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INFLUENCE OF THE INTRINSIC GROWTH RATE AND THE INTRA-COMPETITION COEFFICIENT ON THE TYPE OF STABILITY BETWEEN TWO MUTUALISTICALLY INTERACTING LEGUMES

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ABSTRACT

Mathematical modelling is one of the techniques that can be used to model the mutualistic interaction between two competing legumes for limited growth resources. In particular numerical simulation has been successfully used to predict the simultaneous effect of the changes in the intrinsic growth rate and the intra-competition coefficient parameter value on the type of stability. On the basis of this method, a dominant stable co-existence steady-state solution has been observed.

INTRODUCTION

Recently, the mathematical technique of a numerical simulation has been utilized to model the type of stability of a mutualistic interaction with only changes in the intrinsic growth rate parameter value of the cowpea legume when other model parameter values were fixed (Ekaka-a et al. 2014). Other related ideas of the application of a numerical simulation can be found in the work of Ekaka-a (2009) and several cited publications therein presented (Baker and Rihan, 1995; Baker et al. 2005; Baker, 1995; Bart and Hill, 2005).

However, the open mathematical problem of examining the simultaneous effect of changes in the intrinsic growth rate and the intra-competition coefficient of the cowpea legume on the type of stability remains a daunting inter-disciplinary research question. This problem which forms the core of the present motivation can be tackled using the method of a numerical simulation. The significance of the carrying capacities and the contribution of the groundnut legume to enhance the growth of the cowpea legume have the potential of providing capacity-building knowledge that will help in farming decisions, crop planting and crop production in the sequel.

Mathematical Formulation

The work of Ekaka-a, Nafo and Lale (2014), considered the following system of continuous nonlinear first order ordinary differential equations that describes the dynamics between cowpea and groundnut in the perspective of a mutualistic interaction

$$\frac{dC(t)}{dt} = C(t)[\alpha_1 - \beta_1 C(t) + \gamma_1 G(t)] \qquad (1)$$

$$\frac{dG(t)}{dt} = G(t)[\alpha_2 - \beta_2 G(t) + \gamma_2 C(t)] \qquad (2)$$

Here, the model parameter values α_1 and α_2 specify the intrinsic growth rates having the precise values of 0.0225 and 0.0446 in the units of grams over the permitted area of land space; the model parameter values β_1 and β_2 specify the intra-competition coefficients having the precise values of 0.006902 and 0.0133; the model parameter values γ_1 and γ_2 specify the inter-competition coefficients having the precise values of 0.0018 and 0.01. The independent variable t is specified as the unit of days.

METHOD OF ANALYSIS

For a fixed value of the inter-competition coefficient of the cowpea legume when the intrinsic growth rate and the intra-competition coefficient parameter values are varied, the stability of the co-existence steady-state solution is predicted by using the technique of a MATLAB simulation program. The details of this simulation analysis are presented in the following Table 1 and Table 2.

RESULTS AND DISCUSSION

For a fixed inter-competition coefficient of cowpea value of 0.001 [Table 1] when the intrinsic growth rate of the cowpea legume ranges from 0.000225 to 0.00225 and when the intra-competition coefficient of the legume ranges from 0.000069 to 0.00069, the type of stability is called degenerate because the range of the intra-competition coefficient parameter value is smaller than 0.0007515188.

Table 1: Assessing the impact of the intrinsic growth and the intra-competition of cowpea on the stability characteristic due to a fixed inter-competition coefficient of cowpea value of 0.001 for $0.000225 \leq \alpha_1 \leq 0.00225$ and $0.000069 \leq \beta_1 \leq 0.00069$

Example	α_1	β_1	steadc1	steadc2	ev1	ev2	type of stability
1	0.000225	0.000069	-5.24	-0.59	-0.0026	1.075964e-002	Deg
2	0.000450	0.000138	-6.20	-1.31	-0.0031	2.131377e-002	Deg
3	0.000675	0.000207	-7.39	-2.21	-0.0034	3.431506e-002	Deg
4	0.000900	0.000276	-8.94	-3.37	-0.0037	5.099837e-002	Deg
5	0.001125	0.000345	-11.01	-4.92	-0.0040	7.329464e-002	Deg
6	0.001350	0.000414	-13.93	-7.12	-0.0043	1.046722e-001	Deg
7	0.001575	0.000483	-18.34	-10.44	-0.0045	1.521447e-001	Deg
8	0.001800	0.000552	-25.80	-16.05	-0.0047	2.324108e-001	Deg
9	0.002025	0.000621	-41.15	-27.59	-0.0050	3.974342e-001	Deg
10	0.002250	0.000690	-90.85	-64.95	-0.0052	9.317621e-001	Deg

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In this set of ten (10) numerical predictions, the characteristic of the steady-state solution is primarily degenerate satisfying the inequality $\beta_1 <$ 0.00075188. This result implies that a co-existence steady-state solution can be classified as degenerate provided the intra-competition coefficient of the cowpea legume is not bigger than 0.00075188 in the scenario when the intrinsic growth rate of the cowpea legume ranges from the low value of 0.00025 to the high value of 0.00225 and when the intra-competition coefficient of the cowpea legume ranges from the low value of 0.000069 to the high value of 0.00069. The data displayed in Table 2 show that the degenerate co-existence steady-state solution is lost when the intrinsic growth rate of the cowpea legume ranges from 0.002475 to 0.0045 and when the intra-competition coefficient of the cowpea legume ranges from 0.000759 to 0.00138.

Table 2: Assessing the impact of the intrinsic growth and the intra-competition of cowpea on the stability characteristic due to a fixed inter-competition coefficient of cowpea value of 0.001 for $0.002475 \leq \alpha_1 \leq 0.0045$ and $0.000759 \leq \beta_1 \leq 0.001380$

Example	α_1	β_1	steadc1	steadc2	ev1	ev2	type of stability
11	0.002475	0.000759	794.03	600.36	-0.0054	-8.582269e+000	Stable
12	0.002700	0.000828	79.27	62.96	-0.0056	-8.973489e-001	Stable
13	0.002925	0.000897	43.19	35.82	-0.0059	-5.093352e-001	Stable
14	0.003150	0.000966	30.33	26.16	-0.0061	-3.711425e-001	Stable
15	0.003375	0.001035	23.74	21.20	-0.0063	-3.002582e-001	Stable
16	0.003600	0.001104	19.73	18.19	-0.0065	-2.571388e-001	Stable
17	0.003825	0.001173	17.03	16.16	-0.0068	-2.281439e-001	Stable
18	0.004050	0.001242	15.09	14.70	-0.0070	-2.073108e-001	Stable
19	0.004275	0.001311	13.63	13.60	-0.0072	-1.916192e-001	Stable
20	0.004500	0.001380	12.50	12.75	-0.0074	-1.793753e-001	Stable

The dominant stable co-existence steady-state solutions have been observed for the following model parameter values such as $\gamma_1 = 0.001, \gamma_2 = 0.01, \beta_2 = 0.0133$ provided the intra-competition coefficient satisfies the inequality $\beta_1 >$ 0.00075188. In contrast, the ten (10) instances of the degeneracy characteristic of the co-existence steady-state solutions are predicted to occur when $\beta_1 <$ 0.00075188. Since the stability of the co-existence steady-state solution is unanimously sustained when the value of the intra-competition coefficient is bigger than 0.00075188, it follows that as the intrinsic growth rate and the intra-competition coefficient parameter values range from 0.004725 to 0.03375 and from 0.001449 to 0.010353, their corresponding co-existence steady-state solution is said to be stable.

The implied range of data has demonstrated that out of the one hundred and fifty (150) numerical simulations, ten (10) instances have shown the incidence of a degenerate steady-state solution whereas one hundred and fourty (140) instances have shown the incidence of a dominant stable type of stability in the scenario of a mutualistic interaction between the cowpea and groundnut legumes having two negative eigenvalues as specified in the mathematical theory of stability. The influence of the intrinsic growth rate and the intra-competition coefficient of the cowpea legume for a fixed value of the inter-competition coefficient of the cowpea legume has been succinctly quantified using the technique of a numerical simulation analysis. One would expect this contribution to provide a further insight on an aspect that may concern the planting and production of both cowpea and groundnut. In summary, when two legumes interact mutualistically, a stable type of stability for its co-existence steady-state solution is a dominant phenomenon with the expectation of its rich application in the ecosystem functioning and stability. Other extended mathematical analyses of this present analysis will be looked into in the near future.

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