

# Maximum herd efficiency in meat production

## I. Optima for slaughter mass and replacement rate

C.Z. Roux\*

Department of Genetics, University of Pretoria, Pretoria, 0002 Republic of South Africa

Received 29 June 1990; accepted 24 May 1991

Profit rate for a meat production enterprise can be decomposed into the unit price for meat and herd efficiency. Optimal slaughter mass maximizes herd efficiency which, at its maximum, can be expressed in terms of a product of powers of growth and reproduction efficiencies. Likely regions of optimality for slaughter mass are indicated for cattle, sheep, pigs and chickens. Herd efficiency is tabled in terms of percentages of reproduction efficiency and the allometric slope between body mass and cumulate feed intake. Optimal replacement involves either the minimum or maximum rate that can be achieved, and depends on the relative costs and output involved in the keeping of different age classes of reproduction animals. Finally, the relationship between replacement rate and herd age structure is explained.

Die winsverhouding by vleisproduksie kan opgebreek word in die eenheidsprys vir vleis en in kuddedoeltreffendheid. Kuddedoeltreffendheid word deur 'n optimale slagmassa gemaksimiseer, en kan by sy maksimum uitgedruk word as die produk van magte van groei- en reprodüksiedoeltreffendhede. Die verwagte optimale gebiede vir slagmassa is vir beeste, skape, varke en hoenders afgelei. Kuddedoeltreffendheid word getabelleer in terme van persentasies van reprodüksiedoeltreffendheid en die allometriese helling tussen liggaansmassa en geakkumuleerde voerinnam. Optimale vervanging behels haalbare minimum of maksimum tempo's en berus op die relatiewe kostes en produksie wat aan verskillende ouderdomgroepe teeldiere verbonde is. Die verwantskap tussen vervangingstempo en kuddo-ouderdomstruktuur word aangedui.

**Keywords:** Breeding objectives, herd efficiency, meat production, replacement rate, slaughter mass.

\* Present address: Animal and Dairy Science Research Institute, Private Bag X2, Irene, 1675 Republic of South Africa.

### Introduction

In the modern meat production enterprise, nutritional, ecological, biotechnological, physiological, behavioural, genetical and breeding principles should be integrated for maximum profit subject to the ethical requirement for the humane treatment of animals. That there is a penalty to be paid for anything less than such a holistic approach is evident from the magnitude of stress-related problems and diseases such as pale, soft, exudative muscle in pigs and ascites in chickens. Due to varying circumstances in different species and environments, such a holistic approach to meat production is probably very difficult to formulate in its entirety. Instead, the approach followed here is to identify possible biological principles and sources of profit, which can then be fitted into a breeding and management programme as dictated by prevailing circumstances.

The maximization of profit, with the whole production enterprise taken into consideration, is probably the simplest possible breeding objective. The main disadvantage is that profit can fluctuate widely according to the market forces of supply and demand, whereas breeding improvement is generally a long-term enterprise requiring stable goals.

Define herd profit rate as income per year from the sale of young animals (feeders, market animals), divided by total feed costs of the whole herd as well as constant costs per animal. Herd efficiency is total live mass for slaughter divided by costs. The development in the next section will show that the relative increase in profit rate is approximately equal to the relative increase in value (price) of meat plus the relative increase in total herd cost efficiency. Maximizing herd cost efficiency is, therefore, one of the two ways in which profit

rate can be maximized. The advantage to considering herd cost efficiency as the most important objective in breeding improvement and growth manipulation, is that it depends primarily on simple assumptions on the relative cost of feed and management for growth and reproduction, that enter into the calculation of herd efficiency at optimal slaughter mass. Relative costs are probably more stable than prices, with the additional advantage that it is easy to conceptualize the effect of variations in them on herd efficiency. Furthermore, if changes in product value are important, it is easy to join them to herd cost efficiency for the prediction of profit.

According to Dickerson (1970; 1976), breeding strategies should be evaluated in terms of total herd or life cycle efficiency, and not only for a convenient part of the cycle or herd. To the best of knowledge, Wallace (1955) was the first to discover that an optimal slaughter mass, when feed efficiency is at a maximum value, exists when both the feed requirements of ewes and lambs are taken into account. Taylor *et al.* (1985) developed formulae from the growth and feed efficiency equations of Parks (1982), which enabled them to evaluate different breeding strategies in relation to optimal slaughter masses for beef cattle. A similar approach to that of Taylor *et al.* (1985) can be followed by using the allometric autoregressive approach developed by Roux (1974; 1976; 1981). The mathematical development in terms of the allometric form has the advantage of greater simplicity than that of Taylor *et al.* (1985), allowing the evaluation of breeding procedures and selection goals in a very simple fashion with a minimum of restrictive assumptions.

## Assumptions and Theory

Reproduction and growth can best be compared in monetary units. Furthermore, a comparison in financial terms entails no loss in generality, since feed or energetic efficiency can be obtained from cost efficiency by an appropriate simple conversion of units. Let (i)  $w$  be the cost of raising (including feed, housing, health care, etc.), from conception to mating, a reproducing female, or the cost of obtaining a female at mating time to include in the reproducing herd; let (ii)  $x$  be the cost of feed, care and fertilization from mating to weaning of offspring of a first parity female, minus the portion of the preweaning feed cost attributed to market offspring in equation (3), below; let (iii)  $y$  be the cost of feed, care and fertilization from one weaning to the next for the older females, minus the preweaning feed cost attributed to market offspring in equation (3), below; and let (iv)  $z$  be the value of a mature female. Define replacement rate as the proportion of first parity females in a herd and denote it by  $R$ . The total reproduction cost ( $A$ ) of the herd is then:

$$\begin{aligned} A &= (1 - R)y + R(w + x - z) \\ &= y + R(w + x - y - z), \end{aligned} \quad (1a)$$

under the assumption that all replaced females are sold for the amount  $z$ . If the mortality rate is of substantial magnitude, it may be advantageous to replace  $z$  by  $z' = z(1 - \text{mortality rate})$ . The replacement rate per year would then be equal to number of parities per year multiplied by  $R$ .

Define reproduction rate as the ratio of number of offspring surviving to either slaughter or first mating to total number of females exposed to breeding. Denote, furthermore, the reproduction rate by  $r_1$  for first parity females and by  $r_2$  for the rest of the dams. It follows that the average reproduction rate ( $r$ ) of the herd is:

$$r = r_1 R + r_2 (1 - R). \quad (1b)$$

The reproductive or maternal overhead cost per market animal ( $Q$ ) is then:

$$\begin{aligned} Q &= A / (r - R) \\ &= [y + R(w + x - y - z)] / [r_2 - R(1 - r_1 + r_2)], \end{aligned} \quad (1c)$$

independent of the time units involved, since they cancel in the ratio. If they are important, other costs that do not vary with market animal mass can also be included in  $Q$ .

Denote by  $f$  the cumulate feed cost of a market animal to reach the body mass  $m$ , and let  $v$  be the unit value (price) for meat on the hoof. Define herd profit rate ( $Pr$ ) as the ratio of income over costs, so that:

$$Pr = vm / (f + Q), \quad (2a)$$

with  $Q$  defined in (1c).

The ratio

$$e_h = m / (f + Q), \quad (2b)$$

will be denoted by the term herd cost efficiency, since both reproduction and growth costs are incorporated. Let  $\Delta Pr$  represent an increase in the profit rate, and define  $\Delta v$  and  $\Delta e_h$  likewise. Then, for small increases in  $\Delta v$  and  $\Delta e_h$ , such that their product can be neglected in relation to the other quantities, it follows that:

$$\Delta Pr / Pr = \Delta v / v + \Delta e_h / e_h. \quad (2c)$$

Equations (2a) and (2c) show that, after allowing for possible changes in  $v$ , profit rate can be maximized from the maximization of herd cost efficiency  $e_h$ .

Note that the maximization of (2a) is equivalent to the minimization of its inverse, cost per unit of output value. Hence, all the results of this series of papers, including those on optimal slaughter mass, carry over to the inverse formulation. The only aspect that needs handling with care follows since the relative decrease in cost per unit of product ( $\Delta c_h / c_h$ ) is equal to  $(1 + \Delta e_h / e_h)^{-1} - 1$ . This expression is only approximately equal to  $-\Delta e_h / e_h$  for small values of  $\Delta e_h / e_h$ , say 7% or less.

In equation (2b) the income from culled females is included in  $Q$ . This income could also have been included in the numerator with  $m$ , but such a definition unfortunately leads to mathematically intractable results. In some situations the two definitions of profit rate or herd efficiency lead to similar conclusions. The situations where the two definitions lead to disparate results will be noted in the discussion of equation (11). The essential difference between (2b) and its possible alternative, (11), is that (2b) accentuates the profit from the sale of young meat.

The definition of  $Q$  is closely related to the expenses associated with the female herd by Dickerson (1976). The most important differences are a separation of feed cost into maintenance and above-maintenance components by Dickerson, and that in the calculation of  $Q$ , the offspring feed expense incorporated in (2b) is subtracted from the feed expense of the female herd, since it is associated with offspring size. In this respect the definition of  $Q$  corresponds to that of Taylor *et al.* (1985).

According to Roux (1974; 1976; 1981) and Roux & Meissner (1984), body mass ( $m$ ) during growth is related to the cost of cumulate feed intake ( $f$ ):

$$m = \alpha_m (f / \alpha_f)^b, \quad (3)$$

where  $(\alpha_f, \alpha_m)$  is any convenient point on the line (3) relating  $f$  and  $m$ , such as the point representing the maximum body mass of an animal for a given growth phase, and the cost of the amount of feed ingested, from conception onwards, to reach that mass. The exponent,  $b$ , is a constant for a given animal or group of animals during a particular growth phase. For example, from Roux & Meissner (1984),  $b = 0,56$  for steers from the Bonsmara breed with a predicted limit mass ( $\alpha_m$ ) of about 990 kg on about 9 800 kg dry material, which can be appropriately converted to monetary units ( $\alpha_f$ ).

From (2b) and (3) it follows by substitution that herd cost efficiency is equal to:

$$e_h = 1 / [(\alpha_f / \alpha_m) (m / \alpha_m)^{1/b-1} + Q / m] \quad (4a)$$

or

$$e_h = 1 / [(m / \alpha_m)^{1/b-1} e_g^{-1} + (m / \alpha_m)^{-1} e_r^{-1}]. \quad (4b)$$

The ratio  $e_g = \alpha_m / \alpha_f$  is the cumulate growth cost efficiency at a given point of comparison, and  $e_r = \alpha_m / Q$  is the reproduction cost efficiency at that point. In many situations it will be convenient to regard  $\alpha_m$  either as the maximum or limit mass during a growth phase, or a given percentage thereof.

For a given animal,  $e_g$  and  $e_r$  are constants. The form of the growth efficiency curve dictates  $b < 1$ , so that the term  $(m / \alpha_m)^{1/b-1}$  increases as  $m$  increases to  $\alpha_m$ , and  $(m / \alpha_m)^{-1}$

decreases under the same circumstances. Hence,  $e_g^{-1}$  becomes more important in  $e_h$  with increasing  $m$  and  $e_r^{-1}$  less important. These opposing tendencies of the two terms in  $e_h$ , suggest that there may be an optimal slaughter mass  $o < m \leq \alpha_m$ , for which  $e_h$  will be a maximum.

The optimal value of  $m$ , when herd cost efficiency is a maximum, can be obtained from (4a) by differentiation of  $e_h$  and equating the differential  $de_h/dm$  to zero. The mass at maximum herd cost efficiency is:

$$\begin{aligned} m/\alpha_m &= [bQ/(1-b)\alpha_f]^b \\ &= [be_g/(1-b)e_r]^b. \end{aligned} \quad (5)$$

It follows from (3) and (5) that herd cost efficiency ( $e_h$ ) attains a maximum when:

$$bm/f = (1-b)m/Q,$$

i.e. at the intersection of the lines,  $bm/f$  and  $(1-b)m/Q$ , representing weighted cumulate growth and reproduction efficiencies. At this intersection, the opposing tendencies of  $m/f$  and  $m/Q$ , associated with  $m/(f+Q)$ , are in equilibrium.

The substitution of (5) in (4a) gives the maximum herd cost efficiency in two equivalent forms:

$$e_h(\max) = b^b (1-b)^{1-b} (Q/\alpha_f)^b (\alpha_m/Q) \quad (6a)$$

or

$$e_h(\max) = b^b (1-b)^{1-b} (\alpha_m/\alpha_f)^b (\alpha_m/Q)^{1-b}. \quad (6b)$$

In situations where no risk of confusion exists,  $e_h(\max)$  will simply be denoted by  $e_h$ .

As will become clear in the discussion of Table 1, (5) will sometimes give values of  $m > \alpha_m$ , in which case one may be forced to slaughter animals near the limit mass,  $\alpha_m$ . For  $m = \alpha_m$ , (4b) gives:

$$e_h(m = \alpha_m) = 1/(e_g^{-1} + e_r^{-1}). \quad (6c)$$

Equation (6c) is then the maximum achievable herd efficiency. From the algebraical fact that the harmonic mean is always less than or equal to the geometric mean, it follows that:

$$e_h(m = \alpha_m) \leq e_h(b = 1/2), \quad (6d)$$

with  $e_h(b = 1/2)$  equal to the maximum herd efficiency of (6b) with  $b = 1/2$ . Equality will hold in (6d) only if  $e_g = e_r$ .

### Slaughter mass as a percentage of limit mass

From equation (5) it follows that the percentage value of  $m/\alpha_m$  depends on the value of  $b$  and the ratio of reproduction overheads to offspring feed cost ( $Q/\alpha_f$ ). Values of  $m/\alpha_m$  in relation to  $b$  and  $Q/\alpha_f$  are given in Table 1. From Table 1 it follows, for example, with  $Q/\alpha_f = 1/4$  and  $b = 0,7$ , that the slaughter mass at maximum efficiency (optimal slaughter mass) is  $m = 0,69\alpha_m$ . For given  $b$ -values, lower  $Q/\alpha_f$  values strictly indicate lower percentage values of  $\alpha_m$  for optimal slaughter masses. For a given  $Q/\alpha_f$  value, intermediate values of  $b$  give the lowest optimal slaughter values. The empty values in Table 1 have optimal slaughter values greater than the limit mass, which may mean that slaughter mass values near the maximum mass will give the maximum achievable herd efficiency. An example for cattle and sheep follows below, showing that maximum achievable herd efficiencies may often be very close to the maximum.

**Table 1** Optimal slaughter mass as a percentage of limit mass

| $Q/\alpha_f$ | 1/9 | 1/4 | 1/2 | 3/4 | 1   | 2   | 3   |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| $b$          |     |     |     |     |     |     |     |
| 0,1          | 64  | 70  | 75  | 78  | 80  | 86  | 90  |
| 0,2          | 49  | 57  | 66  | 72  | 76  | 87  | 94  |
| 0,3          | 40  | 51  | 63  | 71  | 78  | 95  | 107 |
| 0,4          | 35  | 49  | 64  | 76  | 85  | 112 | -   |
| 0,5          | 33  | 50  | 71  | 87  | 100 | -   | -   |
| 0,6          | 34  | 56  | 84  | 107 | -   | -   | -   |
| 0,7          | 39  | 69  | 111 | -   | -   | -   | -   |
| 0,8          | 52  | 100 | -   | -   | -   | -   | -   |
| 0,9          | 100 | -   | -   | -   | -   | -   | -   |

Dickerson (1978) gives proportions of feed going to different facets of production and reproduction in different species, which can be used as a first approximation to the  $Q/\alpha_f$  ratios. It is assumed here that the cost of gestation and lactation is determined by the size of offspring and is therefore included in  $f$  in (2a), analogous to the assumption of Taylor *et al.* (1985). Hence, the  $Q/\alpha_f$  ratio ranges from about 1/9 in chickens to 1/4 in pigs, to approximately 1 in cattle and 1 1/3 in sheep.

### Slaughter mass for cattle and sheep

For cattle, Dickerson (1978) gives  $Q = \alpha_f$ , approximately, while  $b$  lies between 0,5 and 0,6, from Roux & Meissner (1984). Hence, (4) gives a slaughter mass of  $\alpha_m$  to  $1,3\alpha_m$ . If slaughter masses need to be below the values indicated by (4), it follows from (6d) and similar magnitudes of  $e_g$  and  $e_r$  that the theory associated with (6a, 6b) remains applicable to a fair degree of approximation.

From (4b) and under the assumptions that  $Q = \alpha_f$  or  $e_r = e_g$ , and  $b = 0,55$ , the values of  $e_h/e_h(\max)$  % can be tabled against  $m/\alpha_m$  as follows:

|                   |     |     |     |     |      |
|-------------------|-----|-----|-----|-----|------|
| $m/\alpha_m$      | 0,7 | 0,8 | 0,9 | 1,0 | 1,1  |
| $e_h/e_h(\max)$ % | 92  | 96  | 99  | 100 | 100. |

This shows that  $e_h$  has a reasonably flat surface, so that, if seasonal variation in prices and feed quality is kept in mind, probably no great loss would be incurred if slaughter takes place for  $m > 0,8\alpha_m$ . Roux & Meissner (1984) defined limit mass as the mass of an animal which is difficult to exceed. Thus, while animals tend to get fatter as they grow older, especially on concentrate feeds, this derivation suggests that cattle should be slaughtered when their body masses tend to be temporarily stationary in early maturity. For sheep, Dickerson (1978) gives  $Q > \alpha_f$ , hence it follows from Table 1 that sheep should be slaughtered as late as is commensurate with meat grading requirements.

### Pig slaughter mass

From Tess *et al.* (1983) it seems plausible to approximate  $Q = \alpha_f/4$ . For  $b = 0,72$  from Siebrits *et al.* (1986), slaughter mass is then predicted by  $m = 0,73\alpha_m$ , from (4). From Siebrits *et al.* (1986),  $\alpha_m$  lies between 157 and 206 kg, suggesting optimal slaughter masses between 115 and 150 kg.

This is somewhat higher than the customary slaughter mass of 90 kg in South Africa. However, a plot of  $e_h/e_h(\max)\%$  against  $m/\alpha_m$  for  $e_r = 4e_g$  (since  $Q = \alpha_f/4$ ) and  $b = 0,72$ , according to (4b) reveals:

|                   |     |     |     |     |     |
|-------------------|-----|-----|-----|-----|-----|
| $m/\alpha_m$      | 0,3 | 0,4 | 0,5 | 0,7 | 0,9 |
| $e_h/e_h(\max)\%$ | 84  | 93  | 97  | 100 | 99. |

Hence, no great loss would be involved by slaughtering at  $0,5\alpha_m$  or between 80 and 100 kg.

#### The allometric slope and herd efficiency

Let  $\Delta e_h/e_h$  be the relative gain in herd efficiency and  $\Delta b$  the gain in the allometric slope. It follows from (6b) that:

$$\Delta e_h/e_h = [bQ/(1-b)\alpha_f]^{\Delta b} - 1. \quad (7)$$

Note that both  $\Delta e_h$  and  $\Delta b$  can only be positive if:

$$bQ/(1-b)\alpha_f > 1$$

or from (5), if for slaughter mass ( $m$ ),

$$m > \alpha_m.$$

A slaughter mass larger than the limit mass is impossible, so that (7) implies that  $b$  and  $e_h$  are negatively related for all animals with (allowable) optimal slaughter masses less than their limit masses ( $m < \alpha_m$ ), if  $e_g$  and  $e_r$  (from 4b) remain constant. For animals with  $m = \alpha_m$ , (7) and (6c) indicate that  $e_h$  and  $b$  are unrelated.

The expression for maximum herd efficiency ( $e_h$ ) from (6a) can be factored into a term containing  $b$  and the  $Q/\alpha_f$  ratio and a term consisting of reproduction efficiency ( $e_r = \alpha_m/Q$ ). It is, therefore, natural to table  $e_h$  in Table 2 in terms of percentages of  $e_r$ , according to different values of  $b$  and the  $Q/\alpha_f$  ratio. The empty values in Table 2 correspond to optimal slaughter masses of more than 100% of limit mass in Table 1. The decline of herd efficiency with increases in  $b$  in Table 2 is in agreement with the conclusions emanating from (7). This is an astonishing result, since an increase in  $b$  is associated with an increase in ordinary fixed period growth efficiency. An experiment to test this prediction is under way at the Animal and Dairy Science Research Institute (Irene).

A selection experiment on rats (Scholtz *et al.*, 1990) suggests that the possibility for selection progress in  $b$  is limited, since after four generations of gains according to

**Table 2** Herd efficiency as a percentage of reproduction efficiency

| $Q/\alpha_f$ | 1/9 | 1/4 | 1/2 | 3/4 | 1  | 2  |
|--------------|-----|-----|-----|-----|----|----|
| $b$          |     |     |     |     |    |    |
| 0,1          | 58  | 63  | 67  | 70  | 72 | 77 |
| 0,2          | 39  | 46  | 52  | 57  | 61 | 70 |
| 0,3          | 28  | 36  | 44  | 50  | 54 | 67 |
| 0,4          | 21  | 29  | 39  | 45  | 51 | -  |
| 0,5          | 17  | 25  | 35  | 43  | 50 | -  |
| 0,6          | 14  | 22  | 34  | -   | -  | -  |
| 0,7          | 12  | 21  | -   | -   | -  | -  |
| 0,8          | 10  | 20  | -   | -   | -  | -  |
| 0,9          | 10  | -   | -   | -   | -  | -  |

prediction, progress ceased. Furthermore, considerable antagonism between natural and artificial selection sometimes forced the differences between the high and low lines back to the initial level. Consequently it seems reasonable to suppose that a fair degree of genetic or physiological canalization for the allometric slope,  $b$ , may exist. Forcing  $b$  beyond its canalized borders by selection leads to a decline in reproduction and viability.

#### Optimal replacement rate

From (1) and (6), reproduction efficiency is:

$$e_r = \alpha_m [r_2 - R(1 - r_1 + r_2)]/[y + R(w + x - y - z)], \quad (8)$$

from which it follows that, if:

$$1 - r_1 + r_2 > 0 \text{ and } w + x > y + z, \quad (9)$$

then the replacement rate ( $R$ ) must be as small as possible for  $e_r$  to be as large as possible. If the feed costs  $x$  and  $y$  are approximately equal, then  $w + x > y + z$  means that the value of a young female ( $w$ ) must be greater than that of an old female ( $z$ ). In countries with meat grading systems emphasizing meat quality, this may often be true, since young meat is more tender than meat from old animals.

However, it can also be shown that (9) can be replaced by the more general condition:

$$(1 - r_1)y > (z - w - x)r_2, \quad (10)$$

for  $R$  to be as small as possible. It should be remembered that  $Q > 0$  in (1) for the foregoing theory to be applicable. If the inequality sign in (10) is reversed,  $R$  should be taken as large as possible for maximum herd efficiency.

In contrast to the results flowing from (9) and (10), Taylor *et al.*'s (1985) general recommendation is that  $R$  should be as large as possible. Part of the explanation is that they considered lean tissue production, regardless of meat quality considerations. Hence, inevitably the value of an old female must be larger than that of a young female, or  $z > w$ , in present notation. If  $z > w$ , then the reversal of the inequality in (10) becomes more likely. The other part of the explanation depends on Taylor *et al.*'s (1985) definition of overall efficiency,  $e_o$ , which can be transcribed to:

$$e_o = [(r - R)vm + Rz]/[(r - R)f + y + R(w + x - y)] \quad (11)$$

in terms of the symbols defined in (1a), in contrast to herd profit rate from (2a):

$$Pr = [(r - R)vm]/[(r - R)f + y + R(w + x - y - z)],$$

where  $v$  is the unit price of meat, included here to render the different quantities comparable. Note that both  $e_o$  and  $Pr$  are equal to  $rvm/(rf + y)$  for  $R = 0$ , indicating approximate equality also for small replacement rates ( $R$ ). However, with replacement rates equal to their maximum values, i.e. equal to reproduction rates ( $R = r$ ) in Taylor *et al.*'s (1985) female single sex system,  $e_o = rz/[y + r(w + x - y)]$ , while  $Pr = 0$ , as it should, since there are no young animals for sale or slaughter.

Taylor *et al.*'s (1985) overall efficiency is the appropriate definition when systems are of interest where the possibility may exist that  $e_o(R = r) > e_o(R = 0)$ , i.e. where

$$z/[y + r(w + x - y)] > m/(rf + y), \text{ or for } r = 1, \text{ where}$$

$$z/(w + x) > vm/(f + y).$$

Since the cost of lactation and pregnancy may be proportional to offspring size and therefore part of  $f$  rather than  $Q$  in (2a), it may be that  $z/(w+x)$  and  $vm/f$  do not differ much in circumstances where meat quality is ignored and where the slaughter mass is near to the limit mass. It is, then, clear that a system with  $R = r$  will be more profitable than one with a small replacement rate due to the cost  $y$  of keeping a breeding female from weaning to weaning. An example of such a system is Taylor *et al.*'s (1985) single-sex, bred-heifer system.

It is important to note that the maximization of  $R$  does not allow terminal crossbreeding in species with low reproduction rates.

### Replacement rate and herd age structure

Let  $p_i$  be the proportion of females retained in the herd from their  $i$ -th mating to their  $i+1$ -th mating. Then a discrete aging model from Karlin (1968) can be used to obtain the herd age structure that will be stochastically induced with the passage of time by a certain set of probabilities,  $p_i$ ,  $i = 1, 2, \dots, n$ , under the assumption of a constant herd size.

This herd structure is:

|                    |       |         |            |                          |                          |
|--------------------|-------|---------|------------|--------------------------|--------------------------|
| Mating number      | 1     | 2       | 3          | $i$                      | $n$                      |
| Proportion of herd | $1/g$ | $p_1/g$ | $p_1p_2/g$ | $p_1p_2 \dots p_{i-1}/g$ | $p_1p_2 \dots p_{n-1}/g$ |

where

$$g = 1 + p_1 + p_1p_2 + \dots + p_1p_2 \dots p_{i-1} + \dots + p_1p_2 \dots p_{n-1}.$$

The replacement rate ( $R$ ) is equal to the proportion of females in the first mating group, so that:

$$R = 1/g. \quad (12a)$$

If one starts off with a herd of females at their first mating, it may be necessary for many years to elapse before the stochastically induced mating structure for a constant herd size is approximated, but eventually it will be reached. In some circumstances it may be advantageous to aid the stochastic process by management practice. With relatively large probabilities,  $p_i$ , the herd age structure will consist of age classes of about equal size. In such a situation:

$$R = 1/n, \quad (12b)$$

approximately. Even if it is advantageous to buy replacements for the reproduction herd every  $n$  years, the average replacement rate will be given by (12b).

In the situation where  $p_1 \neq p_2 = p_3 = \dots = p_n = p$ :

$$g = 1 + p_1(1 - p^{n-1})/(1 - p) \quad (12c)$$

This portrays the situation where young females are less fertile than the older ones in the herd, as may happen when the repeatability of fertility is of reasonable magnitude and infertile females are culled. If young females can be tested or examined for pregnancy and the non-pregnant ones can be sold for slaughter near their optimal slaughter masses,  $p_1 = 1$  for herd age structure purposes. If  $p_1 = p$ :

$$g = (1 - p^n)/(1 - p). \quad (12d)$$

### Discussion

The main practical value of the results thus far is the indication where optimal slaughter masses are likely to be for different species of farm animals, and the delineation of conditions when maximum or minimum replacement rates would maximize herd efficiency. However, the main applications of the present approach are deferred, by necessities of space, to succeeding articles (Roux, 1992a; 1992b; Roux & Scholtz, 1992). For ease in cross-referencing, the equations will be numbered consecutively in the different articles.

### References

- DICKERSON, G.E., 1970. Efficiency of animal production – molding the biological components. *J. Anim. Sci.* 10, 849.
- DICKERSON, G.E., 1976. The choice of selection objectives in meat-producing animals. In: Meat Animals. Growth and productivity. Eds. Lister, D., Rhodes, D.N., Fowler, V.R. & Fuller, M.F., Plenum Press, New York. p. 449.
- DICKERSON, G.E., 1978. Animal size and efficiency: Basic concepts. *Anim. Prod.* 27, 367.
- KARLIN, S., 1968. A first course in stochastic processes. Academic Press, New York.
- PARKS, J.R., 1982. A theory of feeding and growth of animals. Springer-Verlag, Berlin.
- ROUX, C.Z., 1974. The relationship between growth and feed intake. *Agroanimalia* 6, 49.
- ROUX, C.Z., 1976. A model for the description and regulation of growth and production. *Agroanimalia* 8, 83.
- ROUX, C.Z., 1981. Animal growth in the context of time series and linear optimal control systems. *S. Afr. J. Anim. Sci.* 11, 57.
- ROUX, C.Z., 1992a. Maximum herd efficiency in meat production. II. The influence of growth and reproduction. *S. Afr. J. Anim. Sci.* 22, 6.
- ROUX, C.Z., 1992b. Maximum herd efficiency in meat production. III. Feeder-breeder dimorphism. *S. Afr. J. Anim. Sci.* 22, 11.
- ROUX, C.Z. & MEISSNER, H.H., 1984. Growth and feed intake patterns. 1. The derived theory. In: Herbivore nutrition in the subtropics and tropics. Eds. Gilchrist, F.M.C. & Mackie, R.I., The Science Press, Craighall. p. 672.
- ROUX, C.Z. & SCHOLTZ, M.M., 1992. Maximum herd efficiency in meat production. IV. Crossbred reproduction and constant slaughter mass. *S. Afr. J. Anim. Sci.* 22, 16.
- SCHOLTZ, M.M., ROUX, C.Z., DE BRUIN, D.S. & SCHOEMAN, S.J., 1990. Medium-term responses to and changes in fitness with selection for parameters of the allometric-autoregressive model. *S. Afr. J. Anim. Sci.* 20, 65.
- SIEBRITS, F.K., KEMM, E.H., RAS, M.N. & BARNES, P.M., 1986. Protein deposition in pigs as influenced by sex, type and livemass. 1. The pattern and composition of protein deposition. *S. Afr. J. Anim. Sci.* 16, 23.
- TAYLOR, S.C.S., MOORE, A.J., THIESSEN, R.B. & BAILEY, C.M., 1985. Efficiency of food utilization in traditional and sex-controlled systems of beef production. *Anim. Prod.* 40, 401.
- TESS, M.W., BENNETT, G.L. & DICKERSON, G.E., 1983. Simulation of genetic changes in life cycle efficiency of pork production. 1. A bio-economic model. 2. Effects of components on efficiency. 3. Effects of management systems and feed prices on importance of genetic components. *J. Anim. Sci.* 56, 336.
- WALLACE, L.R., 1955. Factors influencing the efficiency of feed conversion by sheep. *Proc. Nutr. Soc.* 14, 7.