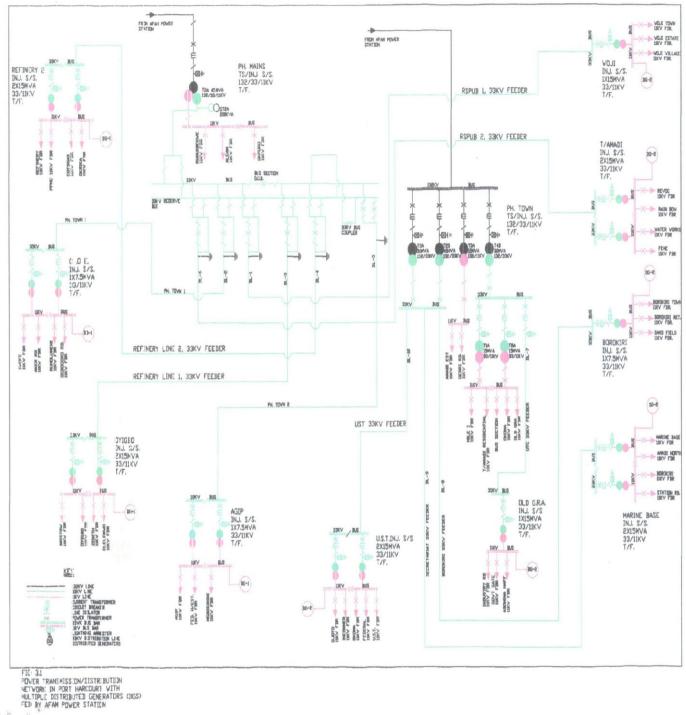
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