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NUMERICAL MODELLING OF THE IMPACT OF THE CARRYING CAPACITY (S) ON THE STABILITY OF TWO INTERACTING MUTUALISTIC POPULATIONS: ISSUES FOR THE RECOVERY OF STABILITY LOSS

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ABSTRACT

The impact of the changes in the carrying capacities for two mutualistically interacting legumes on the type of stability is a formidable mathematical problem. The method of quantifying this impact is called the technique of a numerical simulation. The loss of stability can be recovered provided the value of the inter-competition coefficient of the cowpea legume is correctly chosen. The results of this study which we have not seen elsewhere are presented and discussed.

INTRODUCTION

While the notions of the carrying capacities and stability are not new within the ecological and mathematical literatures (Ekaka-a, 2009; Ekaka-a, Nafo and Lale, 2014; Kot, 2001; Pielou, 1977; Murray, 1993; Renshaw, 1991; Damgaard, 2004; Addicot, 1981; Albrecht, 1974; Ahmad and Lazer, 1998), the studies that specify the effect of the changing carrying capacities on the type of stability is not a popular type of research investigation. While the carrying capacity defines the maximum size of the population that can support the growth of the population over the length of the growing period in the appropriate unit of days, the stability of two interacting populations can be either said to be primarily stable or unstable. The type of stability can also be classified as either degenerate (that is when either or both of the co-ordinates defining the positive unique co-existence steady-state solution carries a negative sign) or sitting on the cusp. It is against this background that we have proposed to study this mathematical problem of ecology using the method of a numerical simulation.

Mathematical Formulation

Following Ekaka-a, Nafo and Lale (2014), we have considered the following system of continuous nonlinear first order ordinary differential equations that describes the dynamics between cowpea and groundnut in the perspective of a mutualistic interaction

$$\frac{dC(t)}{dt} = C(t)[\alpha_1 - \beta_1 C(t) + \gamma_1 G(t)] \qquad (1)$$

$$\frac{dG(t)}{dt} = G(t)[\alpha_2 - \beta_2 G(t) + \gamma_2 C(t)] \qquad (2)$$

Here, the model parameter values α_1 and α_2 specify the intrinsic growth rates having the precise values of 0.0225 and 0.0446 in the units of grams over the permitted area of land space; the model parameter values β_1 and β_2 specify the intra-competition coefficients having the precise values of 0.006902 and 0.0133; the model parameter values γ_1 and γ_2 specify the inter-competition coefficients having the precise values of 0.0018 and 0.01. The independent variable t is specified as the unit of days.

Method of Analysis

The model parameter values of the two intrinsic growth rates were varied in conjunction with a change in the value of the inter-competition coefficient of the cowpea legume in order to study the type of stability that can be predicted in a mutualistic interaction between two legumes. The results that we have obtained using this method are presented and discussed next. For the purpose of this study, the notations CC (1) and CC (2) stand for the carrying capacities of the cowpea and groundnut legumes; the notations CSS (1) and CSS (2) stand for the co-existence steady-state solutions of the cowpea and groundnut legumes; the notations EV1 and EV2 stand for the eigenvalues while the notation TOS stands for the type of stability.

RESULTS AND DISCUSSION

In the context of a mutualistic interaction between two legumes, we have found a dominant stable steady-state solution when the value of the inter-competition coefficient of cowpea is 0.0005 and the value of the inter-competition coefficient of groundnut is 0.01 (Table 1) due to the changes in the carrying capacities of these two interacting legumes

Table 1: The influence of the carrying capacity (s) on the stable steady-state solution when the value of the inter-competition coefficient of cowpea is 0.0005 and the value of the inter-competition coefficient of groundnut is 0.01

Example	CC (1)	CC (2)	α_1	α_2	CSS (1)	CSS (2)	EV1	EV2	TOS
1	0.0002	0.0004	0.0326	0.0335	0.04	0.06	-0.00024	-8.360568e-004	Stable
2	0.0004	0.0009	0.0652	0.0671	0.07	0.12	-0.00047	-1.672114e-003	Stable
3	0.0007	0.0013	0.0978	0.1006	0.11	0.18	-0.00071	-2.508171e-003	Stable
4	0.0009	0.0018	0.1304	0.1341	0.15	0.25	-0.00094	-3.344227e-003	Stable
5	0.0011	0.0022	0.1630	0.1677	0.19	0.31	-0.00118	-4.180284e-003	Stable
6	0.0014	0.0027	0.1956	0.2012	0.22	0.37	-0.00142	-5.016341e-003	Stable
7	0.0016	0.0031	0.2282	0.2347	0.26	0.43	-0.00165	-5.852398e-003	Stable
8	0.0018	0.0036	0.2608	0.2683	0.30	0.49	-0.00189	-6.688455e-003	Stable
9	0.0020	0.0040	0.2934	0.3018	0.33	0.55	-0.00212	-7.524512e-003	Stable
10	0.0022	0.0045	0.3260	0.3353	0.37	0.61	-0.00236	-8.360568e-003	Stable
11	0.0025	0.0049	0.3586	0.3689	0.41	0.68	-0.00260	-9.196625e-003	Stable
12	0.0027	0.0054	0.3912	0.4024	0.44	0.74	-0.00283	-1.003268e-002	Stable
13	0.0029	0.0058	0.4238	0.4359	0.48	0.80	-0.00307	-1.086874e-002	Stable
14	0.0032	0.0062	0.4564	0.4695	0.52	0.86	-0.00331	-1.170480e-002	Stable
15	0.0034	0.0067	0.4890	0.5030	0.56	0.92	-0.00354	-1.254085e-002	Stable
16	0.0036	0.0071	0.5216	0.5365	0.59	0.98	-0.00378	-1.337691e-002	Stable
17	0.0038	0.0076	0.5542	0.5701	0.63	1.04	-0.00401	-1.421297e-002	Stable
18	0.0041	0.0080	0.5868	0.6036	0.67	1.10	-0.00425	-1.504902e-002	Stable
19	0.0043	0.0085	0.6194	0.6371	0.70	1.17	-0.00449	-1.588508e-002	Stable
20	0.0045	0.0089	0.6520	0.6707	0.74	1.23	-0.00472	-1.672114e-002	Stable

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An increase in the inter-competition coefficient to enhance the growth of the cowpea legume during a growing season can generate a dominant incidence of a degenerate steady-state solution that does that provide any biological meaning in terms of ecosystem functioning and stability. The details of this novel contribution are presented in Table 2. Our numerical simulation analysis has shown that the dominant stable steady-state solution can be totally lost in a scenario when the value of the inter-competition coefficients is the same while the intrinsic growth rates are undergoing some sort of changes very likely to be driven by some ecological disturbance such as a high sea level rise that is related to climate change impact. This chaotic observed can be recovered to a full stability when the value of the inter-competition coefficient of cowpea is 0.002 and the value of the inter-competition coefficient of groundnut is 0.01. The corresponding results of this analysis are displayed in Table 3.

Table 2: The influence of the carrying capacity (s) on the stable steady-state solution when the value of the inter-competition coefficient of cowpea is 0.01 and the value of the inter-competition coefficient of groundnut is 0.01

Example	CC (1)	CC (2)	α_1	α_2	CSS (1)	CSS (2)	EV1	EV2	TOS
1	0.0002	0.0004	0.0326	0.0335	-0.91	-0.65	-0.00032	1.522677e-002	Degenerate
2	0.0004	0.0009	0.0652	0.0671	-1.82	-1.30	-0.00064	3.045354e-002	Degenerate
3	0.0007	0.0013	0.0978	0.1006	-2.73	-1.95	-0.00095	4.568031e-002	Degenerate
4	0.0009	0.0018	0.1304	0.1341	-3.63	-2.60	-0.00127	6.090708e-002	Degenerate
5	0.0011	0.0022	0.1630	0.1677	-4.54	-3.25	-0.00159	7.613385e-002	Degenerate
6	0.0014	0.0027	0.1956	0.2012	-5.45	-3.90	-0.00191	9.136062e-002	Degenerate
7	0.0016	0.0031	0.2282	0.2347	-6.36	-4.55	-0.00223	1.065874e-001	Degenerate
8	0.0018	0.0036	0.2608	0.2683	-7.27	-5.20	-0.00254	1.218142e-001	Degenerate
9	0.0020	0.0040	0.2934	0.3018	-8.18	-5.85	-0.00286	1.370409e-001	Degenerate
10	0.0022	0.0045	0.3260	0.3353	-9.08	-6.50	-0.00318	1.522677e-001	Degenerate
11	0.0025	0.0049	0.3586	0.3689	-9.99	-7.14	-0.00350	1.674945e-001	Degenerate
12	0.0027	0.0054	0.3912	0.4024	-10.90	-7.79	-0.00381	1.827212e-001	Degenerate
13	0.0029	0.0058	0.4238	0.4359	-11.81	-8.44	-0.00413	1.979480e-001	Degenerate
14	0.0032	0.0062	0.4564	0.4695	-12.72	-9.09	-0.00445	2.131748e-001	Degenerate
15	0.0034	0.0067	0.4890	0.5030	-13.63	-9.74	-0.00477	2.284016e-001	Degenerate
16	0.0036	0.0071	0.5216	0.5365	-14.54	-10.39	-0.00509	2.436283e-001	Degenerate
17	0.0038	0.0076	0.5542	0.5701	-15.44	-11.04	-0.00540	2.588551e-001	Degenerate
18	0.0041	0.0080	0.5868	0.6036	-16.35	-11.69	-0.00572	2.740819e-001	Degenerate
19	0.0043	0.0085	0.6194	0.6371	-17.26	-12.34	-0.00604	2.893086e-001	Degenerate
20	0.0045	0.0089	0.6520	0.6707	-18.17	-12.99	-0.00636	3.045354e-001	Degenerate

Table 3: The influence of the carrying capacity (s) on the stable steady-state solution when the value of the inter-competition coefficient of cowpea is 0.002 and the value of the inter-competition coefficient of groundnut is 0.01

Example	CC (1)	CC (2)	α_1	α_2	CSS (1)	CSS (2)	EV1	EV2	TOS
1	0.0002	0.0004	0.0326	0.0335	0.05	0.07	-0.00026	-1.097891e-003	Stable
2	0.0004	0.0009	0.0652	0.0671	0.11	0.15	-0.00053	-2.195781e-003	Stable
3	0.0007	0.0013	0.0978	0.1006	0.16	0.22	-0.00079	-3.293672e-003	Stable
4	0.0009	0.0018	0.1304	0.1341	0.22	0.30	-0.00105	-4.391563e-003	Stable
5	0.0011	0.0022	0.1630	0.1677	0.27	0.37	-0.00131	-5.489453e-003	Stable
6	0.0014	0.0027	0.1956	0.2012	0.32	0.45	-0.00158	-6.587344e-003	Stable
7	0.0016	0.0031	0.2282	0.2347	0.38	0.52	-0.00184	-7.685234e-003	Stable
8	0.0018	0.0036	0.2608	0.2683	0.43	0.59	-0.00210	-8.783125e-003	Stable
9	0.0020	0.0040	0.2934	0.3018	0.49	0.67	-0.00236	-9.881016e-003	Stable
10	0.0022	0.0045	0.3260	0.3353	0.54	0.74	-0.00263	-1.097891e-002	Stable
11	0.0025	0.0049	0.3586	0.3689	0.60	0.82	-0.00289	-1.207680e-002	Stable
12	0.0027	0.0054	0.3912	0.4024	0.65	0.89	-0.00315	-1.317469e-002	Stable
13	0.0029	0.0058	0.4238	0.4359	0.70	0.96	-0.00341	-1.427258e-002	Stable
14	0.0032	0.0062	0.4564	0.4695	0.76	1.04	-0.00368	-1.537047e-002	Stable
15	0.0034	0.0067	0.4890	0.5030	0.81	1.11	-0.00394	-1.646836e-002	Stable
16	0.0036	0.0071	0.5216	0.5365	0.87	1.19	-0.00420	-1.756625e-002	Stable
17	0.0038	0.0076	0.5542	0.5701	0.92	1.26	-0.00446	-1.866414e-002	Stable
18	0.0041	0.0080	0.5868	0.6036	0.97	1.34	-0.00473	-1.976203e-002	Stable
19	0.0043	0.0085	0.6194	0.6371	1.03	1.41	-0.00499	-2.085992e-002	Stable
20	0.0045	0.0089	0.6520	0.6707	1.08	1.48	-0.00525	-2.195781e-002	Stable

Similarly, we have found a dominant stable steady-state solution when the value of the inter-competition coefficient of cowpea is 0.0025 and the value of the inter-competition coefficient of groundnut is 0.01 (Table 4) due to the changes in the carrying capacities of these two interacting legumes.

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Table 4: The influence of the carrying capacity (s) on the stable steady-state solution when the value of the inter-competition coefficient of cowpea is 0.0025 and the value of the inter-competition coefficient of groundnut is 0.01

Example	CC (1)	CC (2)	α_1	α_2	CSS (1)	CSS (2)	EV1	EV2	TOS
1	0.0002	0.0004	0.0326	0.0335	0.06	0.08	-0.00027	-1.215870e-003	Stable
2	0.0004	0.0009	0.0652	0.0671	0.12	0.16	-0.00054	-2.431741e-003	Stable
3	0.0007	0.0013	0.0978	0.1006	0.18	0.24	-0.00081	-3.647611e-003	Stable
4	0.0009	0.0018	0.1304	0.1341	0.25	0.32	-0.00108	-4.863482e-003	Stable
5	0.0011	0.0022	0.1630	0.1677	0.31	0.40	-0.00135	-6.079352e-003	Stable
6	0.0014	0.0027	0.1956	0.2012	0.37	0.48	-0.00162	-7.295222e-003	Stable
7	0.0016	0.0031	0.2282	0.2347	0.43	0.56	-0.00189	-8.511093e-003	Stable
8	0.0018	0.0036	0.2608	0.2683	0.49	0.64	-0.00216	-9.726963e-003	Stable
9	0.0020	0.0040	0.2934	0.3018	0.55	0.72	-0.00243	-1.094283e-002	Stable
10	0.0022	0.0045	0.3260	0.3353	0.61	0.80	-0.00269	-1.215870e-002	Stable
11	0.0025	0.0049	0.3586	0.3689	0.68	0.88	-0.00296	-1.337457e-002	Stable
12	0.0027	0.0054	0.3912	0.4024	0.74	0.96	-0.00323	-1.459044e-002	Stable
13	0.0029	0.0058	0.4238	0.4359	0.80	1.04	-0.00350	-1.580632e-002	Stable
14	0.0032	0.0062	0.4564	0.4695	0.86	1.12	-0.00377	-1.702219e-002	Stable
15	0.0034	0.0067	0.4890	0.5030	0.92	1.20	-0.00404	-1.823806e-002	Stable
16	0.0036	0.0071	0.5216	0.5365	0.98	1.28	-0.00431	-1.945393e-002	Stable
17	0.0038	0.0076	0.5542	0.5701	1.05	1.36	-0.00458	-2.066980e-002	Stable
18	0.0041	0.0080	0.5868	0.6036	1.11	1.44	-0.00485	-2.188567e-002	Stable
19	0.0043	0.0085	0.6194	0.6371	1.17	1.52	-0.00512	-2.310154e-002	Stable
20	0.0045	0.0089	0.6520	0.6707	1.23	1.60	-0.00539	-2.431741e-002	Stable

On the application of a numerical simulation analysis, we have found some instances of dominant stable and degenerate type of stability due to changes in the intrinsic growth rate parameter values and a few changes in the inter-competition coefficient of the cowpea legume that has acted as a control parameter to recover the loss of full stability. We would expect this present contribution to complement other types of mathematical analyses on the ecological systems of several complexities.

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