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Teachers' mathematics pedagogical content knowledge and quality of early mathematics instruction in Turkey

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It is a well-known fact that societies today need to provide quality mathematics education to individuals from their early years in order to compete on a global scale. Although there is a growing interest in early mathematics, there are still some obstacles regarding quality mathematics instruction. Two of the most important obstacles are the mathematics pedagogical content knowledge (MPCK) and early mathematics teacher qualification of pre-school teachers. The purpose of this study was to evaluate the relationship between pre-school teachers' classroom mathematics practices and their MPCK. The study included 55 pre-school teachers. Classroom Observation of Early Mathematics – Environment and Teaching (COEMET) was adapted into Turkish. The teachers' MPCK was assessed using the Survey of Pedagogical Content Knowledge in Early Childhood Mathematics (SPECKECM). The results show that the teachers' MPCK was at a medium level, being most successful regarding shapes and least in patterns. The COEMET scores were positively and statistically significantly correlated to the total score of the SPECKECM. Moreover, teachers' pedagogical content knowledge (PCK) scores successfully predicted the total COEMET scores.

Keywords: early mathematics; instructional quality; pedagogical content knowledge

Introduction

Global competition in the 21st century is based on a productive and innovative economy. An innovative economy demands a competent and satisfactory workforce that excel in mathematics (National Research Council [NRC], 2009). In this sense, mathematics is critical for all members of all societies in the world.

The National Council of Teachers of Mathematics (NCTM) and the National Association for the Education of Young Children (NAEYC) report that high-quality, challenging, and accessible mathematics education for children aged 3 to 6 is vital for future mathematics learning (NAEYC & NCTM, 2002/2010). Also, many studies show that early mathematics learning experiences contribute to children's learning and later achievement, in many fields, especially in mathematics (Bailey, Siegler & Geary 2014; Claessen & Engel, 2013; Ten Braak, Lenes, Purpura, Schmitt & Størksen, 2022; Watts, Duncan, Siegler & Davis-Kean, 2014).

Early mathematics education should include stimulating activities and learning environments that offer experiences to expand children's knowledge and develop mathematical concepts and skills. Thus, mathematics education is linked to the child, teacher, and environment in which teaching and learning opportunities take place (Björklund, Van den Heuvel-Panhuizen & Kullberg, 2020).

Literature Review

International assessment and evaluation examinations, such as the Program for International Student Assessment (PISA) of the Organisation for Economic Co-operation and Development (OECD), are conducted to provide countries with comparative data on their education systems. PISA measures the mathematical literacy, science literacy, and reading skills of 15-year-old students in 3-year cycles. PISA 2018 results indicate that Turkey's average score in mathematics had increased by 34 points compared to 2015 and reached 454. Turkey's success rate in mathematics literacy was 48.7% in PISA 2015. It has increased to 63.4% in the PISA 2018 report. According to the PISA 2018 results Turkey is the country that has increased its score the most, but still remained below the OECD average (OECD average = 489) in mathematics (Millî Eğitim Bakanlığı, 2019).

Current studies in the field of early mathematics show that Turkey has some issues with teachers' pedagogical content knowledge (PCK) and mathematics teaching. Some of these issues are that teachers do not have sufficient knowledge of mathematical concepts and skills (Pekince & Avcı, 2016). Most teachers think that early mathematics cover only numbers (Fırat & Dinçer, 2018) and shapes (Yazlik & Öngören, 2018). Teachers experience difficulties in planning mathematics activities (Pekince & Avcı, 2016), and they do not sufficiently include mathematical activities in their daily plans (Pekince & Avcı, 2016). The mathematics activities they plan are mostly teacher centred (Erincik, 2020) and at anti-participation level (Pekince & Avcı, 2016). Teachers evaluate whether students have learned mathematical concepts but cannot do the evaluation systematically (Tarim & Bulut, 2006). They are weak in evaluating mathematics activities in general (Erincik, 2020; Koç, 2017) and they have limited knowledge in mathematics teaching methods (Koç, 2017). Furthermore, they do not find their own education sufficient for pre-school mathematics teaching (Tarim & Bulut, 2006), and need support in mathematics education (Koç, 2017). The research findings presented above show that pre-school and kindergarten teachers working in Turkey experience difficulties regarding both content and strategies in teaching mathematics. This can lead to serious obstacles in teaching early mathematics effectively.

Early mathematics knowledge is the strongest predictor of children's mathematics achievement in later school life (Watts et al., 2014). Despite the importance of the subject, research on the relationship between early childhood teachers' PCK in mathematics and the quality of instruction is quite limited (Corrigendum, 2015). To fill this gap in the literature, we investigated whether there was a significant relationship between teachers' mathematics pedagogical content knowledge (MPCK) and the quality of instruction. Teachers' PCK is evaluated according to certain indicators, such as how they teach content like number sense, patterns, ordering, shapes, spatial sense, and comparisons. The quality of instruction, however, is evaluated with indicators such as the characteristics of class culture and specific mathematical activities. Examining teachers' PCK of mathematics and the quality of instruction with these indicators can shed light on the characteristics of teachers' mathematics teaching and how young children's understanding of mathematics can be supported better.

Our literature review revealed that there was no valid and reliable tool to observe the quality of early mathematics practices in Turkey. Therefore, the quality of early mathematics practices in Turkey has not been investigated thus far. To close this gap in research, the Classroom Observation of Early Mathematics – Environment and Teaching (COEMET) has been adapted into Turkish as part of this study. There are four important reasons for using COEMET. The first is based on research. COEMET was created based on a body of research on the characteristics and teaching strategies of effective teachers of pre-school mathematics (Clements, Sarama & DiBiase, 2004; Sarama, Clements, Wolfe & Spitler, 2016). Another powerful feature is that other measurement tools like High-Impact Strategies for Early Mathematics ([HIS-EM] The Early Math Collaborative, 2011) evaluate only mathematics activity, while COEMET is a more holistic observation tool that evaluates class elements, class culture and special mathematics activity. COEMET also allows data collection with on-site observation. The last reason is that COEMET is not related to any specific curriculum (Kilday & Kinzie, 2009).

Among others, one goal of teacher education is to develop teachers' PCK because it has been associated with teaching quality. Studies addressing PCK in educational studies have assumed that the more teachers have PCK the more appropriate educational methods and practices they employ in classroom settings (Kulgemeyer & Riese, 2018). Additionally, Gropen, Kook, Hoisington and Clark-Chiarelli (2017) show that teachers with higher PCK tend to assess children's learning better, promote the use of scientific inquiry and plan in-depth investigation in science learning in early childhood classrooms. Evidence from these studies indicate that teachers' PCK is an indicator of their teaching quality.

To summarise, current literature shows significant problems in the field of pre-school mathematics in Turkey. Despite the importance of the issue, the number of studies is quite limited in Turkey and the rest of the world. The most prominent of these issues is what and how teachers teach mathematics. In addition, no study that evaluated the quality of the mathematics environment and activities offered by early childhood teachers in Turkey, and the teachers' PCK on mathematics, could be found (Bağci & Ivrendi, 2016; Çiltas, Güler & Sözbilir, 2012; Yıldız Altan, Genç Çopur & Dağlıoğlu, 2021).

In this context, it was thought that the results of observational research were especially needed for teachers who worked in early childhood classrooms to support children's mathematical skills. In addition, it was also thought that the findings of this study would contribute significantly, not only to early childhood mathematics practices but also to early childhood teacher education. Three research questions guided our study:

1) Is the Turkish translation of COEMET a valid and reliable tool to measure the quality of mathematics teaching in pre-school classrooms?
2) Is there a significant relationship between pre-school teachers' classroom mathematics practices and their PCK scores?
3) What proportion of variance in the quality of mathematics instruction can be accounted for by PCK in early childhood classrooms?

Theoretical Framework

Young children are born mathematicians (NAEYC, 2014). They can construct some mathematical ideas and strategies by themselves. However, they cannot excel in mathematics without deliberate and high-quality education (Clements, 2001; Ginsburg & Ertle, 2008). One of the critical elements for high-quality mathematics education is teachers' PCK (Gervasoni, Hunter, Bicknell & Sexton, 2012; MacDonald, Davies, Dockett & Perry, 2012). According to Ginsburg and Amit (2008), pre-school teachers who have a deep understanding of mathematics and PCK can provide mathematics education that will help children to think mathematically in their daily activities.

The PCK concept was proposed by Shulman in 1986. It is defined as knowing what to teach to which age group and integrating it with the knowledge of how to teach. To establish a solid mathematical foundation in children, teachers should have deep knowledge and experience of mathematics, as well as knowing what and how children can learn mathematics (Zhang, 2015).

Therefore, an effective mathematics education begins with teachers having a good level of PCK (Bukova-Güzel, Canturk-Günhan, Kula, Özgür & Elçi, 2013; Jang, 2013). Effective teachers are knowledgeable about child development, learning styles, and effective teaching strategies, and they use these in designing learning environments (Copple & Bredekamp, 2009).

Researchers draw attention to the connection between young children's curiosity, meaningful learning, and achievement in mathematics with their teachers' pedagogical approach and the quality of teaching (Katz, 2015). Clements and Sarama (2002) also claim that mathematics should be rooted in and developed from children's activities. Early mathematics education often takes place in natural learning environments, that is, it is embedded in what children are already doing (Gasteiger, 2012; Van Oers, 2010). In this respect, early mathematics education is different from school mathematics education. Early mathematics teaching requires encouraging children to recognise and think about numbers and mathematical structures in daily life, capturing the mathematics-related clues in children's actions and speech, and providing learning experiences by transforming these everyday situations into mathematically appropriate learning opportunities (Ginsburg, Lee & Boyd, 2008; McCray & Chen, 2012). In societies where early childhood education is regarded as an integral part of the education system, the attention has shifted to its quality (Cerezci, 2020). High-quality early mathematics teaching and learning experiences can only be provided by qualified teachers (Sarama et al., 2016). What and how the teacher teaches early mathematics is one of the important indicators that determines the quality of the teacher (Gervasoni et al., 2012; MacDonald et al., 2012). Therefore, early childhood teachers should spontaneously promote mathematical learning processes in these informal learning situations.

Methodology

In this study, quantitative methods were used. The data were collected through observing and surveying the 55 pre-school classrooms/teachers. Rasch analysis, Pearson correlation analysis, and regression analysis were used to analyse the data.

Participants

Fifty-five female pre-school teachers participated in the study. The 55 classrooms were clustered in 20 schools within one urban and one suburban school district in one of the Aegean region provinces of Turkey.

All participating teachers were female, ranging in age between 25 and 51 years: 41 years old and above (45.50%), 36 to 40 years old (38.20%), 31 to 35 years old (8.50%), and 25 to 30 years old (6.50%). Regarding education, 89.00% held a bachelor's degree; 63.00% held a bachelor's degree in early childhood education, and 26.00% held a bachelor's degree in child development). There were 16 to 20 children in 65.50%, 10 to 15 children in 21.80%, and 21 and above children in 12.70% of the observed classrooms. There were no assistant teachers in 91.00% of the classes. Half of the classes consisted of 4-year-olds and the other half consisted of 5- to 6-year-old children.

Instruments

Three data collection tools were used in this study: 1) a demographic information survey (developed by the researchers); 2) a questionnaire (*the Survey of Pedagogical Content Knowledge in Early Childhood Mathematics - SPECKECM)*; 3) an observation form (*Classroom Observation of Early Mathematics – Environment and Teaching – COEMET*).

The demographic information survey consists of a set of questions (items) to gain background information of the participant teachers and their classrooms, such as their gender, age, graduation degree, teaching experience, type of school, location of the school, classroom size and children's age groups. Teachers were requested to complete the survey prior to the classroom observation.

SPECKECM was developed by Smith (1998) to measure early childhood teachers' PCK in mathematics on six sub-category areas including *number sense*, *ordering*, *shapes*, *pattern*, *spatial sense*, and *comparison*. It has 15 multiple-choice items. Each item consists of three possible answers, one correct and two incorrect. Smith (2000) reported a good range of reliability (Cronbach's alpha = .70) and validity for SPECKECM by administering the survey to 400 early childhood teachers. SPECKECM was adapted into Turkish by Aksu and Kul (2017). Aksu and Kul (2017) reported KR-20 reliability scores as .71. In addition, confirmatory factor analysis was done to determine its validity. The results were χ^2 (75) = .95, RMSEA = .00, CFI = .99. For our study, the KR-20 reliability score was computed as .73. Therefore, the instrument was regarded as a reliable tool to measure pre-school teachers' PCK of mathematics.

COEMET was developed by Sarama and Clements (2007) to assess the quality of mathematics instruction including characteristics and teaching strategies of effective teachers in early childhood mathematics from pre-K to Grade 2. It is a classroom observation tool that evaluates classroom culture and the use of mathematics activities and is not connected to any curriculum (Clements & Sarama, 2008). It includes 28 items about how the teacher interacts with the children and uses teachable mathematics moments, how mathematics is displayed in the classroom, how confident the teacher appears in teaching mathematics. More specifically, it consists of four main sections: a) classroom elements, b) classroom culture, c) specific mathematics activities and d) other classroom elements.

The section on classroom elements includes four items about the general structure of the classroom (e.g., the number of running computers, the number of mathematics activities and their duration). The classroom culture is about the characteristics of the interaction among students and teachers, their knowledge about mathematics, enthusiasm for mathematics ideas and teachable moments in the classroom. It consists of two sub-sections, namely, environment and interaction (an example item: teacher used teachable moments) and personal attributes of teachers (example item: the teacher seems knowledgeable and confident in mathematics). The specific mathematics activities are about children sharing their ideas, mathematical focus, and teachers facilitating their responses, expectations and supporting their conceptual understanding. It comprises seven sub-sections as mathematical focus (example item: the mathematics activity is developmentally appropriate), organisation, teaching approaches and interaction (example item: the pace of activity is suitable for children's needs and levels), expectations (example item: the teacher has big and realistic expectations regarding children's mathematics ideas), eliciting children's solution methods (example item: the teacher facilitated children's responses), supporting children's conceptual understanding (example item: the teacher supported the listener's understanding), extending children's mathematical thinking (example item: the teacher elaborated on children's ideas) and assessment and instructional adjustment (example item: the teacher listened to children and took notes). The other classroom elements are non-scored items dealing with the number of students, adults and volunteers. The items in the COEMET were in a 5-point Likert scale (1: *strongly disagree*, to 5: *strongly agree*) except those in the classroom environments and the other classroom elements.

For research purposes, we translated COEMET into Turkish. Back-translations of all translations were done by two experts working in the field of educational sciences and English language to ensure that the quality of the translations was good. Based on the reviews and opinions of the experts, the final Turkish version of the COEMET was constructed. After constructing the final version, the first author ran five pilot observations in five classes. Because there was only one observer (rater) and video recording in the classroom was not allowed, the intra-rater reliability coefficient was calculated to determine the reliability of COEMET. To calculate the intra-rater coefficient, the five observations in the pilot study were used because the same classrooms were included in the data collection. As the COEMET has Likert-type items, the weighted Cohen's Kappa coefficient, which takes the magnitude of disagreement between the scores, was calculated as .89.

Previous studies in which the COEMET was used reported its reliability score as .70 cut-off value (e.g., Cronbach alpha as .97 in Clements & Sarama, 2008). Detailed information about the COEMET can be found in Sarama and Clements (2007).

Data Collection

The first author collected the observational data via COEMET since she was well experienced and trained by the developers of the tool to use the COEMET. The observations were conducted while the teachers were leading the mathematics activities. Each observation session took at least 3 class hours (150 minutes or more). The observer arrived in the classroom before class started and minimised her participation in order not to disturb the class culture. All observations were conducted over a period of 2.5 months (from 25 February to 15 May 2019).

The first two main sections of the COEMET were assessed once for each observation. The third main section was assessed for each mathematics activity.

Data related to the SPECKECM were collected via a paper-pencil form. Before observing the classroom, the SPECKECM form was handed out to each teacher who was given 10 minutes to complete it.

Data Analysis

To investigate the first research question (to measure the validity and reliability of the Turkish translation of COEMET), Rasch analysis was used. Unlike classical test theory, Rasch analysis uses logits rather than raw scores (Rasch, 1961). Logit is the unit of Rasch measurement that indicates the ratio of probability to give a correct response (responder's trait/ability level) (Bond & Fox, 2015). In Rasch measurement it is argued that most of affective variables in educational research are treated as an interval scale (Likert-scale items, strongly disagree etc.) which depends on the responder's understanding of *strongly disagree* (Bond & Fox, 2015). Thus, in the Rasch measurement model it is argued that Likert-scale items should be treated as an ordinal scale (Alpaslan, Özgür & Rıdvan, 2021).

In Rasch analysis, raw ordinal scores are transformed to the logits that indicate the item's difficulty in an linear scale (Elhan & Atakurt, 2005). A negative value for logits shows that the item is very easy to agree with (or easy item) whereas a positive value is for a difficult item (Bond & Fox, 2015). All analysis was run in Winsteps 3.80 software.

To address the second research question, Pearson correlation analysis was used. Firstly, we checked the normality of the data by examining the skewness and kurtosis values and results from the Shapiro-Wilk normality tests. For all sub-factors, kurtosis ranged from -0.99 to .12 and skewness from -0.48 to .0.37. Also, Shapiro-Wilk yielded a non-significant difference from the normality, which indicated the data had a normal distribution. For the third research question, regression analysis was used to test how much variance of COEMET could be explained by teachers' PCK scores.

Results

The purpose of this study was to examine the relationship between teachers' PCK (the SPECKECM scores) and the quality of mathematics teaching (the COEMET scores) and to understand how much variance of the COEMET scores is explained by teachers' PCK scores.

The results of the study are arranged under three headings: (a) Rasch analysis of the Turkish version of the COEMET (cf. Table 1 and Figure 1); (b) correlation between the COEMET and SPECKECM (cf. Tables 2 and 3); (c) regression between the COEMET and SPECKECM (cf. Table 4).

Findings regarding Research Question 1: Validity and Reliability Results of COEMET

Rasch analysis of the COEMET

Rasch analysis is a useful tool to examine the construct validity and reliability of a measurement. The fit of the items to the other items and responders was examined using Rasch analysis. Rasch fit statistics provide various useful information on detecting irrelevant items and items that are consistent with the underlying constructs. Point measure correlations in Rasch statistics in Table 1, for example, show whether items work together as a single construct. This demonstrates whether items correlate with their level of agreeability, and also, the magnitude of the correlations shows which items contribute more to the construct and which ones less (Planinic, Ivanjek & Susac, 2010). In addition, item infit/outfit mean square (MNSQ) and separation coefficients indicate items measuring a concept other than that estimated by the construct (Planinic et al., 2010).

Person reliability was .88 and item reliability was .92 for the COEMET. Rasch analysis provides a useful tool including a measure table and a Wrigth map which locates the items and person in the same interval scale. Table 1 represents the measure table of the Rasch analysis for COEMET. Figure 1 shows the Wrigth map of the COEMET. The measure table includes useful information about agreeability or attitudes of teachers and classroom culture in pre-school classrooms. In Table 1, the measure column indicates the degree of agreement or disagreement. For instance, item 27 measured the degree to which the teacher took notes and listened to the children (strongly disagree was graded as 1 and strongly agree was 5). Positive value indicates that the observer was less likely to agree that teachers took notes and listened to the children. Item 4 and item 5B were out of the scale on the positive side, which indicates that the observer did not note that children used mathematics software (item 4) and the mathematics area/centre had three sides (item 5B). Infit MNSQ and outfit MNSQ values indicate how well the data fit the Rasch model (Boone, Staver & Yale, 2013). Suggested values should be between .5 and 1.5 logits. As seen in Table 1, except for item 4 and item 5B, items fitted well in the Rasch model. For construct validity, correlation between items should be higher than .30 (Alpaslan et al., 2021). As seen in Table 1, the point measurement correlations between items in the COEMET were higher than the .30 cut-off point, except for item 5A, item 5B, item 4, and SBM.

Table 1 The measure of Rasch analysis for COEMET

Item	Total score	Measure	Model *SE*	Infit MNSQ	Outfit MNSQ	Point measure correlation
5B-EI	0	6.61	1.79	0	0	0
5A-EI	11	3.36	0.29	1.56	0.99	0.42
SBM	15	3.33	.24	1.47	1.18	.22
4-EI	17	3.28	0.07	0.83	1.13	0.09
5C-EI	21	2.63	0.22	1.94	1.18	0.49
5D-EI	24	2.49	0.21	1.13	1.00	0.53
5E-EI	28	2.32	0.2	1.53	1.97	0.52
SB	38	2.25	.15	.82	1.01	.16
27-AIA	81	2.18	0.23	1.22	1.31	0.35
19-ECSM	117	0.79	0.18	0.78	0.74	0.75
23-CCU	133	0.30	0.17	1.24	1.14	0.62
28-AIA	139	0.12	0.17	0.86	0.83	0.66
17-EX	143	0.00	0.17	0.81	1.05	0.68
26-ECMT	144	-0.03	0.17	0.9	0.86	0.77
2-EI	145	-0.05	0.01	1.08	0.86	0.47
1-EI	147	-0.11	0.01	0.54	0.51	0.75
15-OTAI	147	-0.12	0.01	0.72	0.72	0.7
20-ECSM	147	-0.12	0.17	1.00	1.00	0.78
21-ECSM	152	-0.26	0.17	0.95	0.96	0.69
25-ECMT	154	-0.32	0.17	0.9	0.88	0.66
3-EI	150	-0.36	0.14	0.65	0.73	0.63
9-PAT	150	-0.36	0.14	0.85	1.04	0.65
5-EI	154	-0.44	0.14	0.97	1.01	0.62
22-SCCU	158	-0.44	0.17	0.81	0.78	0.66
6-EI	159	-0.46	0.14	1.04	1.22	0.55
12-OTAI	165	-0.66	0.14	0.91	1.05	0.57
8-PAT	169	-0.74	0.15	0.6	0.68	0.8
14-OTAI	173	-0.83	0.15	1.08	1.23	0.62
16-OTAI	174	-1.03	0.15	0.62	0.63	0.66
24-SCCU	178	-1.07	0.18	0.69	-0.66	0.76
18-EX	180	-1.14	0.18	1.49	1.37	0.69
7-PAT	187	-1.15	0.16	0.69	0.76	0.76
10-MF	188	-1.18	0.16	0.60	0.67	0.65
13-OTAI	206	-1.68	0.18	0.83	0.82	0.5
11-MF	210	-1.81	0.18	0.88	0.81	0.51

Note. EI: Environment and interaction, PAT: Personal attributes of teachers, MF: Mathematical focus, OTAI: Organisation, teaching approaches and interaction, EX: Expectations, ECSM: Eliciting children's solution methods, CCU: Children's conceptual understanding, SCCU: Supporting children's conceptual understanding, ECMT: Extending children's mathematical thinking, AIA: Assessment and instructional adjustment, SB: The classroom has a smart board, and SBM: Used smart board for mathematics.

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                |
                |
                |T Item5A   Item4 SBM
  3             +
                |
            X   |  Item5C
                |  Item5E   Item5D
                |  SB   Item27
  2             +
                |
            X T|S
            X   |
            X   |
  1         X   +
          XXX   |  Item19
     XXXXXXXX S|
            X   |
                |  Item23   Item28
  0        XX  +M Item17   Item26   Item2    Item1    Item15   Item20
         XXXX   |
           XX M|  Item21   Item25   Item3    Item5    Item6    Item9    Item22
         XXXX   |  Item12
        XXXXX   |  Item8    Item14
 -1    XXXXXX   +  Item16   Item24   Item18
          XXX   |  Item10   Item7
      XXXXXXX S|
        XXXXX   |S Item13
                |  Item11
 -2             +
         <less>|<freq>
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Figure 1 The Wrigth map of the COEMET
Note. Item 5B was not displayed in Figure 1 for its simplicity. X represents 1 observation.

In Figure 1, the frequency of the observations regarding the items in the COEMET decreases from bottom to top. The least observed items were item 5A, item 5B, item 4 and SBM and the most observed items were 11, 13, 7 and 10 respectively. Item 5B could not be included in the analysis since none of the observed classrooms had a three-sided mathematics centre (measure = 6.61). In addition, it was observed that very few pre-school classrooms had mathematics centres (measure = 3.36). Moreover, it was determined that the use of smart boards for mathematics (SBM) was exceedingly rare (measure = 3.33). It was found that the variety of materials in mathematics centres was quite low (measure = 2.63). It was observed that children rarely used mathematics centres to play games (measure = 2.49) and that these were not developmentally appropriate (measure = 2.32). The teachers rarely observed and listened to the children, or took notes (took notes in small groups only; measure = 2.18).

The teachers seldom asked children to share, clarify, and justify their ideas (measure = .79). The teachers supported the children's understanding moderately (measure = .30). The teachers did not adapt tasks and discussions to accommodate the range of children's abilities and development (measure = .12).

The teachers did not have high or realistic mathematical expectations of the children (measure = .00) at a high level. They sometimes encouraged mathematical reflection (measure = -.03). The teachers' and other staff's involvement in the activity was low (measure = -.12 and measure = -.05). The teachers also actively interacted with and was responsive to the children (measure = -0.11). The teachers' involvement in the activity was low. The teachers sometimes facilitated children's responses (measure = -.12).

The teachers sometimes encouraged children to listen to and evaluate others' thinking/ideas (measure = -.26) and elicited key mathematics ideas during and/or towards the end of the activity. The teachers sometimes built on and/or elaborated on children's mathematical ideas and strategies (measure = -.32). The teachers used teachable moments as they occurred to develop mathematical ideas (measure = -.36). The environment showed signs of mathematics (measure = -.44). Children's mathematics work and/or other signs of mathematical thinking were on display. It can be said that the children's mathematics activities were exhibited in the classroom (measure = -.46). The teachers showed curiosity and/or enthusiasm for mathematical ideas and or connections to other ideas or real-world situations (measure = -.36). The teachers supported the describer's thinking (measure = -.44). They began the mathematics activities by engaging and focusing on children's mathematical thinking (measure = -.66). The teachers showed that they believed that mathematics learning could and should be enjoyable (measure = -.74). The teachers' management strategies enhanced the quality of the activity (measure = -.83). The teaching strategies used were appropriate for the development levels/needs of the children and the purposes of the activity (measure = -1.03). The teachers provided "just enough" support (measure = -1.07). The teachers acknowledged and/or reinforced children's efforts, persistence and/or concentration (measure = -1.14).

The teachers facilitated children's actions at an appropriate level, providing adequate, not too little and not too much, help or information. They also displayed an understanding of mathematics concepts (measure = -1.18). They appeared to be knowledgeable about mathematics (measure = -1.15). It was determined that the pace of the teachers was appropriate for the pace of the children (measure = -1.68). The mathematical content was appropriate for the developmental levels of the children in the class. Teachers mostly performed developmentally appropriate practices (measure = -1.81).

Findings regarding Research Question 2: Relationship between COEMET and SPECKECM

In Table 2, descriptive statistics of the COEMET and SPECKECM are given. Also, reliability values of Cronbach's alpha for the COEMET and KR-20 for the SPECKECM are displayed. The reliability values indicate that all sub-factors are above the cut-off value of .70, which is the recommended cut-off values for an acceptable reliability. As seen in Table 2, the highest mean value was found in the mathematical focus sub-factor, indicating that the teachers tended to conduct the activities that were appropriate to children's developmental level. However, the lowest mean value was in the assessment and instructional adjustment sub-factor, implying that the teachers were not likely to make any change or adjustment during their activities based on the children' needs. For the SPECKECM, the teachers were most successful with shapes (1.77 out of 3.00) and least successful with patterns (1.00 out of 3.00).

Table 2 Descriptive statistics of the COEMET and SPECKECM

Variables	*M*	*SD*	Skewness	Kurtosis	Reliability
COEMET	2.97	.67	-0.07	-0.54	.77
Environment and interaction	3.02	0.89	-0.20	-0.58	.79
Personal attributes of teachers	3.09	1.09	-0.10	-0.68	.82
Mathematical focus	3.84	0.76	-0.23	-0.23	.75
Organisation, teaching approaches and interaction	3.36	0.78	-0.26	-0.57	.89
Expectations	3.15	1.03	-0.73	-0.44	.78
Eliciting children's solution methods	2.49	1.02	-0.03	-1.07	.79
Supporting children's conceptual understanding	2.85	0.83	-0.54	0.03	.86
Extending children's mathematical thinking	2.75	0.87	-0.54	-0.17	.80
Assessment and instructional adjustment	2.22	0.83	0.37	-0.25	.88
SPECKECM	7.81	2.71	-0.38	-.37	
Number sense	1.24	0.96	0.15	-1.01	.92
Patterns	1.00	0.88	0.84	0.32	.94
Ordering	1.25	0.89	-0.04	-0.94	.90
Shapes	1.77	0.98	1.63	1.33	.91
Spatial sense	1.13	0.77	-0.23	-1.27	.92
Comparisons	1.51	0.66	-1.02	-0.07	.89

Pearson correlations were computed to examine the relations between the SPECKECM and COEMET dimensions and are presented in Table 3. For the COEMET sub-factors, all correlations were positive and significant. The strongest correlation was between the personal attributes of teachers and the organisation, teaching approaches and interaction ($r = .72$, $p < .01$), implying that the higher the personal attributes of teachers were, so were the organisation, teaching approaches and interaction. The second strongest correlation was between supporting children's conceptual understanding and extending children's mathematical thinking ($r = .71$, $p < .01$), implying that the more the teachers supported children's conceptual understanding the more they extended children's mathematical thinking. The third strongest correlation was between the expectations and eliciting children's solution methods ($r = .70$, $p < .01$). The lowest correlation coefficient was between the expectations and assessment and instructional adjustment ($r = .27$, $p < .05$). As for the SPECKECM, some of the correlations were significant and others were not. The highest correlation coefficient was between spatial sense and ordering ($r = .34$, $p < .05$) and comparisons and spatial sense ($r = .34$, $p < .05$), implying that the knowledge on spatial sense and ordering and comparisons was related to each other. It was followed by spatial sense and pattern ($r = .33$, $p < .05$) and shapes and number ($r = .31$, $p < .05$).

Table 3 Correlation coefficients amongst variables

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1)	Total of SPECKECM	1															
2)	Number	.57**	1														
3)	Pattern	.49**	-.11	1													
4)	Spatial sense	.71**	.19	.33**	1												
5)	Ordering	.56**	.27**	.23*	.34**	1											
6)	Shapes	.53**	.31*	.03	.17	.12	1										
7)	Comparisons	.52**	.16	.17	.34**	.13	.22*	1									
8)	Environment and interaction	.05	.06	.02	.06	.18	-.03	-.01	1								
9)	Personal attributes of teachers	.33**	.24*	.01	.20*	.20*	.25*	.11	.62**	1							
10)	Mathematical focus	.23*	.01	.28**	.15	.09	.23*	.02	.38**	.48**	1						
11)	Organisation, teaching approaches and interaction	.09	-.07	.18	.19	.01	.01	.06	.50**	.72**	.44**	1					
12)	Expectations	.16	-.05	.27**	.10	.01	.02	.23*	.38**	.61**	.41**	.65**	1				
13)	Eliciting children's solution methods	.20*	.03	.19	.28	-.08	.13	.17	.40**	.62**	.46**	.59**	.70**	1			
14)	Supporting children's conceptual understanding	.20*	-.01	.31**	.29	-.02	.01	.18	.43**	.48**	.46**	.52**	.55**	.66**	1		
15)	Extending children's mathematical thinking	.34**	.10	.24*	.35**	.19	.08	.16	.56**	.65**	.47**	.58**	.52**	.67**	.71**	1	
16)	Assessment and instructional adjustment	.45**	.26*	.22*	.40**	.27*	.10	.28*	.32**	.45**	.32**	.42**	.27*	.38**	.37*	.41**	1

Note. *$p < .05$, **$p < .01$.

As for the correlations between the COEMET and SPECKECM, it was found that some of the correlation coefficients were statistically significant, but some were not. Except for environment and interaction, organisation, teaching approaches and interaction, and expectations, dimensions of the COEMET were positively and statistically significantly correlated to the total point of the SPECKECM. This shows that the teachers' PCK was associated with their practices of the COEMET. The strongest correlation was between the total score of SPECKECM and assessment and instructional adjustment (r = .45, p < .01). Also, the dimension, assessment and instructional adjustment, was related to all the dimensions of the SPECKECM, except shapes. This result implies that when the teachers' PCK had increased they were more likely to adjust the assessment and instructional practices based on the children's needs. Given that assessment and instructional adjustment was the least observed COEMET dimension in the sample of this study, it can be said that this may be because of teachers' unsatisfactory PCK. Extending children's mathematical thinking was also related to the total scores of the SPECKECM (r = .34, p < .05), indicating that the more mathematics knowledge teachers had the more they extended children's mathematical thinking. Environment and interaction and organisation, teaching approaches and interaction were not significantly correlated to any dimension of the SPECKECM.

Findings regarding Research Question 3: Regression Analysis between SPECKECM and COEMET

To test how much variance of COEMET could be explained by teachers' PCK scores, a regression analysis was run. The analysis shows that teachers' PCK successfully predicted the total COEMET score (F(1,53) = 5.41, p < .05). A 10% variance in COEMET was explained by teachers' PCK (R^2 = .10). The standardised regression coefficient showed that the PCK positively contributed to the COEMET score (β = .34), indicating that a higher PCK score resulted in a higher COEMET score.

Table 4 Results of the regression test

Predictor	B	β	t	β (%95 CI)
PCK	.078	.034	2.39*	(.07–.59)

Note. *< .05

Discussion

The first research question of the study was: Is the Turkish translation of COEMET a valid and reliable tool to measure the quality of mathematics teaching in pre-school classrooms? To examine the first research question, 55 teachers were observed and the fit of items to the other items and responders was examined using the Rasch analysis. All the items in COEMET, except item 4 and item 5B, fitted well in the Rasch model with .88 person reliability and .92 item reliability. Therefore, those items were removed from the final analysis.

Computers have become a well-known tool in pre-school education; a tool that can support the teaching and learning process (Clements & Sarama, 2003). Studies also indicate that pre-school children are successful and eager to use educational software and that they benefit from these activities to enhance their own learning (Clements & Nastasi, 1993; Clements & Sarama, 2003; Wang & Ching, 2003). Studies in the United States of America (USA) show that teachers use mathematics software in their classroom activities at a rate of 33% (Sarama, 2002; Sarama & DiBiase, 2004). However, there were no computers for teachers and children to use mathematics software and apps in any of the observed classrooms. It was thus clear that computers and mathematics software and apps, which are important educational materials of the 21st century classrooms, were not used in the Turkish pre-school and kindergarten classrooms. As for the use of smart boards (SBs) in mathematics activities (SBM), the data show that even if there was a smart board in the classroom, teachers almost never used it for mathematics activities. Hacısalihoğlu Karadeniz (2014) had similar findings and states that although pre-school teachers generally had positive attitudes towards the use of technology in mathematics, they could not use technology sufficiently during mathematics activities and they usually focused on teacher-centred methods. Orçan-Kaçan and Kimzan (2017) also state that the use of technological tools in pre-school education varies depending on the school's technological infrastructure and teacher competencies. The lack of early mathematics software and applications available in Turkish is regarded as another reason for not using computers and smart boards in pre-school education (Kol, 2012).

It was observed that very few of the classrooms had mathematics centres (item 5). It was also found that most of the available mathematics centres were physically and developmentally unsuitable for children to play in and that the level of diversity of the material was not ideal. Although the National Pre-school Education Programme implemented in Turkey emphasises the importance of learning centres in pre-school classrooms and lists that several learning centres are required, mathematics centres are not among them (Millî Eğitim Bakanlığı, 2013). This inadequacy of the national programme seems to constitute the general basis of the findings in our research. Because of large class sizes and having no mathematics centres and no teacher aides in the pre-school classrooms, learning activities mostly took place in large groups in Turkey. Therefore, the children in these classrooms had little chance to practise their mathematical thinking individually or in small groups. Other studies delivered similar results. For example, Büyüktaşkapu Soydan (2019) examined the types of activities (individual-small/large groups) that pre-school teachers included in their daily plans. The research revealed that the planned mathematics activities were mostly large group activities and there were far less individual and small group mathematics activities. Different reasons are given why teachers prefer large group activities to small group and individual activities. Sadik and Dikici-Siğirtmaç (2016) state that teachers find individual and small group activities more difficult to handle compared to large group activities. Dogan-Burc (2006) and Öztürk and Gangal (2016) also claim that teachers prefer large group activities as a way to manage the class, to cope with undesirable behaviour and to maintain silence in the classroom. However, in small groups children can receive individualised attention and instruction that may not be possible in large group activities. Teachers can also better observe how individual children perform tasks and how they interact with other children (Wasik, 2008). For a counting activity that requires the teacher to ask each child to count and demonstrate one-to-one correspondence, for example, a group of three to four children is best (Klein, Wirth & Linas, 2004).

The teachers participating in this study seemed to believe that learning mathematics could and should be enjoyable. Pre-school teachers' thoughts and feelings about mathematics and mathematics education play a vital role in their teaching. Therefore, teachers who think that mathematics is fun and give positive messages about it in the classroom, can be more successful in using natural learning environments that can be associated with daily situations (Gasteiger, 2014; Van Oers, 2010).

One of the biggest obstacles in introducing mathematical concepts to children is that teachers usually do not have the necessary mathematical knowledge to teach the mathematics concepts (Dombro, Jablon & Stetson, 2011; NRC, 2009). We observed that the teachers appeared to be highly knowledgeable about mathematics, displayed a remarkably high level of understanding of mathematical concepts and were also very good at appropriating the mathematical content to the developmental level of the children in the classrooms. Although the participant teachers seemed to have a good command of the concepts of mathematics and could adapt the content in accordance with the children's level, our and other studies conducted in Turkey show that pre-school teachers' mathematics PCK is low (e.g., Dagli, 2018). Also, research findings show that pre-school teachers understand a limited number of mathematical concepts (e.g., Firat & Dinçer, 2018). Based on these findings, it becomes clear that since teachers are constantly teaching these limited numbers of concepts, they become competent in teaching them and appropriating their teaching strategies to the developmental level and needs of the children.

By asking the right mathematical questions, the teacher can also create a supportive atmosphere in which the children further develop their mathematical thinking and start thinking like mathematicians (Mason, 2000). In a kindergarten context, Carlsen, Erfjord and Hundeland (2010) found that the kindergarten teachers' frequent use of questions enabled children's participation in the learning activities. The participant teachers in this study began mathematics activities by engaging and focusing children's mathematical thinking. Although the teachers presented this behaviour, their scores on this item were relatively lower compared to other items further down the Wright map (cf. Figure 1). Considering the importance of teachers' interest in children's mathematical thoughts for learning, the teacher should have shown more of this behaviour since early mathematics learning is related to the quality of mathematics experiences in the environment (Klibanoff, Levine, Huttenlocher, Vasilyeva & Hedges, 2006). Despite factors such as large class sizes and the absence of assistant teachers in the classrooms, teachers planned and implemented mathematics activities that were suitable for the children's pace and developmental levels. Although it was observed that teachers were competent in their practices related to mathematics, they remained weak in observing and listening to children and taking appropriate notes (they took notes only in small group activities). It was also observed that teachers did not encourage children to share their ideas and talk about their ideas. The teacher's management strategies, on the other hand, enhanced the quality of the activity. This finding suggests that the teachers' use of effective classroom management strategies support effective mathematics practices.

In addition, it was also observed that teachers interacted with children and used teachable moments to improve children's mathematical thinking moderately. Some other studies also show that the level of child participation in mathematics activities was usually at the non-participation level (Pekince & Avcı, 2016). The teachers used traditional teacher-centred methods in implementing mathematics activities instead of active learning methods based on new approaches (Baki & Hacisalihoğlu Karadeniz, 2013). They usually assume an explanatory role and they cannot keep children cognitively active. According to Kaya and Aytar (2012), there is a gap between pre-school teachers' thoughts (self-perception) about using a child-centred approach and their actual practices.

The teachers in this study did not support the children's mathematical thinking by asking them to share, clarify, and justify their ideas and facilitate their responses at a high level. The teachers did not encourage children to listen to and evaluate others' thinking/ideas either. Bağci and Ivrendi (2016) conducted a study on pre-school teachers' opinions about teacher competencies and in-service training needs and found that pre-school teachers were highly competent in terms of teaching profession special area competencies. Kök, Çiftçi and Ayik (2011) also found similar results. However, these were self-report studies. Observational studies, however, show that teachers need to be supported in following and implementing the programme (Aysu & Aral, 2016), and the teachers' quality should be increased by arranging seminars, courses and in-service training programmes related to the pre-school education programme (Köksal, Balaban Dağal & Duman, 2016).

The best way to support children's learning and development is to observe the details of both the planned and the child-initiated learning processes and to use the observations to enhance classroom practices (Neaum, 2016). Observing children's in-class activities and taking notes regarding these observations are important for evaluating and supporting their learning and development (Gullo & Hughes, 2011). The teachers in this study, however, missed an important opportunity to improve the quality of mathematics activities by not spending enough time to observe, listen and provide feedback about the children's learning and discussing mathematical concepts with them as suggested by Gifford (2004).

One of the roles of an effective pre-school teacher is to provide children with the necessary support when they need it (Copple & Bredekamp, 2009). The teachers in this study provided "just enough" support to facilitate children's actions at the appropriate level. This was done by providing adequate (not too little, not too much) help or information. On the other hand, the teachers' level of elaborating on children's mathematical ideas and strategies was not high. There is evidence in the literature that mathematical thinking and competencies are not developed without the provision of a learning opportunity (Bergqvist & Lithner, 2012; Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge & Gould, 2005). If children have access to a guide, they are more likely to advance in their mathematical thinking, especially if the guide asks key questions (Laine, Näveri, Pehkonen, Ahtee & Hannula, 2018; Van Oers, 2010). The teachers in this study also did not encourage mathematical reflections at a high level. Literature recommends that pre-school teachers support children's mathematical thinking by making connections, modelling, posing questions like, what would happen if, and providing children with regular and meaningful opportunities to gain experience and use mathematics (Frye, Baroody, Burchinal, Carver, Jordan & McDowell, 2013). However, the participant teachers did not pay enough attention to using these types of instructional skills in their classrooms. It was also revealed that in almost none of the classrooms the teachers listened to the children's answers and took notes regarding the teaching-learning processes. The teachers' not observing, listening, and recording children's learning enough might be as a result of the large class sizes in Turkey. Research on class size (and ratio) produced inconsistent findings; many studies found no significant associations between class size and child outcomes (Burchinal, Zaslow, Tarullo, Votruba-Drzal & Miller, 2016; Howes, Burchinal, Pianta, Bryant, Early, Clifford & Barbarin, 2008). However, when evaluating these studies, it should be taken into count that an average pre-school classroom in the USA consists of 20 students with a 1:10 teacher-child ratio (Barnett, Vasileiou, Djemil, Brooks & Young, 2011). The situation is quite different in Turkish pre-school classrooms. Although the *Regulation Regarding Preschool Education and Primary Education Institutions* of the Ministry of National Education emphasises that the number of children in a pre-school classroom should be between 10 and 20, it is also stated in the same document that more students can be admitted if needed, without specifying a maximum limit (Millî Eğitim Bakanlığı, 2015). The regulation leads to an increase in the number of children in pre-school education classrooms in a way that is not conducive for the developmental and learning needs of the children. For example, Borland, Howsen and Trawick (2005) state that in classrooms with large numbers of children teachers' teaching skills and time need to be divided among many children and this decreases student achievement because teacher-student interaction is less. The current practices regarding class sizes in Turkey appear to reduce the children's developmental and learning opportunities.

The second research question was, "Is there a significant relationship between pre-school teachers' classroom mathematics practices and their PCK scores?" For the effective implementation of the pre-school mathematics curriculum, pre-school teachers should have certain competencies. Shulman (1986) defined PCK as the transformation of content knowledge into teaching knowledge. In this context, PCK has been regarded as a basic prerequisite for effective mathematics teaching for many years. In this study, the MPCK of the teachers was found to be at a medium level. This finding is also supported by previous research. For example, Bilgen (2018) examined the MPCK of in-service and pre-service pre-school teachers. It was found that both in-service and pre-service pre-school teachers had a medium level of MPCK, and the in-service teachers scored slightly higher than the pre-service teachers. While the PCK levels of the in-service teachers differed depending on the years of service, there was a significant difference in the MPCK of the teacher candidates based on gender, grade level, academic achievement, taking a mathematics education course, and believing in the necessity of mathematics education in the pre-school period. Both in-service and pre-service pre-school teachers considered mathematics education necessary in the pre-school period to prepare for primary (elementary) education and to acquire basic mathematics skills. These skills include number, counting, operation, pattern, matching and concrete-based activities in pre-school mathematics. In addition, in-service teachers considered themselves more competent in pre-school mathematics than teacher candidates.

The teachers' medium level MPCK can be explained by their attitudes towards mathematics and how much importance they attach to mathematics. Kowalski, Pretti-Frontczak and Johnson (2001) and Lee (2006), for example, claim that teachers consider developing socio-emotional skills more important than developing early academic skills, including mathematics. The previous research finding also supports the idea that the teachers should encourage children's natural tendency towards mathematics in their daily lives (Baroody, 2000; Clements & Sarama, 2007). In Millî Eğitim Bakanlığı's programme book (2013), the importance of developing mathematical inquiry skills in children by way of mathematics activities is also underlined and to do that it is recommended that children should be given examples that they may encounter in daily life.

It was also revealed in this research that there was a strong relationship between teachers' MPCK and their personal attributes, mathematical focus, supporting children's conceptual understanding, and the dimensions of environment and interaction. Our finding is supported by those of previous studies (Askew, 1999; Baumert, Kunter, Blum, Brunner, Voss, Jordan, Klusmann, Krauss, Neubrand & Tsai, 2010; Ma, 1999; Monk, 1994; Nye, Konstantopoulos & Hedges, 2004) showing the existence of a strong relationship between pre-school teachers' MPCK and the quality of their mathematics teaching practices. However, these studies have been carried out with different age groups that included elementary and secondary education. A limited number of studies included pre-school teachers. In their studies, McCray (2008) and McCray and Chen (2012) found that the mathematics PCK of pre-school teachers was positively correlated with teaching practices and children's mathematical outcomes. However, in these studies, the quality of mathematics instruction was examined only through "maths-related teacher language." In our study, the quality of mathematics instruction was examined through classroom observation by means of a research-based measurement tool.

It is known that teachers' mathematical PCK is an important variable for their practices in the classroom (Ball, Lubienski & Mewborn, 2001; Shulman, 1987). Therefore, teachers with higher MPCK have a high chance of performing quality mathematics practices. Pedagogical competence is the teacher's ability to manage learning, which consists of planning, implementing, and evaluating student learning outcomes. If the teachers want to fulfil their professional role, they must have these capabilities (Rahman, 2014). The NCTM (2022) also states that the teachers should actively introduce children to mathematical concepts, methods, and language through a variety of appropriate experiences and research-based teaching strategies. They should guide children in seeing connections with other topics, and encourage them to communicate and explain their thoughts while interacting with mathematics in depth and on an ongoing basis. However, both past and current research studies revealed that most of the pre-school teachers did not possess a high level of MPCK to provide quality learning opportunities for young learners (Ball, 1990; Hill, Schilling & Ball, 2004) and they often found that their knowledge and skills were not adequate to teach mathematics (Copley, 2004; Li, 2021; Wilkins, 2008).

The studies conducted in Turkey also provided similar results. For example, Köksal et al. (2016) examined the opinions of pre-school teachers about the pre-school education programme. It was found that the teachers' views on the implementation of the pre-school education programme varied according to the level of the programme they graduated from (associate degree, bachelor's degree, or post-graduate degree), their professional seniority, the type of school they worked at and the in-service training they had received. We also claim that the functionality of the programme will increase as teachers' ability to conduct the pre-school education programme improves by means of seminars, courses, and in-service training programmes. By including technology in the pre-school programme its effectiveness will improve qualitatively as well as quantitatively. Moreover, Özsirkinti, Akay and Yilmaz-Bolat (2014) evaluated the opinions of pre-school teachers about the pre-school education programme, and it was found that there were problems in putting together learning centres in the classrooms due to the physical structure of the classrooms and the number of children. Although teachers displayed positive attitudes towards the changes in the programme, they were not provided with in-service training to help them implement these better.

In summary, as Clements and Sarama (2015) state, mathematics, which is at the centre of cognition, is today also the main component of innovative educational approaches such as science, technology, engineering, and mathematics (STEM). Societies that offer qualified mathematics education to individuals from the early years will be among the leading societies. Undoubtedly, qualified teachers are the most important element of qualified early mathematics teaching. The most important indicator of teacher qualification is the level of teachers' PCK. In this study, it was seen that teachers' MPCK was moderate. Although it seems that some of the existing problems related to the quality of early mathematics education can be solved by providing quality education to enhance teachers' MPCK both in pre-service and in-service years, a more comprehensive approach is needed to rectify the structural problems (e.g., large classes, lack of assistant teachers, inadequate materials and technological tools for instruction) affecting the quality of education. Only in this way can we raise individuals who possess the required 21st century skills.

Conclusion

In this study, it was found that computers were not used in observed mathematics activities and that few classrooms had mathematics learning centres. These findings show that standards regarding physical organisation should be improved in pre-school education classrooms in Turkey. In addition, it was observed that despite their positive attitude towards mathematics, pre-school teachers did not transfer their positive beliefs to their classroom practices. Moreover, teachers' MPCK was found to be at a moderate level. To increase teachers' MPCK to a higher level, supportive education is needed both for pre-service and in-service teachers. Pre-school teachers are the key to effective and high-quality mathematics education in pre-school classrooms.

Authors' Contributions

MOK collected data and co-authored the manuscript. NDA wrote the manuscript, except the data analysis section. MMA conducted all statistical analyses and wrote the data analysis section.

Notes

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