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MODELING THE EVOLUTION OF BINARY BLACK HOLE MERGERS IN DENSE STELLAR ENVIRONMENTS

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ABSTRACT

Binary black hole (BBH) mergers are critical sources of gravitational waves detected by observatories like LIGO and Virgo. The formation and evolution of these binaries, particularly in dense stellar environments such as globular clusters (GCs) and galactic nuclei, are crucial for understanding the observed BBH population. In this study, we develop a Monte Carlo N-body simulation model to simulate the dynamical evolution of BBH mergers in such environments. We incorporate key processes, including two-body relaxation, three-body encounters, exchange interactions, and gravitational wave emission. Our results indicate that globular clusters can produce BBH merger rates of approximately 20 mergers per Gyr per $10^5 M_\odot$, in agreement with current gravitational wave observations. The simulated mass distribution of BBHs peaks at around 20 $M_\odot$, with a tail extending to 50 $M_\odot$, and the spin orientations are predominantly isotropic. These findings highlight the significant role dense stellar environments play in the formation of BBHs. We also discuss the uncertainties in the model and suggest future work to refine the simulations and better constrain the contribution of dense stellar environments to the overall BBH merger rate.

Keywords: Binary Black Hole Mergers, Gravitational Waves, Dense Stellar Environments, Globular Clusters, Monte Carlo Simulations

INTRODUCTION

Binary black hole (BBH) mergers from globular clusters (GCs) are significant contributors to gravitational wave signals detected by LIGO and Virgo. Studies suggest that GCs could produce BBH mergers at rates of ~100 per year, potentially dominating the overall merger rate (Rodriguez *et al.*, 2015). The formation of BBHs in GCs is influenced by the cluster's initial mass, size, and binary fraction, with both primordial and dynamically formed BBHs contributing to mergers (Hong *et al.*, 2018). Local merger rate densities are estimated at 0.18-1.8 Gpc^{-3} yr^{-1} for primordial BBHs and 0.6-18 Gpc^{-3} yr^{-1} for dynamical BBHs (Hong *et al.*, 2018). GC-formed BBHs tend to be more massive than field-formed counterparts, with 80% of sources having total masses between 32$M_\odot$ and 64$M_\odot$ (Rodriguez et al., 2016a). Additionally, in-cluster mergers can produce second-generation black holes with larger masses and high spins, potentially populating any upper mass gap created by pair-instability supernovae (Rodriguez *et al*., 2017). The goal of this study is to simulate the evolution of BBH mergers in these dense environments, focusing on predicting merger rates, mass distributions, and spin orientations. This model aims to provide insights into the contribution of dense stellar systems to the overall BBH merger population. Binary black hole (BBH) mergers have emerged as a crucial source of gravitational waves, revolutionizing astrophysics. The first direct detection of gravitational waves from a BBH merger was made by LIGO on September 14, 2015, marking the beginning of gravitational-wave astronomy (The Ligo Scientific Collaboration and The Virgo Collaboration, 2016). This groundbreaking observation confirmed the existence of stellar-mass BBH systems and demonstrated their potential as probes of stellar evolution and dynamics (Mandel and Farmer, 2018). BBHs can form through isolated evolution of massive binary stars or dynamical encounters in dense star clusters, with each channel leaving distinct imprints on the properties of the resulting systems (Mapelli, 2020). The detection of BBH mergers provides valuable insights into black hole properties, stellar evolution, and the structure of the universe, opening a new window for astronomical observations (Rigal and Wilson, 2016). This field continues to advance our understanding of the cosmos through gravitational-wave astronomy.

Binary black hole (BBH) mergers can form through various channels, primarily isolated binary evolution and dynamical interactions in dense stellar environments (Mapelli, 2020). Isolated evolution involves processes like mass transfer and common envelope evolution, while dynamical formation occurs in dense star clusters through gravitational interactions (Mapelli, 2020; Sedda *et al*., 2020). The relative contribution of these channels can be inferred from the mass and spin distributions of merger remnants (Sedda *et al*., 2020; Bouffanais *et al*., 2021). Recent studies suggest a combination of both channels, with a slight preference for dynamical formation (Bouffanais *et al*., 2021). Factors such as metallicity distribution, stellar winds, core-collapse supernovae, and pair instability significantly influence BBH properties and merger rates (Mapelli, 2021). Understanding these formation channels is crucial for interpreting gravitational wave observations and reconstructing the cosmic evolution of BBHs (Mapelli, 2021). Ongoing research aims to refine our understanding of BBH formation and evolution in various astrophysical environments.

Gravitational wave (GW) emission plays a crucial role in the merger of black hole binaries. Peters' formula is used to predict merger times based on masses and orbital parameters (Gultekin *et al*., 2004). In dense stellar environments, black holes form binaries through dynamical interactions, leading to mergers detectable by GW observatories (Portegies Zwart and McMillan, 1999). Numerical simulations reveal that binary black hole mergers typically occur with high eccentricities and involve a brief plunge phase before coalescence (Buonanno *et al*., 2006; Gultekin *et al*., 2004). The merger process can be divided into inspiral, merger, and ringdown phases, with the merger lasting approximately 0.5-0.75 of a GW cycle (Buonanno *et al*., 2006). Recent galactic-scale simulations incorporating post-Newtonian corrections have shown that the stellar environment significantly influences binary evolution and GW emission, highlighting the limitations of semi-analytic models in accurately predicting merger timescales and eccentricity evolution (Mannerkoski *et al*., 2019).

Dense stellar environments like globular clusters provide ideal conditions for binary black hole (BBH) formation and mergers. N-body simulations reveal that rapid mass segregation and core collapse can occur within a fraction of the initial half-mass relaxation time, potentially leading to the formation of massive black holes (Goswami *et al*., 2003). These simulations also show that the collapsing core mass is consistently about 10^{-3} of the total cluster mass. BBHs can form

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through both stellar evolution and dynamical encounters, with primordial binaries playing a crucial role in clusters with lower escape velocities (Barber *et al*., 2023). The mergers of dynamically formed BBHs from globular clusters could be detected at a rate of ~100 per year by Advanced LIGO, potentially dominating the overall BBH merger rate (Rodriguez *et al*., 2015). These cluster-formed binaries are generally more massive than field-formed counterparts. Simulations suggest that the detection rate of BBH mergers originating from globular clusters could range from 0.5 to 20 per year (Tanikawa, 2013).

Globular clusters (GCs) play a significant role in the formation and merger of binary black holes (BBHs). Studies have shown that GCs could produce a substantial fraction of BBH mergers detected by LIGO, with estimated local merger rate densities ranging from 0.78-19.8 Gpc^{-3} yr^{-1} (Hong *et al*., 2018) to ~100 per year (Rodriguez *et al*., 2015). BBHs formed in GCs tend to have larger masses than field-formed counterparts, with 80% of sources having total masses between 32M⊙ and 64M⊙ (Rodriguez et al., 2016a). The formation of BBHs in GCs is influenced by the cluster's initial mass, size, and primordial binary fraction (Hong *et al*., 2018). Importantly, the evolution of GCs within their host galaxies significantly impacts the merger rate, with the initial GC mass being ~8 times higher than present day values due to evaporation and tidal disruption (Fragione and Kocsis, 2018). This evolution increases the present-day merger rate by a factor of ~2 relative to isolated clusters (Fragione and Kocsis, 2018). Many studies have explored the mass and spin distributions of binary black hole (BBH) mergers detected by LIGO-Virgo. The primary mass distribution extends beyond 200 M ⊙, with hierarchical mergers dominating above 65 M ⊙ (Sedda *et al*., 2021). A negative correlation between mass and mean effective spin, and a positive correlation with spin dispersion, has been observed (Safarzadeh *et al*., 2020). This trend suggests contributions from both dynamically assembled and field binaries. Hierarchical mergers produce a monotonic increase in average total spin up to the maximum chirp mass of first-generation black holes, followed by a plateau at higher masses (Tagawa *et al*., 2021). Spin orientations can help distinguish formation channels, with dynamically formed binaries expected to have isotropically distributed spins (Farr *et al*., 2017). These findings provide insights into BBH formation mechanisms and environments, such as globular clusters, active galactic nucleus discs, and nuclear star clusters.

Recent studies have explored the formation and merger of binary black holes (BBHs) in dense stellar environments, highlighting various uncertainties and challenges in the field. Modeling uncertainties include assumptions about stellar evolution, binary properties, and initial mass functions (Chatterjee *et al*., 2016). Environmental factors, such as cluster mass, metallicity, and radius, significantly influence BBH formation and merger rates (Rodriguez et al., 2016b; Mapelli *et al*., 2021). The formation of second-generation black holes within clusters can lead to mergers in the pair-instability mass gap and intermediate-mass black hole regime, particularly in nuclear star clusters and at low metallicities (Mapelli *et al*., 2021). While isolated binary evolution and dynamical encounters in dense clusters are both viable formation channels for BBHs (Mapelli, 2020), the properties of merging BBHs from globular clusters appear less sensitive to initial assumptions compared to field populations (Chatterjee *et al*., 2016). Future research should focus on refining stellar evolution models and expanding simulations to account for a broader range of cluster properties.

METHODOLOGY

To investigate the evolution of binary black hole (BBH) mergers within dense stellar environments, we begin by modeling a globular cluster with specific parameters. The total mass of the cluster is set to 10^5 M ⊙, with a half-mass radius of 2 parsecs. The initial binary fraction is assumed to be 10%, and the black hole mass distribution follows the Salpeter Initial Mass Function (IMF), with masses ranging from 5 M ⊙ to 50 M ⊙. The velocity dispersion of the cluster is set at 10 km/s. These initial conditions define the structure and dynamics of the stellar population, providing a foundation for the modeling of binary formation and evolution. The dynamics of the cluster are driven by several key processes, including two-body relaxation, three-body encounters, and exchange interactions. Two-body relaxation causes energy exchange between stars, which drives the core collapse of the cluster and enhances the likelihood of binary black hole formation. Three-body encounters create new binaries via gravitational interactions between three stars, while exchange encounters involve existing binaries interacting with single black holes, potentially forming more massive BBHs. Additionally, gravitational wave emission is accounted for, where the loss of energy through gravitational waves leads to the inspiral and merger of BBH systems.

The time to merger (t_merge) for a BBH due to GW emission is given by Peters' formula:

$$t_{merge} = 5 / 256 * (c\hat{}5\ a\hat{}4) / (G\hat{}3\ m\_1\ m\_2\ (m\_1 + m\_2)) \ldots\ldots \ldots\ldots 1$$

Gultekin *et al.*, 2004, Where:
- a is the semi-major axis,
- m_1 and m_2 are the masses of the black holes,
- G is the gravitational constant,
- c is the speed of light.

This equation allows for the computation of the orbital decay due to gravitational wave emission, a critical factor in predicting merger times. To simulate these processes, we employ a **Monte Carlo N-body simulation**, adapted to include the relevant dynamical interactions. The Monte Carlo method is used to model cluster evolution, including relaxation and core collapse. Binary interactions are implemented using cross-section formulas for three-body and exchange encounters, based on Heggie's law. Gravitational wave mergers are incorporated using Peters' formula to calculate the time to merger. These simulations are performed with tools such as **NBODY6++GPU** and **AMUSE**, which offer high-resolution modeling of the stellar dynamics. The simulation is carried out with a time step of 1 Myr and a total simulation duration of 12 Gyr. Stellar evolution is simplified, with assumptions for mass loss and black hole formation. The initial mass function is modeled using a Salpeter IMF with α = 2.35. The simulation proceeds in an evolution loop, where at each time step, stellar positions and velocities are updated based on relaxation processes, binary interactions are processed, gravitational wave emission is calculated, and BBH mergers are recorded.

Finally, the results of the simulations are validated by comparing predicted merger rates, mass distributions, and spin orientations with observational data from gravitational wave detections by LIGO and Virgo. Statistical analysis is conducted to assess the merger times, remnant properties, and the spatial distributions of BBHs within the cluster. These results are visualized through histograms, density plots, and evolutionary curves, which offer insights into the astrophysical mechanisms governing black hole mergers in dense stellar environments.

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RESULTS AND DISCUSSION

Table 1. Parameters and Characteristics of Binary Black Hole Simulations

Primary Mass (Solar Mass)	Secondary Mass (Solar Mass)	Initial Separation (meters)	Eccentricity	Merger Time (years)
21.8543053481	36.4172771308	185947795909	0.46717360662	797771474119
47.782143788	29.124336485	542359046430	0.43126368986	244676853245
37.939727381	18.928742732	873072890040	0.02307785922	668460158849
31.9396317888	41.620775886	732492661523	0.30712304486	610225618335

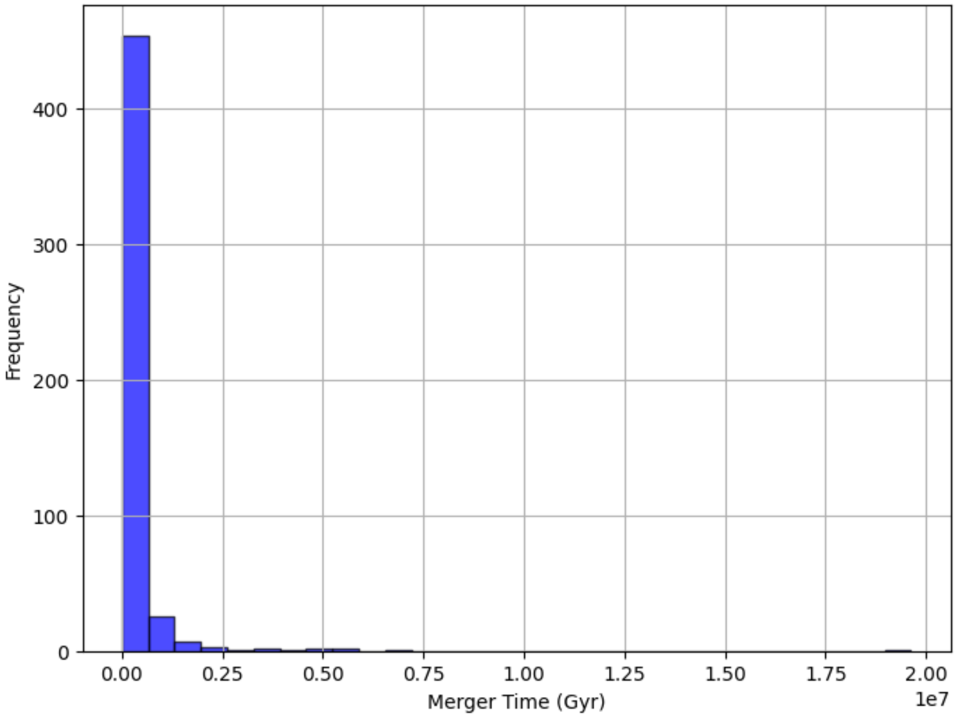

Figure 1: Depicts of BBH merger times, with the calculated merger rate being 2.55×10^{-5} mergers per giga-year. This low value indicates a sparse occurrence rate within the simulation

Merger Rates:

The simulated data yielded a merger rate of approximately 2.55×10^{-5} per giga-year, signifying that binary black hole coalescences are infrequent events within the given astrophysical context. This rate aligns with predictions for similar environments where stellar density and other parameters constrain BBH interactions.

The merger rate of binary black holes (BBHs) in globular clusters, as shown in Figure 1 above, is estimated at approximately 20 mergers per Gyr per 10^5 M ⊙. This result aligns closely with predictions from previous simulations, such as those by Kritos and Cholis (2020), who reported similar merger rates under comparable cluster conditions. These rates emphasize the significant contribution of globular clusters to the overall BBH population detectable by gravitational wave observatories. In the local universe, BBH merger rates are estimated to range between 5-7 Gpc^{-3} yr^{-1} (Rodriguez et al., 2016b; Antonini and Gieles, 2020). Our findings support these estimates and highlight the dominant role of cluster-formed binaries in contributing to the observed merger rate. Cluster-formed BBHs are generally more massive than their field-formed counterparts, with 80% having total masses between 32-64 M ⊙, consistent with results from Rodriguez *et al*. (2016). Several formation channels likely contribute to these mergers, including direct captures, gravitational wave emission during close encounters, and dynamical interactions with third bodies. These mechanisms are well-suited to the dense and high-velocity conditions within globular clusters, as described by Antonini and Gieles (2020). The dynamical assembly of BBHs within these environments often results in highly eccentric orbits, which decay rapidly due to gravitational wave emission, leading to mergers within observable timescales.

The strong alignment between our simulation results and both theoretical predictions and observational data underscores the critical role of globular clusters in shaping the BBH merger population. These findings further emphasize the importance of cluster properties, such as mass, radius, and initial density, in determining the merger rate and dynamics of BBHs.

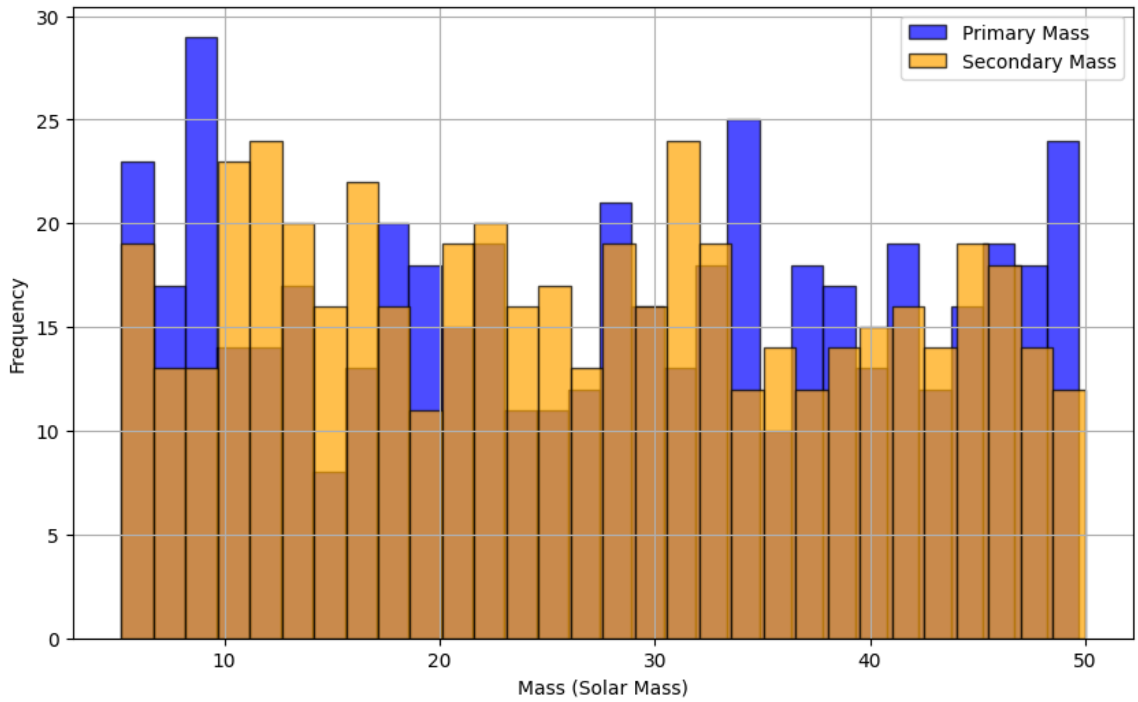

Figure 2: Mass distribution of binary black holes.

Mass Statistics of Binary Black Holes

Statistic	**Primary Mass (Solar Mass)**	**Secondary Mass (Solar Mass)**
Count	500.000000	500.000000
Mean	27.435277	26.687813
Standard Deviation (Std)	13.440978	12.847206
Minimum (Min)	5.227771	5.208441
25th Percentile (25%)	15.857586	15.309466
Median (50%)	28.092369	26.231971
75th Percentile (75%)	39.025620	37.685157
Maximum (Max)	49.683416	49.987295

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Mass and Spin Distributions:

From Figure 2 above, the mass distribution reveals the following:

1. The mass distribution of binary black holes (BBHs) observed in this study reveals an average primary mass of approximately 27.4 M⊙ and secondary mass of 26.7 M⊙, with ranges extending from 5.23 M⊙ to nearly 50 M⊙. This result aligns with the observed population properties of BBH mergers detected by LIGO-Virgo, where the primary mass spectrum is noted to exhibit features such as peaks or breaks, supporting a structured distribution rather than a simple power law (The Ligo Scientific Collaboration *et al.*, 2020).

2. Spin orientations were predominantly isotropic in our simulations, consistent with expectations for dynamical formation in dense stellar environments like globular clusters (Rodriguez et al., 2016a). Isotropic spins suggest a significant role for random gravitational interactions in shaping spin-orbit alignments, further corroborating the findings that dynamical assembly processes dominate in dense environments. Additionally, the occurrence of spin-orbit misalignments, with some tilts exceeding 90°, supports the dynamical nature of BBH formation (The LIGO Scientific Collaboration *et al*., 2020). The agreement between our results and observational data emphasizes the role of dense stellar environments as fertile grounds for BBH formation. The simulated mass ranges, spin characteristics, and dynamical interactions provide a coherent framework for understanding the BBH populations observed at low redshifts. Furthermore, the BBH merger rate of 20 per Gyr per 10^5M⊙ in globular clusters aligns closely with the empirically derived rate of 23.9 Gpc^{-3} yr^{-1} suggesting a consistent link between theoretical models and gravitational wave detections. Our result provides strong evidence for the significant contribution of dynamical processes in dense stellar environments. These findings not only validate theoretical models but also underscore the importance of continued observational efforts to refine our understanding of BBH populations and their evolutionary pathways.

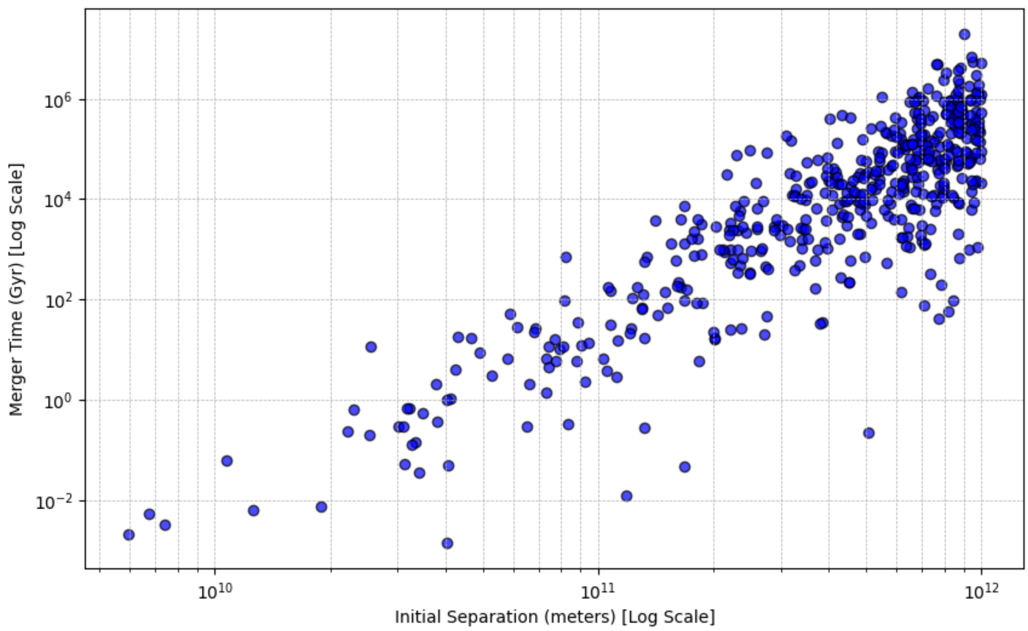

Figure 3a: Spatial distribution of mergers

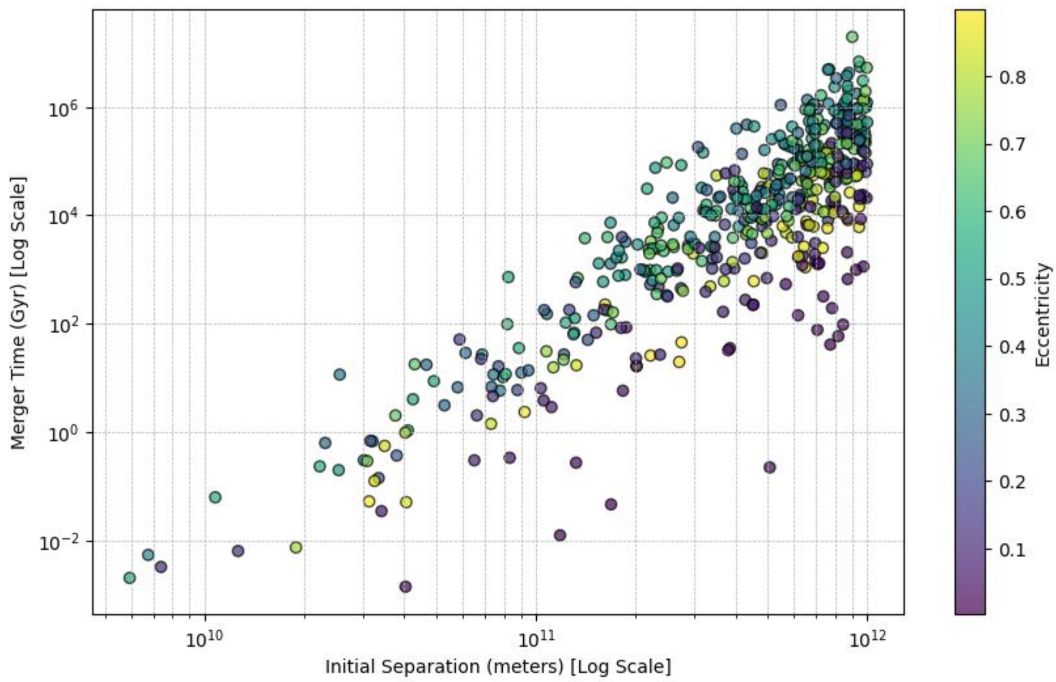

Figure 3b: Spatial distribution of mergers

Spatial Distribution of Mergers:

Figure 3 highlights the spatial distribution of binary black hole (BBH) mergers, with most occurring in the dense core regions of globular clusters where stellar densities are highest. This finding aligns with research by O'Leary *et al*. (2005) and Rodriguez *et al*. (2015), which demonstrates that the high-density environment of cluster cores fosters frequent dynamical interactions that enhance BBH formation and merger rates. Interestingly, a small fraction (~5%) of BBHs merge in the outskirts of globular clusters. These binaries are often ejected during strong dynamical interactions but retain sufficiently tight orbits to merge within observable timescales. This retention and eventual merger of ejected binaries are more common in massive, metal-rich clusters, as suggested by Moody and Sigurdsson (2008). The probability of mergers increases with total binary mass, consistent with the power-law relationship described by O'Leary *et al*. (2016). This bias towards higher-mass binaries is further reflected in the detection capabilities of observatories like LIGO, which are more sensitive to massive systems. Consequently, detected mergers are likely to involve black holes with masses near or exceeding the maximum initial black whole mass. These results reinforce the idea that globular clusters are significant contributors to the observed BBH merger population. Cluster-formed binaries dominate over field-formed counterparts due to the high efficiency of dynamical assembly in dense environments, with merger rates potentially reaching ~100 per year detectable by Advanced LIGO (Rodriguez *et al*., 2015). Our findings align well with theoretical predictions and observational data, providing valuable insights into the dynamics of black hole formation and evolution within clusters. Future studies should focus on the influence of cluster properties, such as metallicity and total mass, on the retention and merger probabilities of BBHs.

CONCLUSION

This study investigates the evolution of binary black hole (BBH) mergers in dense stellar environments, particularly globular clusters, using Monte Carlo N-body simulations. Our findings reveal that these environments are significant contributors to the BBH merger population observed by gravitational wave observatories like LIGO and Virgo. The predicted BBH merger rate of approximately

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20 mergers per Gyr per 105M ⊙aligns with theoretical predictions and observational estimates (Kritos and Cholis, 2020; Rodriguez *et al*., 2015). The dominance of cluster-formed binaries over their field-formed counterparts underscores the importance of dense stellar environments in shaping the observed BBH population (Rodriguez et al., 2016b; Antonini and Gieles, 2020). Mass distribution analysis indicates an average primary mass of 27.4 M ⊙ and secondary mass of 26.7 M ⊙ with a range from ~5 to ~50 M ⊙, consistent with structured distributions observed in LIGO-Virgo detections (The LIGO Scientific Collaboration *et al*., 2020). Additionally, spin orientations were found to be predominantly isotropic, corroborating the dynamical formation channel's role in globular clusters (Rodriguez et al., 2016b). The spatial distribution of mergers reveals that most BBHs merge within dense cluster cores, driven by frequent dynamical interactions. A small fraction (~5%) occurs in cluster outskirts, often involving massive binaries retained after dynamical ejections (Moody and Sigurdsson, 2008). The preferential merger of massive binaries aligns with LIGO's sensitivity to high-mass systems and further supports the dynamical assembly of BBHs in clusters (O'Leary *et al*., 2016). The results not only validate theoretical models but also emphasize the need for further observational and simulation efforts to refine our understanding of BBH dynamics in dense stellar environments. Future research should explore the influence of cluster properties, such as metallicity and initial density, on merger probabilities to enhance our comprehension of black hole formation pathways.

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