

A Hybridized Centroid Technique for 3D Molodensky-Badekas Coordinate Transformation in the Ghana Geodetic Reference Network using Total Least Squares Approach

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Abstract

The Molodensky-Badekas model is one of the similarity transformation models used in Ghana for transferring Global Positioning System (GPS) coordinates from the geocentric World Geodetic System 1984 (WGS 84) ellipsoid to the local non-geocentric coordinate system, and vice versa. The objective of the Molodensky-Badekas model is to introduce a centroid to cater for the correlation that exists between the parameters when used over a small portion on the earth surface. However, the Molodensky-Badekas model performance depends on a particular centroid method adopted and the adjustment technique used. By virtue of literature covered, it was realised that the arithmetic mean centroid has been the most widely used. In view of this, the present study developed and tested two new hybrid centroid techniques known as the harmonic-quadratic mean and arithmetic-quadratic mean centroids. The proposed hybrid approaches were compared with the geometric mean, harmonic mean, median, quadratic mean and arithmetic mean. In addition, the Total Least Squares (TLS) technique was used to compute the transformation parameters with varying centroid techniques to investigate and assess their accuracies in precise GPS datum transformation parameters estimation within the Ghana Geodetic Reference Network. Statistical indicators such as Mean Error (ME), Mean Squared Error (MSE), Standard Deviation (SD), and Mean Horizontal Position Error (MHPE) were used to assess the centroid techniques performance. The results attained show that the Harmonic-Quadratic Mean produced reliable coordinate transformation results within the Ghana geodetic reference network and thus could serve as practical alternative technique to the frequently used arithmetic mean.

Keywords: Coordinate transformation, Molodensky-Badekas model, Centroid, Total Least Squares

1. Introduction

Transfer of coordinates between different reference frames is an indispensable tool in geospatial professions like geodesy, surveying and photogrammetry. Coordinate transformation is a mathematical algorithm that takes coordinates of a point in one reference frame into coordinates of the same point in a second reference frame (Ghilani, 2010). The transformation can result in changes in the position, size and shape of the network of points; this is known as an affine

transformation. If the transformation preserves the shapes as a result of a uniform scale factor in all directions but positions of points do change, then it is a conformal transformation (Constantin-Octavian, 2006). This paper focuses on the 3D conformal transformation of Molodensky-Badekas model with three translations, three rotations, and a scale factor for X, Y, Z coordinates.

The Molodensky-Badekas model is one of the commonest conformal transformations used by researchers in Ghana (Ayer and Tiennah, 2008; Dzidefo, 2011; Ziggah *et al.*, 2013a) and other countries due to their simplicity in application. The model introduces a centroid to cater for the high correlation that exists between the adjusted parameters by relating the parameters to the centroid when applied to a network of points that cover a small portion of the earth surface (Mitsakaki, 2004; Constantin-Octavian, 2006; Mihalache, 2012). The arithmetic mean centroid method is the most widely used approach by most researchers to compute values of centroid coordinates in the implementation of the Molodensky-Badekas model within their respective countries (Kheloufi, 2006; Turgut, 2010; Dzidefo, 2011; Okwuashi and Eyoh, 2012; Stankova *et al.*, 2012; Mihalache, 2012; Ziggah *et al.*, 2013a; Solomon, 2013; Mohammed and Mohammed, 2013). The ramification of the choice of centroid method on the Veis model has been investigated by Ziggah *et al.* (2013b). The authors assessed a variety of centroids that best fits the Ghana Geodetic Reference Network using Veis transformation model. Based on analysis of their results, they concluded that the transformation parameters of the root mean square (quadratic mean) centroid are the most realistic as compared to arithmetic mean, harmonic mean and median centroids.

The present study applies the conventional centroid methods of arithmetic mean, geometric mean, harmonic mean, quadratic mean and median in the implementation of the Molodensky-Badekas model within the Ghana geodetic reference network. The authors developed two hybrid centroid techniques namely arithmetic-quadratic mean and harmonic-quadratic mean to test their suitability compared to the conventional methods in determining parameters by the Molodensky-Badekas model within Ghana's geodetic reference network. This will further create the opportunity for geospatial professionals to know the most precise centroid approach to be applied in transforming points from the World Geodetic System 1984 (WGS 84) to the local War Office coordinate system accurately in Ghana.

2. Study Area and Data Source

In this study, 3D coordinate transformations were carried out in the Ghana geodetic reference network. Ghana's geodetic reference network is a network of monuments erected at points whose coordinates are known and kept at the records section of the Survey and Mapping Division of Lands Commission. Historical evidence shows that in establishing the network observations were made by Captain Gordon Guggisberg, the Governor of the by then Gold Coast, from a pillar in Accra. This was subsequently involved in triangulation nets with other trigonometric points to obtain adjusted latitudes and longitudes of these triangulation points to form the Accra Datum. It is important to note that the Accra datum is based on the War Office 1926 ellipsoid with semi-major axis $a = 6378299.99899832$ m, semi minor axis $b = 6356751.68824042$ m, flattening $f = 1/296$ and a Gold Coast feet to meter conversion factor of 0.304799706846218 (Thomas *et al.*, 2000).

However, with the introduction of Global Navigation Satellite System (GNSS) such as Global Positioning System (GPS) for geodetic surveying, the Ghana Survey and Mapping Division of Lands Commission, embarked on the Land Administration Project (LAP) sponsored by the World Bank, to establish a new geodetic reference network referred to as the golden triangle (Fig. 1). This new geodetic reference network adopted the WGS84 datum through the International Terrestrial Reference Frame 2005 (ITRF2005) coordinates specified at epoch 2007.39 (Kotzev 2013). Three permanently operating reference stations have been established at the vertices of this triangle with nineteen second-order reference stations spatially well distributed (Poku-Gyamfi and Hein, 2006). It

is important to note that the LAP has been divided into phases, with the first phase covering five out of the ten administrative regions in Ghana. These regions namely Ashanti, Greater Accra, Central, Western and Eastern form the first phase of the national GPS network.

Two sets of 19 common points from the LAP in both the local War Office $(\phi, \lambda, h)_{OFF}$ and global WGS 84 $(\phi, \lambda, h)_{WGS}$ system which form the golden triangle as shown in Fig. 1 were used in this study for the coordinate transformation. Here, (ϕ, λ, h) is the geodetic latitude, geodetic longitude and ellipsoidal height respectively.

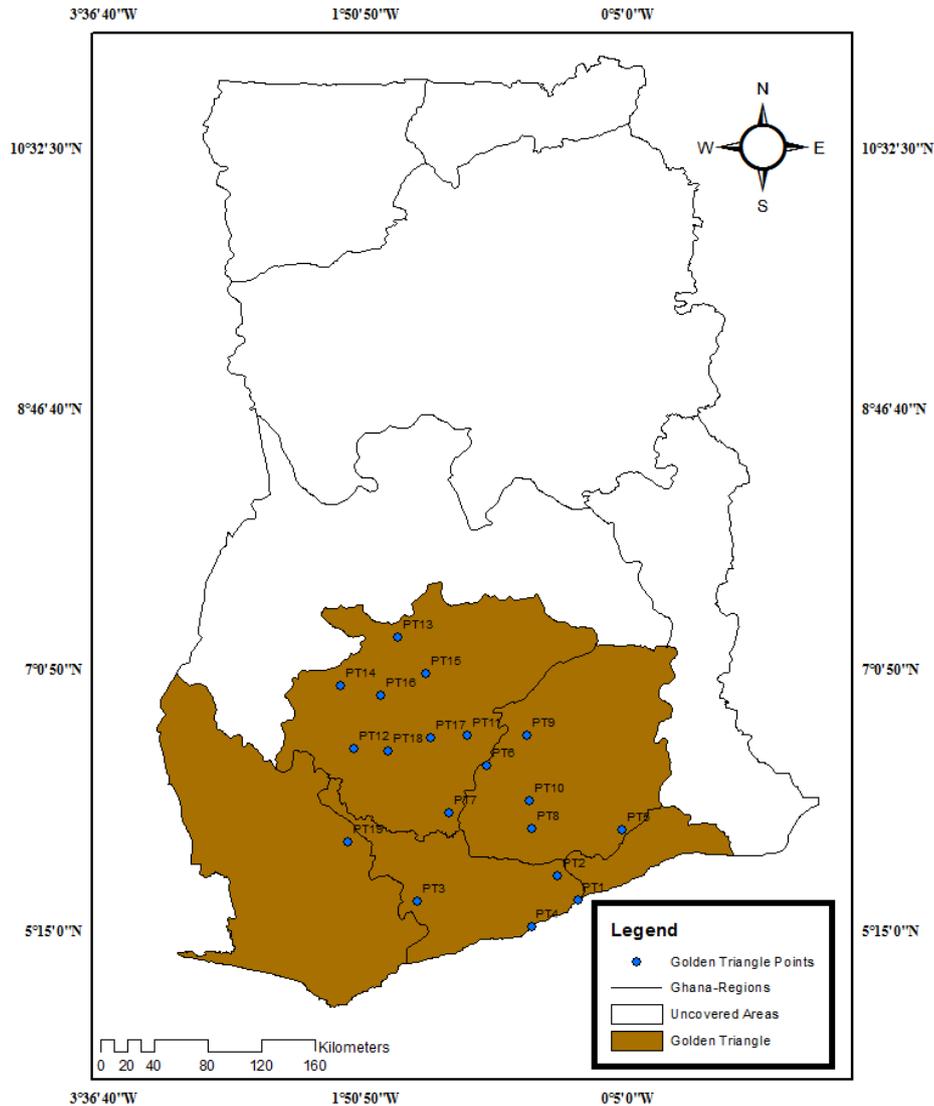


Figure 1. The Study area showing the golden triangle

3. Applied Methods

3.1 Data Conversion

Curvilinear geodetic coordinates (ϕ, λ, h) of common points in both the WGS84 and War Office 1926 system were converted to rectangular cartesian coordinates (X, Y, Z) . This was achieved through Equations 1, 2 and 3 (Schofield and Breach, 2007; Leick *et al.*, 2015) expressed as

$$X = (N + h)\cos\phi\cos\lambda \quad [1]$$

$$Y = (N + h)\cos\phi\sin\lambda \quad [2]$$

$$Z = \{N(1 - e^2) + h\}\sin\phi \quad [3]$$

where $N = \frac{a}{(1 - e^2 \sin^2 \phi)^{\frac{1}{2}}}$, is the radius of curvature in the prime vertical, and $e^2 = 2f - f^2$, is the eccentricity of the ellipsoid. a is semi-major axis of the reference ellipsoid, f is the flattening of the reference ellipsoid and (ϕ, λ, h) is the set of geodetic coordinates.

3.2 Abridged Molodensky Model

Ghana's local geodetic network involved data in geodetic latitude, geodetic longitude and orthometric height; hence Equations 1, 2 and 3 could not be applied straightforwardly. In order to estimate the rectangular coordinates for the War Office 1926 ellipsoid the Abridged Molodensky transformation model was used.

The Abridged Molodensky transformation model convert coordinates directly between two datums by relating the ellipsoidal coordinates of one datum to the other with the assumption that the relative position of the two ellipsoids differs only by translations (Al Marzooqi *et al.*, 2005; Ayer and Tiannah, 2008). It is a model that requires three dimensional geocentric datum shifts $(\Delta X, \Delta Y, \Delta Z)$, the difference between the semi-major axes (Δa) of the two reference ellipsoids and the difference between the flattening (Δf) of the two reference ellipsoids. The Abridged Molodensky transformation is given in curvilinear form by Equations 4 to 6 (Al Marzooqi *et al.*, 2005; Ayer and Tiannah, 2008) as:

$$\Delta\phi = \frac{1}{\rho \sin 1''} (-\Delta X \sin\phi\cos\lambda - \Delta Y \sin\phi\sin\lambda + \Delta Z \cos\phi + (a.\Delta f + f.\Delta a)\sin 2\phi) \quad [4]$$

$$\Delta\lambda = \frac{1}{N \cos\phi \sin 1''} (-\Delta X \sin\lambda + \Delta Y \cos\phi) \quad [5]$$

$$\Delta h = \Delta X \cos\phi\cos\lambda + \Delta Y \cos\phi\sin\lambda + \Delta Z \sin\phi + (a.\Delta f + f.\Delta a)\sin^2\phi - \Delta a \quad [6]$$

with

$$\rho = \frac{a(1 - e^2)}{\sqrt[3]{1 - e^2 \sin^2 \phi}} \quad [7]$$

Here, ρ is the radius of curvature in the meridian, $(\Delta\phi, \Delta\lambda, \Delta h)$ is the set of corrections to transform $(\phi, \lambda, h)_{WGS}$ to $(\phi, \lambda, h)_{OFF}$ and $(\Delta X, \Delta Y, \Delta Z)$ is the set of corrections to transform $(X, Y, Z)_{WGS}$ to $(X, Y, Z)_{OFF}$.

The estimated Δh values were then used to compute h for the War Office 1926 ellipsoid through Equation 8 given by

$$h_{OFF} = h_{WGS} - \Delta h. \quad [8]$$

3.3 Molodensky-Badekas Model

The mathematical expression (Equation 9) relating the two rectangular coordinate systems is given by:

$$\begin{bmatrix} X_W \\ Y_W \\ Z_W \end{bmatrix}_{WGS} = \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix}_{OFF} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} + \eta R \begin{bmatrix} X_F - X_C \\ Y_F - Y_C \\ Z_F - Z_C \end{bmatrix}_{OFF} \quad [9]$$

where

X_C, Y_C and Z_C are the respective centroids of points in the War Office 1926 reference frame;

X_F, Y_F and Z_F are the respective War Office 1926 reference frame coordinates;

X_W, Y_W and Z_W are the respective ITRF 2005 reference frame coordinates;

T_X, T_Y and T_Z are the respective translations along the X, Y and Z axes;

η is the scale factor;

R (Equation 10) is the total rotational matrix; which is given by:

$$R = \begin{bmatrix} \cos \alpha_3 & \sin \alpha_3 & 0 \\ -\sin \alpha_3 & \cos \alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos \alpha_2 & 0 & -\sin \alpha_2 \\ 0 & 1 & 0 \\ \sin \alpha_2 & 0 & \cos \alpha_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_1 & \sin \alpha_1 \\ 0 & -\sin \alpha_1 & \cos \alpha_1 \end{bmatrix} \quad [10]$$

α_1, α_2 and α_3 are the rotation angles.

Equation 9 can be simplified to Equation 11 if the rotation parameters are considered to be small (not more than 10");

$$\begin{bmatrix} X_W \\ Y_W \\ Z_W \end{bmatrix}_{WGS} = \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix}_{OFF} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} + \begin{bmatrix} 1+\eta & R_Z & -R_Y \\ -R_Z & 1+\eta & R_X \\ R_Y & -R_X & 1+\eta \end{bmatrix} \cdot \begin{bmatrix} X_F - X_C \\ Y_F - Y_C \\ Z_F - Z_C \end{bmatrix}_{OFF} \quad [11]$$

The solution of the unknown transformation parameters is obtained by method of Total least squares. To achieve this, Equation 11 was expressed into matrices: the design matrix (A) (Equation 12), observation vector (L) (Equation 13), and the solution matrix (X) (Equation 14).

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -(Z_F - Z_C) & (Y_F - Y_C) & (X_F - X_C) \\ 0 & 1 & 0 & (Z_F - Z_C) & 0 & -(X_F - X_C) & (Y_F - Y_C) \\ 0 & 0 & 1 & -(Y_F - Y_C) & (X_F - X_C) & 0 & (Z_F - Z_C) \end{bmatrix} \quad [12]$$

$$L = \begin{bmatrix} X_{WGS} - X_F - (X_F - X_C) \\ Y_{WGS} - Y_F - (Y_F - Y_C) \\ Z_{WGS} - Z_F - (Z_F - Z_C) \end{bmatrix} \quad [13]$$

$$X = \begin{bmatrix} T_X \\ T_Y \\ T_Z \\ R_X \\ R_Y \\ R_Z \\ \eta \end{bmatrix} \quad [14]$$

3.4 Total Least Squares

Total Least Squares (TLS) is an algorithm created by Golub and Van Loan (1980), which is based on the Errors-in-Variable model. It is a more robust estimator of the solution of a system of equations than the ordinary least squares.

Consider a system of equations in the form of Equation 11 to be solved by least squares (Equation 15):

$$AX \approx L, \text{ where } A \in \mathbb{R}^{m \times n}, X \in \mathbb{R}^{n \times d}, L \in \mathbb{R}^{m \times d}, \text{ and } m \geq n. \quad [15]$$

Unlike ordinary least squares that models out errors in the observation matrix only, TLS considers errors in both the design and observation matrices. Therefore, Equation 15 is expressed in Equation (16) (Akyilmaz, 2007) as

$$(A + e_A) \cdot X = L + e_L, \quad \text{rank}(A) = m < n \quad [16]$$

where e_A and e_L are the errors in the design and observation matrices respectively.

TLS is an iterative algorithm that minimises the errors in Equation 16 until a minimising matrix $[A, L]$ is obtained such that any X which satisfies Equation 15 becomes the TLS solution (Akyilmaz, 2007). The functional relation that is used to compute the TLS solution is given by Equation 17 as

$$[A, L] \cdot [X^T, -1]^T \approx 0 \quad [17]$$

The rank of $[A, L]$ is $m+1$, and must be reduced to m . After the rank reduction, the TLS solution is obtained through (Equation 18):

$$[X^T, -1]^T = \frac{-1}{V_{m+1, m+1}} \cdot V_{m+1}. \quad [18]$$

To solve the TLS problem, the Singular Value Decomposition of the matrix $[A, L]$ is needed. The SVD of $[A, L]$ is given by Equation 19 (Markovsky and Van Huffel, 2007; Ge and Wu, 2012):

$$[A, L] = USV^T \quad [19]$$

where

$S = \text{diag}(\sigma_1, \dots, \sigma_m, \sigma_{m+1}), \sigma_1 \geq \dots \geq \sigma_m \geq \sigma_{m+1}$ be the singular values of $[A, L]$.

$V = [v_1, \dots, v_m, v_{m+1}]$, $V^T V = I_{m+1}$ and $v_i \in \mathbb{R}^{m+1}$

$U = [U_1, U_2]$, $U_1 = [u_1, \dots, u_m]$, $U_2 = [u_{m+1}, \dots, u_n]$, $U^T U = I_n$ and $u_i \in \mathbb{R}^{m+1}$

If $V_{m+1, m+1} \neq 0$, then $AX = L = -1/(V_{m+1, m+1}) \cdot A[v_1, \dots, v_{m+1}]^T$ belongs to the column space of A ; hence X solves the basic TLS problem (Acar *et al.*, 2006; Okwuashi and Eyoh, 2012).

3.5 Conventional Centroid Techniques

Let n be the number of points of War Office 1926 reference frame coordinates (X_F, Y_F, Z_F). The arithmetic, geometric, harmonic, quadratic and median mathematical expressions (Gleb *et al.*, 2009) are given in Equations 20 to 25 respectively.

3.5.1 Arithmetic Mean Centroid (AMC)

$$X_C = \frac{1}{n} \sum_{i=1}^n X_F, Y_C = \frac{1}{n} \sum_{i=1}^n Y_F \text{ and } Z_C = \frac{1}{n} \sum_{i=1}^n Z_F \quad [20]$$

3.5.2 Geometric Mean Centroid (GMC)

$$X_C = \left(\prod_{i=1}^n X_F \right)^{\frac{1}{n}}, Y_C = \left(\prod_{i=1}^n Y_F \right)^{\frac{1}{n}} \text{ and } Z_C = \left(\prod_{i=1}^n Z_F \right)^{\frac{1}{n}} \quad [21]$$

3.5.3 Harmonic Mean Centroid (HMC)

$$X_C = \left(\frac{1}{n} \sum_{i=1}^n \frac{1}{X_F} \right)^{-1}, Y_C = \left(\frac{1}{n} \sum_{i=1}^n \frac{1}{Y_F} \right)^{-1} \text{ and } Z_C = \left(\frac{1}{n} \sum_{i=1}^n \frac{1}{Z_F} \right)^{-1} \quad [22]$$

3.5.4 Quadratic Mean Centroid (QMC)

$$X_C = \left(\frac{1}{n} \sum_{i=1}^n X_F^2 \right)^{\frac{1}{2}}, Y_C = \left(\frac{1}{n} \sum_{i=1}^n Y_F^2 \right)^{\frac{1}{2}} \text{ and } Z_C = \left(\frac{1}{n} \sum_{i=1}^n Z_F^2 \right)^{\frac{1}{2}} \quad [23]$$

3.5.5 Median Centroid (MDC)

Arrange the coordinates in ascending order, such that $X = [X_1, X_2, \dots, X_n]$, $Y = [Y_1, Y_2, \dots, Y_n]$ and $Z = [Z_1, Z_2, \dots, Z_n]$. The median is

For even numbered n; $X_C = \frac{X_k + X_{k+1}}{2}$, $Y_C = \frac{Y_k + Y_{k+1}}{2}$ and $Z_C = \frac{Z_k + Z_{k+1}}{2}$ where $k = \frac{n}{2}$ [24]

For odd numbered n; $X_C = X_k$, $Y_C = Y_k$ and $Z_C = Z_k$ where $k = \frac{n+1}{2}$ [25]

3.6 Developed Hybrid Centroid Approaches

The Arithmetic-Quadratic Mean developed was obtained on the principle applied in Salamin (1975) where Arithmetic-Geometric mean was used to compute numerically the value of π (π). The proposed Harmonic-Quadratic Mean was adopted from Foster and Phillips (1984).

3.6.1 Arithmetic-Quadratic Mean Centroid (AQMC)

Let a and b represent set of numbers. By constructing a sequence of arithmetic means and that of quadratic means, gives Equations 26 as

$$\begin{aligned} a_0 &= a & b_0 &= b \\ a_1 &= \frac{1}{2}(a_0 + b_0) & b_1 &= \left(\frac{b_0^2 + c_0^2}{2} \right)^{\frac{1}{2}} \\ a_1 &= \frac{1}{2}(a_1 + b_1) & b_2 &= \left(\frac{b_1^2 + c_1^2}{2} \right)^{\frac{1}{2}} \\ &\vdots & &\vdots \\ a_{n+1} &= \frac{1}{2}(a_n + b_n) & b_{n+1} &= \left(\frac{b_n^2 + c_n^2}{2} \right)^{\frac{1}{2}} \end{aligned} \quad [26]$$

The iteration continues until $a_{n+1} = b_{n+1}$; this then becomes the Arithmetic-Quadratic mean of the data set.

3.6.2 Harmonic-Quadratic Mean Centroid (HQMC)

Let b and c represent set of numbers. By constructing a sequence of quadratic means and that of harmonic means will give Equation 27.

$$\begin{aligned}
 b_0 &= b & c_0 &= c \\
 b_1 &= \left(\frac{b_0^2 + c_0^2}{2} \right)^{\frac{1}{2}} & c_1 &= \frac{2}{b_0^{-1} + c_0^{-1}} \\
 b_2 &= \left(\frac{b_1^2 + c_1^2}{2} \right)^{\frac{1}{2}} & c_2 &= \frac{2}{b_1^{-1} + c_1^{-1}} \\
 & \vdots & & \vdots \\
 b_{n+1} &= \left(\frac{b_n^2 + c_n^2}{2} \right)^{\frac{1}{2}} & c_{n+1} &= \frac{2}{b_n^{-1} + c_n^{-1}}
 \end{aligned} \tag{27}$$

The iteration continues until $b_{n+1}=c_{n+1}$; this then becomes the Harmonic-Quadratic mean of the data set.

4. Model Performance Assessment

The positional accuracies of each centroid techniques applied were assessed using the mean error (ME), mean squared error (MSE), Mean horizontal position error (MHPE) and standard deviation (SD). The various performance indices are expressed mathematically by Equations 28 to 31 (Ali and Abustan, 2014; Chai and Draxler, 2014) as

$$ME = \frac{1}{n} \sum_{i=1}^n (p_i - q_i), \tag{28}$$

$$MSE = \frac{1}{n} \sum_{i=1}^n (p_i - q_i)^2, \tag{29}$$

$$MHPE = \frac{1}{n} \sum_{i=1}^n \left(\sqrt{\Delta E^2 + \Delta N^2} \right)_i, \tag{30}$$

$$SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (e - \bar{e})^2}, \tag{31}$$

where n is the number of points, p and q are the measured and new projected grid coordinates respectively. Also, e represents the error, estimated as the difference between the measured and new projected grid coordinates while \bar{e} is the mean of the error values.

5. Results and Interpretation

The centroid coordinates (X_c, Y_c, Z_c) used in the derivation of the parameters are shown in Table 1. The introduction of the centroid coordinate in the Molodensky-Badekas model tends to eliminate the correlation of transformation parameters that exists in Bursa-Wolf model when applied to a network of points that cover a small portion of the Earth surface.

Table 1. Estimated Centroid Coordinates from the War Office System

Centroids	X_c (m)	Y_c (m)	Z_c (m)
AMC	6339126.3957023	-133380.2930677	689482.7337759

GMC	6339122.3422794	-113276.9785389	686810.0344144
HMC	6339118.2885957	-81957.5710755	684112.3463096
MDC	6338649.7834986	-142417.4812968	702901.3232139
QMC	6339130.4488619	146570.1204456	692120.3051639
AQMC	6339128.4222823	86761.2382903	690802.7782932
HQMC	6339124.3687317	311382.8524796	688110.5010340

Table 2 shows the parameters determined for transforming data from WGS84 to War Office 1926 datum using the Molodensky-Badekas model and its related standard deviation values. Analysis of Table 2 shows a negative displacement (T_x) along the x -axes of the War office 1926 and WGS84 reference ellipsoids. This further signifies that the two reference ellipsoids moved in the opposite direction along the x -axis. The positive displacements (T_y and T_z) show that the ellipsoids moved in the same direction along the y - and z -axes. The negative scale factor (Table 2) signify that the area of the different geometric shapes of the two reference systems will be reduced or inverted if the scale factor is applied in the model during coordinate transformation. Therefore, this corroborates the fact that, similarity transformation preserves shapes and angles but the lengths of lines and the position of points may change. The rotational parameters (R_x , R_y , R_z) on the other hand caused the change in positions.

Table 2. Summary of Derived Transformation Parameters from the Centroid methods

3D Molodensky-Badekas Model				
Parameter	AMC	GMC	HMC	MDC
Tx (m)	-196.62110±0.25709	-196.61890±0.27367	-196.61578±0.35389	-196.61844±0.26346
Ty (m)	33.36129±0.25709	33.21142±0.26415	32.98111±0.30003	33.455074±0.26165
Tz (m)	322.34374±0.25709	322.31953±0.26433	322.27129±0.30109	322.26706±0.26162
Rx (sec)	0.44514±3.03178E-06	0.44511±3.03178E-06	0.44512±3.03178E-06	0.445153±3.03178E-06
Ry (sec)	-0.00582±5.01208E-06	-0.00579±5.01208E-06	-0.005812±5.01208E-06	-0.00583±5.01208E-06
Rz (sec)	0.02199±4.98865E-06	0.02199±4.98865E-06	0.021989±4.98865E-06	0.021999±4.98865E-06
Scale	-7.16775±2.99101E-06	-7.16775±2.99101E-06	-7.167817±2.99101E-06	-7.16771±2.99101E-06
3D Molodensky-Badekas Model				
Parameter	QMC	AQMC	HQMC	
Tx (m)	-196.59116±1.42704	-196.59756±1.13136	-196.57466±2.22997	
Ty (m)	31.36031±0.87595	31.786261±0.70687	30.170516±1.35491	
Tz (m)	321.72067±0.88686	321.85918±0.71523	321.3936±1.37272	
Rx (sec)	0.44513±3.03178E-06	0.44515±3.03178E-06	0.44516±3.03178E-06	
Ry (sec)	-0.00569±5.01208E-06	-0.00586±5.01208E-06	-0.00580±5.01208E-06	
Rz (sec)	0.02194±4.98865E-06	0.02200±4.98865E-06	0.02193±4.98865E-06	
Scale	-7.16807±2.99101E-06	-7.167624±2.99101E-06	-7.167406±2.99101E-06	

The residuals between the measured and new projected grid coordinates in Eastings and Northings for each centroid technique is shown in Table 3. The Molodensky-Badekas model, like all mathematical models is an approximation of reality, hence it is worth noting that the different centroid techniques applied could not completely absorb and model out distortions in data related to Ghana’s local geodetic network. These distortions (Table 3) could be attributed to the heterogeneous nature of local geodetic networks due to the observational procedures used for its establishment and methods of adjustment used in unifying different smaller networks into a single network. It is important to know that the Ghana War Office 1926 reference frame is no exception.

This is because triangulation was done in the mountainous southern regions whereas traversing was done in the northern territories and other low lying regions from 1922 to 1923 (Kotzev, 2013). The network was then adjusted in smallish figures before combining them together. This resulted in lack of homogeneity between some primary traverses (Kotzev, 2013). In line with this, it can fairly be stated that the Molodensky-Badekas could not effectively model out the distortions in the War Office 1926 reference frame. This has therefore contributed to the large range of differences in ΔE and ΔN . In addition, random errors in both observation data applied in the determination of parameters also contributed to the discrepancies between the measured and new projected grid coordinates. To this effect, the rigorousness of TLS was not fully realised. It is therefore imperative that future studies should consider more consistent and advanced robust techniques in the area of artificial intelligence. Conversely, the overall analysis with reference to Table 3 revealed closely identical values by the various centroid methods.

Table 3. Residuals of Transformed Points (units in metres)

Point	AMC		GMC		HMC		MDC	
	$\Delta E(m)$	$\Delta N(m)$						
P1	-0.55249	1.03022	-0.55242	1.02926	-0.55248	1.02933	-0.55253	1.02926
P2	-0.29423	0.81368	-0.29421	0.81370	-0.29423	0.81370	-0.29421	0.81370
P3	0.27190	-1.55011	0.27187	-1.55013	0.27191	-1.55010	0.27187	-1.55013
P4	0.10175	1.80298	0.10173	1.80300	0.10176	1.80300	0.10173	1.80300
P5	-0.70927	-0.87679	-0.70924	-0.87677	-0.70928	-0.87677	-0.70924	-0.87677
P6	0.09827	-0.87659	0.09831	-0.87664	0.09826	-0.87658	0.09831	-0.87664
P7	0.91940	-0.66448	0.91939	-0.66450	0.91941	-0.66448	0.91939	-0.66450
P8	0.18855	0.51312	0.18854	0.51313	0.18856	0.51314	0.18854	0.51313
P9	0.48216	-0.14261	0.48220	-0.14268	0.48216	-0.14260	0.48220	-0.14268
P10	0.37959	-0.42533	0.37957	-0.42537	0.37959	-0.42532	0.37957	-0.42537
P11	-0.33588	-0.38148	-0.33587	-0.38151	-0.33589	-0.38148	-0.33587	-0.38151
P12	-0.39691	-0.50887	-0.39691	-0.50894	-0.39690	-0.50888	-0.39691	-0.50894
P13	0.54613	1.01654	0.54612	1.01653	0.54612	1.01653	0.54612	1.01653
P14	-0.25305	0.88537	-0.25304	0.88537	-0.25304	0.88536	-0.25304	0.88537
P15	0.80541	0.19197	0.80538	0.19195	0.80540	0.19196	0.80538	0.19195
P16	-0.27828	0.63592	-0.27833	0.63594	-0.27828	0.63592	-0.27833	0.63594
P17	-0.40010	-0.58672	-0.40011	-0.58672	-0.40010	-0.58672	-0.40011	-0.58672
P18	-0.12609	-0.08579	-0.12610	-0.08585	-0.12609	-0.08579	-0.12610	-0.08585
P19	-0.61110	-0.73179	-0.61105	-0.73184	-0.61109	-0.73179	-0.61116	-0.73184
Point	QMC		AQMC		HQMC			
	$\Delta E(m)$	$\Delta N(m)$						
P1	-0.55253	1.02926	-0.55247	1.02926	-0.55253		1.02926	
P2	-0.29421	0.81370	-0.29421	0.81364	-0.29421		0.81359	
P3	0.27198	-1.55013	0.27191	-1.55014	0.27187		-1.55013	
P4	0.10173	1.80300	0.10177	1.80294	0.10173		1.80289	
P5	-0.70924	-0.87677	-0.70924	-0.87683	-0.70924		-0.87688	
P6	0.09831	-0.87664	0.09829	-0.87661	0.09820		-0.87664	
P7	0.91939	-0.66450	0.91942	-0.66451	0.91939		-0.66450	
P8	0.18854	0.51313	0.18858	0.51309	0.18854		0.51313	
P9	0.48220	-0.14268	0.48219	-0.14264	0.48220		-0.14256	
P10	0.37957	-0.42537	0.37962	-0.42537	0.37957		-0.42537	
P11	-0.33587	-0.38151	-0.33586	-0.38150	-0.33587		-0.38151	
P12	-0.39680	-0.50894	-0.39689	-0.50889	-0.39702		-0.50883	
P13	0.54612	1.01642	0.54615	1.01652	0.54612		1.01653	

P14	-0.25304	0.88526	-0.25303	0.88535	-0.25315	0.88537
P15	0.80549	0.19184	0.80543	0.19195	0.80538	0.19195
P16	-0.27822	0.63583	-0.27827	0.63591	-0.27833	0.63594
P17	-0.40011	-0.58684	-0.40008	-0.58674	-0.40011	-0.58672
P18	-0.12599	-0.08585	-0.12607	-0.08581	-0.12610	-0.08574
P19	-0.61105	-0.73184	-0.61109	-0.73181	-0.61116	-0.73184

Figure 2 shows a graphical representation of the use of the mean error to assess the performance of the different centroid methods.

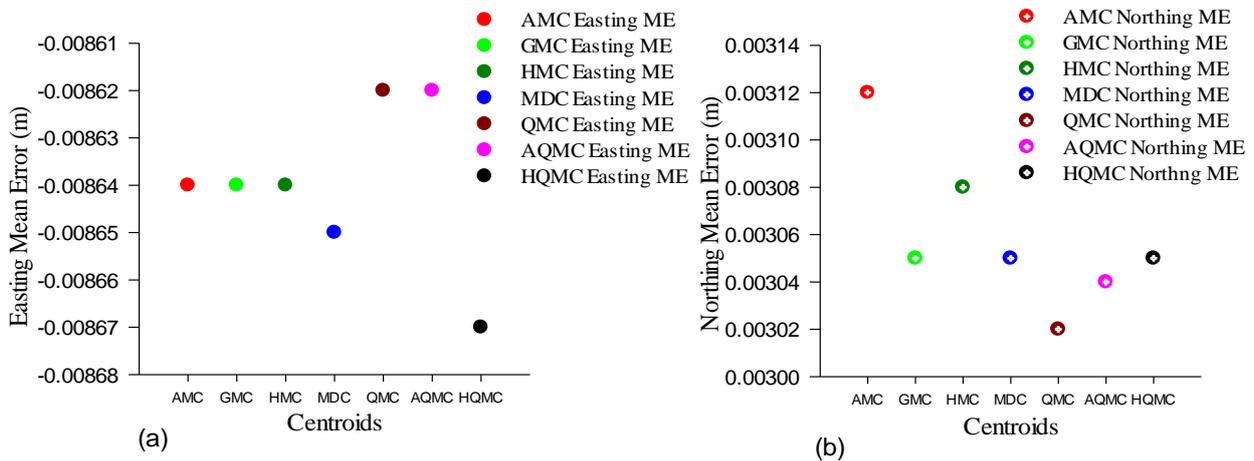


Figure 2. Performance Assessments by Mean Error: (a) Mean Errors in the Eastings, (b) Mean Errors in the Northings. Easting values are depicted by circles; northing values are depicted by circles with crossed centres. AMC values are red, GMC values are green, HMC values are dark green, MDC values are blue, QMC values are dark red, AQMC values are pink and HQMC values are black.

The negative ME values in the Eastings (Figure 2a) signify under fitting. Thus, most of the results attained for new projected grid coordinates are less than their corresponding measured coordinates. The minimum ME of -0.00867 m was realised in the HQMC; while the maximum ME of -0.00862 m was realised in the QM and AQM centroids. In the Northings (Figure 2b), it was observed that a maximum ME of 0.00312 m is incurred if the AMC is applied to the study area. The QMC on the other hand slightly outperformed the AQMC to yield the minimum ME of 0.00302 m. On the basis of the mean error for the Northing coordinates, though the proposed hybrid centroids performed well in the northings, the AQMC was slightly better. However, a general assessment of Figure 2 indicates that the HQMC performed better.

In order to interpret the differences between the measured and new projected grid coordinates, the mean squared errors of each centroid technique was calculated. The MSE as a performance assessment index measures how near the new projected grid coordinates are to their corresponding measured coordinates. The smaller the MSE, the better the centroid technique applied. The performance evaluation of the various centroids by the MSE is shown graphically in Figure 3.

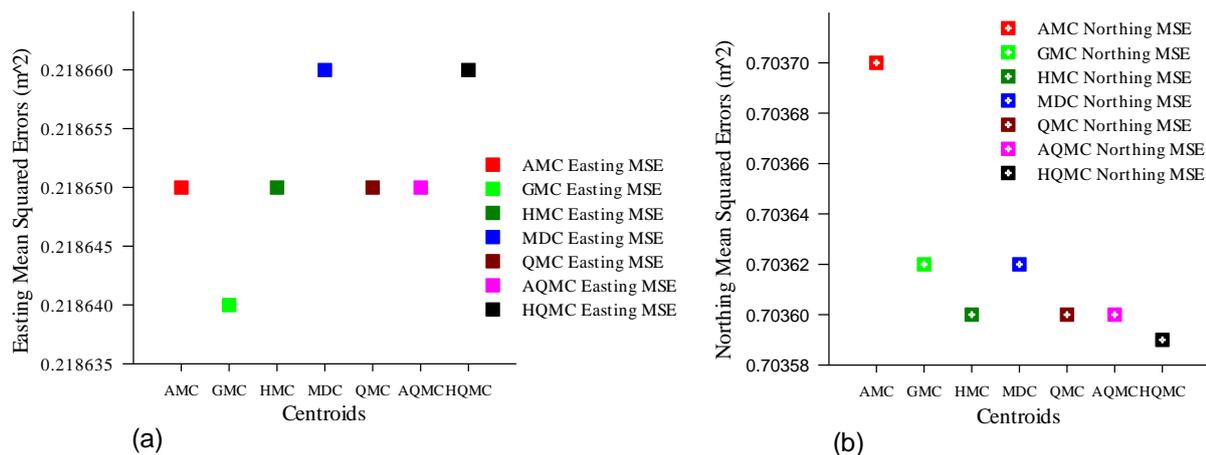


Figure 3. Performance Assessments by Mean Squared Error: (a) Mean Squared Errors in the Eastings, (b) Mean Squared Errors in the Northings. Easting values are depicted by squares; northing values are depicted by squares with crossed centres. AMC values are red, GMC values are green, HMC values are dark green, MDC values are blue, QMC values are dark red, AQMC values are pink and HQMC values are black.

The MSE was used as a criterion to measure the efficiency of the centroid techniques. It is well known that the closer the MSE value to zero the better the performance of the centroid method. In this study, it was observed that the MSE values in the Eastings (Figure 3a) for the centroids were closely related. However, in comparison, the GMC conspicuously attained a minimum MSE of 0.21864 m. A maximum MSE of 0.21866 m in the Eastings was observed from the MDC and HQMC results. With reference to Figure 3b, the HQMC performed slightly better than the other centroid techniques with MSE of 0.70359 m. The AQMC, QMC and HMC produced identical MSE values in the Northings. Evidence from Figure 3b showed that the AMC yielded the maximum MSE of 0.70370 m.

The standard deviation was also calculated for each centroid technique, as shown in Figure 4. This was carried out in order to know the extent of variation of the mean error of each centroid technique from the most probable value.

Considering Figure 4a, a minimum SD in the Eastings was observed from the GMC. It is worth stating that comparable SD values were obtained from the AMC, MDC and the two proposed hybrid centroids. Considering the Northing coordinate, the HQMC outperformed the other centroid techniques with a minimum SD of 0.86178 m as shown in Figure 4b. The AQMC, QMC and HMC produced identical SD values in the Northings. The AMC on the other hand yielded a maximum SD value of 0.86185 m.

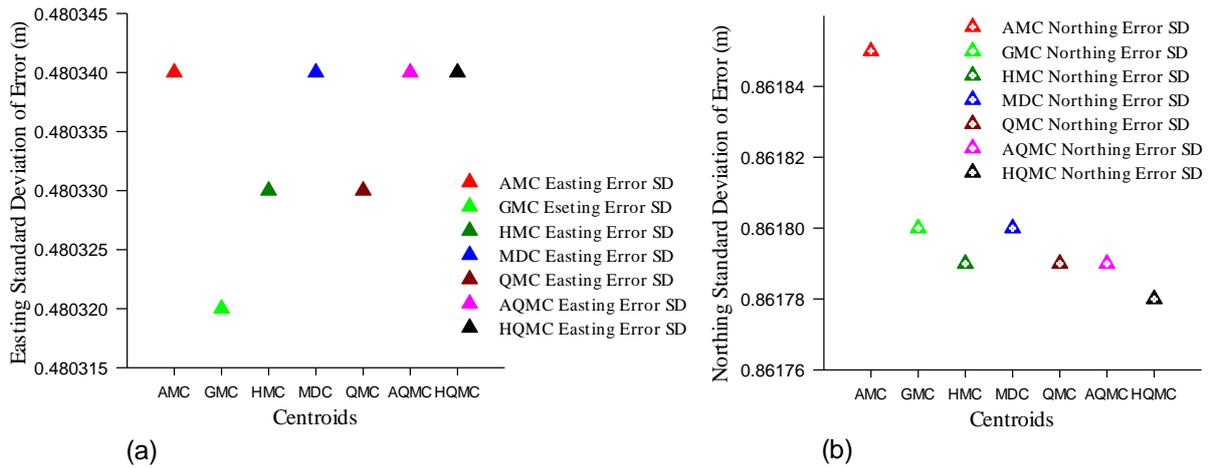


Figure 4. Performance Assessments by Standard Deviation of Error: (a) Standard Deviation of Errors in the Eastings, (b) Standard Deviation of Errors in the Northings. Easting values are depicted by triangles; northing values are depicted by triangles with crossed centres. AMC values are red, GMC values are green, HMC values are dark green, MDC values are blue, QMC values are dark red, AQMC values are pink and HQMC values are black.

Figure 5 is a summary of MHPE of each centroid technique. It measures the average magnitude of horizontal displacement of the new projected grid coordinates from their measured values. The minimum MHPE of 0.88123 m was obtained by the HQM, QM and HM centroids. Again, the weakness of the most frequently used AMC was evident, with a slightly higher value of 0.88128 m.

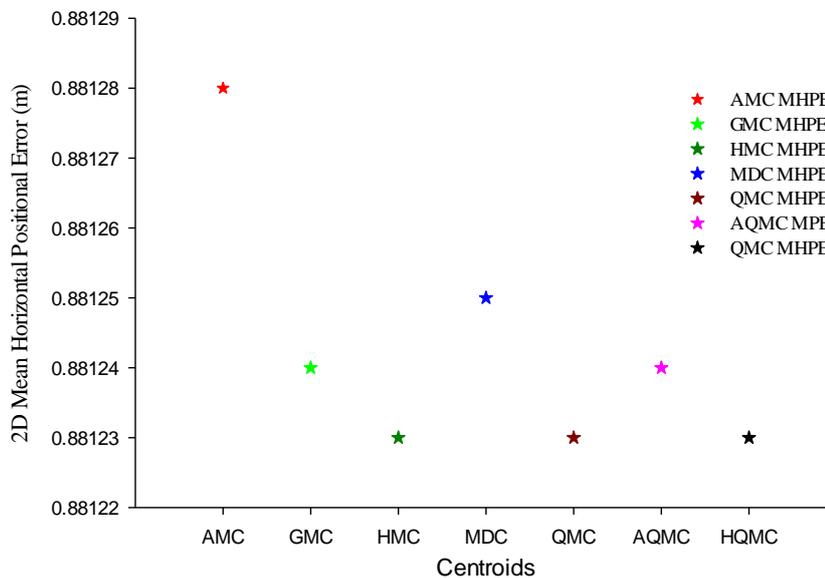


Figure 5. Performance Assessment by Standard Deviation of Error. AMC values are red, GMC values are green, HMC values are dark green, MDC values are blue, QMC values are dark red, AQMC values are pink and HQMC values are black.

The results of the performance assessment indices utilized in this study have been further summarised in Table 4.

Table 4. Results of Performance Assessment of the Centroid Techniques (units in metres)

Centroids		ME	MSE	SD	MHPE
AMC	Eastings	-0.00864	0.21865	0.48034	0.88128
	Northings	0.00312	0.70370	0.86185	
GMC	Eastings	-0.00864	0.21864	0.48032	0.88124
	Northings	0.00305	0.70362	0.86180	
HMC	Eastings	-0.00864	0.21865	0.48033	0.88123
	Northings	0.00308	0.70360	0.86179	
MDC	Eastings	-0.00865	0.21866	0.48034	0.88125
	Northings	0.00305	0.70362	0.86180	
QMC	Eastings	-0.00862	0.21865	0.48033	0.88123
	Northings	0.00302	0.70360	0.86179	
AQMC	Eastings	-0.00862	0.21865	0.48034	0.88124
	Northings	0.00304	0.70360	0.86179	
HQMC	Eastings	-0.00867	0.21866	0.48034	0.88123
	Northings	0.00305	0.70359	0.86178	

The conclusion from the above analysis showed that all the methods applied could produce identical results and thus is applicable for surveying and mapping related works. Hence, the two proposed approaches could serve as an alternative to the existing approaches.

6. Conclusion

The generic mean centroid (arithmetic mean) applied in the Molodensky-Badekas model has been varied by applying the geometric mean, harmonic mean, quadratic mean, median and two proposed hybrid centroids (arithmetic-quadratic mean and harmonic-quadratic mean). Although good coordinate transformation results have been obtained from the Molodensky-Badekas model for years with the arithmetic mean, the field of geodetic engineering demands accuracies to the maximum. In the light of this, the present study objective is to test the suitability of the new proposed hybrid centroid technique to the conventional centroid methods. The analyses conducted in this study based on the statistic performance indicators revealed closely identical results among the centroid methods applied. However, the obtained results in decreasing order show that, the Harmonic-Quadratic Mean, Quadratic Mean, Geometric Mean and Harmonic Mean yielded slightly better results than the more frequently used Arithmetic Mean centroid. On the basis of the results attained, it could be concluded that the proposed Harmonic-Quadratic mean centroid could serve as a practical alternative technique to the frequently used arithmetic mean approach. Finally, this study has established that the Molodensky-Badekas model could not absorb more of the distortions in the Ghana local geodetic datum and thus, its accuracy is also dependent upon the centroid method utilized in the transformation process. In view of this, the authors recommend that for future research work, artificial neural network technology should be applied to test its efficacy within the Ghana geodetic reference network.

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References

- Acar, M, Ozludemir, MT, Akyilmaz, O, and Ayan, T 2006, *Total Least Squares in Geodetic Coordinate Transformation*. American Geophysical Union, San Francisco, CA, USA, 11-15 December 4.
- Akyilmaz, O 2007, 'Total Least Squares Solution of Coordinate Transformation'. *Survey Review*, vol. 39, no. 303, pp. 68-80.
- Ali, MH & Abustan, I 2014, 'A Novel Index for Evaluating Model Performance'. *Journal of Natural Resources and Development*, vol. 4, pp.1-9.
- Al Marzooqi, Y, Fashir, H and Ahmed SI 2005, Derivation of Datum Transformation Parameters for Dubai Emirate. In FIG Working Week 2005 and GSDI-8: From Pharaohs to Geoinformatics, Cairo, Egypt, 16-21, April.
- Ayer, J & Tiannah, T 2008, 'Datum Transformation by the Iterative Solution of the Abridging Inverse Molodensky Formulae'. *The Ghana Surveyor*, vol. 1, no. 2, pp. 59-66.
- Chai, T & Draxler, RR 2014, 'Root Mean Square Error (RMSE) or Mean Absolute Error (MAE)? – Arguments against avoiding RMSE in the literature'. *Geoscientific Model Development*, vol.7, pp.1247-1250.
- Constantin-Octavian, A. 2006, 3D Affine Coordinate Transformations. Master of Science Thesis in Geodesy No. 3091 TRITA-GIT EX 06-004, School of Architecture and the Built Environment, Royal Institute of Technology (KTH), 100 44 Stockholm, Sweden.
- Dzidefo, A. (2011), "Determination of Transformation Parameters between the World Geodetic System 1984 and the Ghana Geodetic Network", Master's Thesis, Department of Civil and Geomatic Engineering, KNUST, Kumasi, Ghana, 97 pp.
- Foster, DME & Phillips, GM 1984, 'The Arithmetic-Harmonic Mean', *Mathematics of Computation*, vol. 42, no.165, pp. 183-191.
- Ge, X & Wu, J 2012, 'A New Regularized Solution to Ill-Posed Problem in Coordinate Transformation'. *International Journal of Geosciences*, vol.3, pp. 14-20.
- Ghilani, CD, 2010, *Adjustment Computations: Spatial Data Analysis*, 5th edition, John Wiley & Sons Inc., Hoboken, New Jersey.
- Gleb, B, Calvo, T and James, S 2009, 'Some Results on Lipschitz Quasi-Arithmetic Means', *IFSA-EUSFLAT*.
- Golub, GH & Van Loan, CF 1980, 'An Analysis of the Total Least Squares Problem', *SIAM Journal of Numerical Analysis*, vol. 17, no. 6, pp. 883-893.
- Kheloufi, N. 2006, Coordinates Transformation by "Zoning Method" for Parameters Computing between WGS84 and North Sahara. In XXIII FIG Congress, Munich, Germany, 8-13 October.
- Kotzev, V 2013, 'Consultancy Service for the Selection of a New Projection System for Ghana'. Draft Final Reports, World Bank Second Land Administration Project (LAP-2), Ghana.
- Leick, A, Rapoport, L and Tatamikov, D 2015, *GPS Satellite Surveying*, 4th edition, Wiley.
- Markovsky, I & Van Huffel, S 2007, 'Overview of Total Least-Squares', *Signal Processing*, vol. 87, no. 10, pp. 2283-2302.
- Mihalache, RM 2012, 'Coordinate Transformations for Integrating Map Information in the New Geocentric European System using Artificial Neuronal Networks'. *GeoCAD*,
- Mitsakaki, C 2004, Coordinate Transformations. In FIG Working Week, Athens, Greece, 22-27 May.
- Mohammed, AEM & Mohammed, NZ 2013, 'WGS84 to Adindan-Sudan Datum Transformation Manipulated by ITRF96'. *International Journal of Multidisciplinary Sciences and Engineering*, vol.4 no. 5, pp.60-64.
- Okwuashi, O & Eyoh, E 2012, '3D Coordinate Transformation using Total Least Squares'. *Academic Research International*, vol. 3, no. 1, pp. 2223-9944.

- Poku-Gyamfi, Y & Hein, GW 2006, Framework for the Establishment of a Nationwide Network of Global Navigation Satellite System (GNSS) – A Cost Effective Tool for Land Development in Ghana. In 5th FIG Conference, Accra, Ghana, 8-11 March.
- Salamin, E 1975, 'Computation of π using Arithmetic-Geometric Mean', *Mathematics of Computation*, vol. 30, no. 135, pp. 565-570.
- Schofield, W & Breach, M 2007, *Engineering Surveying*, 6th edition, Elsevier.
- Solomon, M 2013, Determination of Transformation Parameters for Montserrado County, Republic of Liberia. Master of Science Thesis in Geomatic Engineering, Faculty of Civil and Geomatic Engineering, College of Engineering, KNUST, Kumasi, Ghana.
- Stankova, H, Certona, P and Novosad, M 2012, An Analysis of the possibility of Intersystem Transformation for the Purposes of Geo-referencing Old Mine Workings. *GeoScience Engineering*, vol.8, no.3, pp. 1-12.
- Thomas, G, Sannier, CAD and Taylor JC 2000 'Mapping Systems and GIS: A Case Study using the Ghana National Grid'. *The Geographical Journal*, vol. 166, no. 4, pp.306-311.
- Turgut, B 2010, 'A Back-Propagation Artificial Neural Network Approach for Three-Dimensional Coordinate Transformation'. *Scientific Research and Essays*, vol.5, no. 21, pp.3330-3335.
- Ziggah, YY, Youjian, H, Odutola, CA and Fan, DL 2013a, 'Determination of GPS Coordinate Transformation Parameters of Geodetic data between Reference Datums-A Case Study of Ghana Geodetic Reference Network'. *International Journal of Engineering Sciences and Research Technology*, vol. 2, no. 4, pp. 2277-9655.
- Ziggah, YY, Youjian, H, Odutola, CA and Nguyen, TT 2013b, 'Accuracy Assessment of Centroid Computation Methods in Precise GPS Coordinates Transformation Parameters Determination- A Case Study, Ghana'. *European Scientific Journal*, vol. 9, no. 15, pp. 1857-7431.