South African Journal of Geomatics, Vol. 6. No. 1, April 2017

Performance Evaluation of the Ordinary Least Square (OLS) and Total Least Square (TLS) in Adjusting Field Data: An Empirical Study on a DGPS Data

M. S. Peprah[1], I. O. Mensah[1*]

[1]*Deparment of Geomatic Engineering, University of Mines and Technology, Tarkwa, Ghana.*

*Corresponding authors email: nanaobour20@yahoo.com; mspeprah@st.umat.edu.gh

DOI: http://dx.doi.org/10.4314/sajg.v6i1.5

Abstract

Survey measurements have become the traditional means of obtaining point positions by most surveyors over the centuries. The assertion that a post processed DGPS field data is precise, accurate and can be used to execute any engineering works due to its minimum human errors need to be reviewed; this is because the post processed field data still contains errors and needs to be adjust. Adjustments and computations is one of the main research field in mathematical and satellite geodesy to assess the magnitude of errors and to study their distributions whether they are within or not within the acceptable tolerance. In order to achieve the objective of this study, a DGPS field data was adjusted using the Ordinary Least Square (OLS) and Total Least Square (TLS) techniques. The OLS considers errors only in the observation matrix, and adjusts observations in order to make the sum of its residuals minimum. The TLS considers errors in both the observation matrix and the data matrix, thereby minimizing the errors in both matrices. The limited availability of information on OLS and TLS in adjusting DGPS field data and the uncertainty of which method is optimal, whether the OLS and TLS is the most appropriate technique has called for the need to undertake this study. This study aimed at comparing the working efficiency of the OLS and TLS, assessing their individual accuracy and selecting the most effective method in adjusting DGPS field data. Each model was assessed based on statistical indicators of mean horizontal error (MHE), mean bias error (MBE), mean absolute error (MAE), root mean square error (RMSE), and standard deviation (SD). After applying the OLS and TLS methods independently for the same datasets, it was ascertained that, the OLS method was better in adjusting DGPS field data than the TLS with a MHE and SD of +1.203079 m and +1.663134 m as compared to TLS with MHE and SD of +7.0985507 m *and +2.594045 m respectively. This study will therefore create opportunity for geospatial professionals to know the efficiency of OLS and TLS in solving some of the problems in mathematical and satellite geodesy.*

Keywords: Differential Global Positioning, System, Total Least Square, Ordinary Least Square, Survey Adjustments, Horizontal Position Displacement.

1. Introduction

Survey measurements over the centuries have been the traditional means of measuring and portraying the earth's surface (Ghilani and Wolf, 2012). The measurement can be either direct or indirect (Ghilani, 2010). Survey measurements are one of the core mandate in all areas of geoscientific applications. The fundamental measured quantities in every survey measurements are distances, angles, and elevations. These forms the basis for coordinates determination of positions concerning a specific datum either horizontal or vertical (Annan *et al*., 2016a). From these coordinates positions, other distances and angles that were not obtained directly during field measurements may be computed indirectly. The errors that were present in the original direct observations may propagate by the computational process into the indirect values (Ghilani, 2010). Thus, the indirect measurements contain errors that are functions of the original errors (Ghilani, 2010). In adjusting the field data values, the traditional techniques that are normally used is the classical least squares techniques.

Classical least square techniques are the most widely used methods for adjusting field data of ground points. In the classical least square techniques, adjustment of the observation equations where only the observations are considered as stochastic (Acar *et al*., 2006). In some instances, the design matrix elements contain errors which are usually ignored in the classical least techniques and this ignorance remains as an uncertainty in the solution results (Acar *et al*., 2006).

Differential Global Positioning System (DGPS) is of a higher accuracy than the absolute observation due to the use of reference station where coordinates are known to ascertain accuracy. The techniques used in DGPS observations are static, fast static, stop and go, and real time kinematics. Among DGPS survey techniques, the static method is of a higher accuracy due to some techniques used in the data collection process. Data collected after field survey needs to be processed to obtain a desired result. DGPS data after post processing still contain errors (Ansah, 2016; Okwuashi, 2014). The errors remain in the data could be adjusted. Several adjustment methods exist such as ordinary least squares (Annan *et al*., 2016a; Okwuashi and Eyoh, 2012a), Total least squares (Acar *et al*., 2006; Annan *et al*., 2016b; Okwuashi and Eyoh, 2012b), robust estimation (Wieser and Brunner, 2001), least square collocation (LSC) (Moritz, 1972). Conversely, there is a common belief that data obtain from the GPS instrument are very precise and accurate due to less interference of humans in the collection process and can be used to execute any given task without further adjustments.

The measured field data like any other survey measurement contains errors and need to be adjusted. Usually, adjustment is done using classical approach such as ordinary least approach (Annan *et al*., 2016a; Okwuashi & Eyoh, 2012b), but the ordinary least square which is based on regression analysis considers only the observations to be stochastic (Acar *et al*., 2006) and thus account for errors only in the observation vector. Conversely, there exist errors in both the design and observation matrices which ought to be modelled out (Annan *et al*., 2016a). OLS is the most commonly widely used adjustment method in geodesy. It has been used in many geodetic areas in the recent decades. Notable among them are approximation of the surfaces in engineering structures (Lenda, 2008), finding the relationship between global and Cartesian coordinates (Ziggah, 2012), predictions of local coordinates (Odutola *et al*., 2013), converting GPS data from global coordinate system to the National coordinate system (Dawod *et al*., 2011).

Another numerical method applicable for adjusting field data is the TLS. It is worth mentioning that several studies have been carried out by researchers with the TLS techniques. Notable among them are 3D datum transformation including the weighted scenarios (Amiri-Simkooei and Jazaeri, 2012), measuring data perturbation size (Markovsk *et al*., 2009), localization of robots (Yang, 1997), calibration of robots (Nievergelt, 1994). It was emphasized that OLS which was mostly used in early times gave less accuracies because it assumes errors only in the output variable, y-value, whereas TLS assumes errors in both the input and output variables (Effah, 2015). The total least squares (TLS) was invented to resolve the working efficiency of the OLS (Annan *et al*., 2016a). The TLS have the ability to adjust the errors in both the observation matrix and design matrix (Acar *et al*., 2006) in order to yield a better estimate. The limited availability of technical papers in geodesy on TLS have called for the need to undergone this study. Also, the optimal model in adjusting surveying networks whether the OLS and TLS is good enough is the objective of this present study. Researchers such as (Acar *et al*., 2006; Annan *et al*., 2016a, 2016b; Okwuashi and Eyoh, 2012; OKwuashi, 2014) have applied TLS to solve many scientific problems and they concluded the TLS working efficiency is encouraging. This present study adopted the OLS and TLS in adjusting DGPS network and to propose the models which is optimal for adjusting DGPS network.

Although extensive applications of OLS and TLS have been carried out, limited literature is available in geodesy technical papers on the applications of OLS and TLS, for adjusting DGPS field data especially in developing countries like Ghana where geodesy has not yet reached the advanced stage. In addition, there have been arguments on which method, whether the OLS or TLS model is the most effective based on their various ideologies. Thus, the TLS technique takes into account observational errors on both dependent and independent variables while OLS considers only the independent variable. Therefore, this present study aims at making a comparative study of both methods and selecting the most effective technique. The authors were motivated to embark this study because it is yet to be evaluated in Ghana. This study will also create the opportunity for geospatial professionals to arrive a conclusion on which technique is optimal in adjusting DGPS field after post processing.

2. Study Area and Data Source

The study area (Figure 1) is situated in the mining town of Tarkwa which is the administrative capital of the Tarkwa Nsuaem Municipal Assembly in the Western Region of Ghana. It is found in the Southwest of Ghana with geographical coordinates between longitudes 1° 59′ 00″ W and latitude 5° 18′ 00″ N and is 78 m above mean sea level. It is about 85 km from Takoradi, which is the regional capital, 233 km from Kumasi and about 317 km from Accra (Ziggah, 2012). The topography is generally described as remarkable series of ridges and valleys. The ridges are formed by the Banket and Tarkwa Phyllites whereas upper quartzite and Huni Sandstone are present in the valleys. Surface gradients of the ridges are generally very close to the Banket and Tarkwa Phyllites. Its environs generally lie within the mountain ranges covered by thick forest interjected by undulating terrain with few scarps. The study area has a South-western Equatorial climate with seasons influenced by the moist South-West Monsoon winds from the Atlantic Ocean and the North-East Trade Winds. The mean rainfall is approximately 1500 mm with peaks of more than 1700 mm in June and October. Between November and February, the rainfall pattern decreases to between 20 mm to 90 mm (Forson, 2006). The mean annual temperature is approximately 25 °C with small daily temperature variations. Relative humidity varies from 61 % in January to a maximum of 80 % in August and September (Ziggah, 2012; Seidu, 2004).

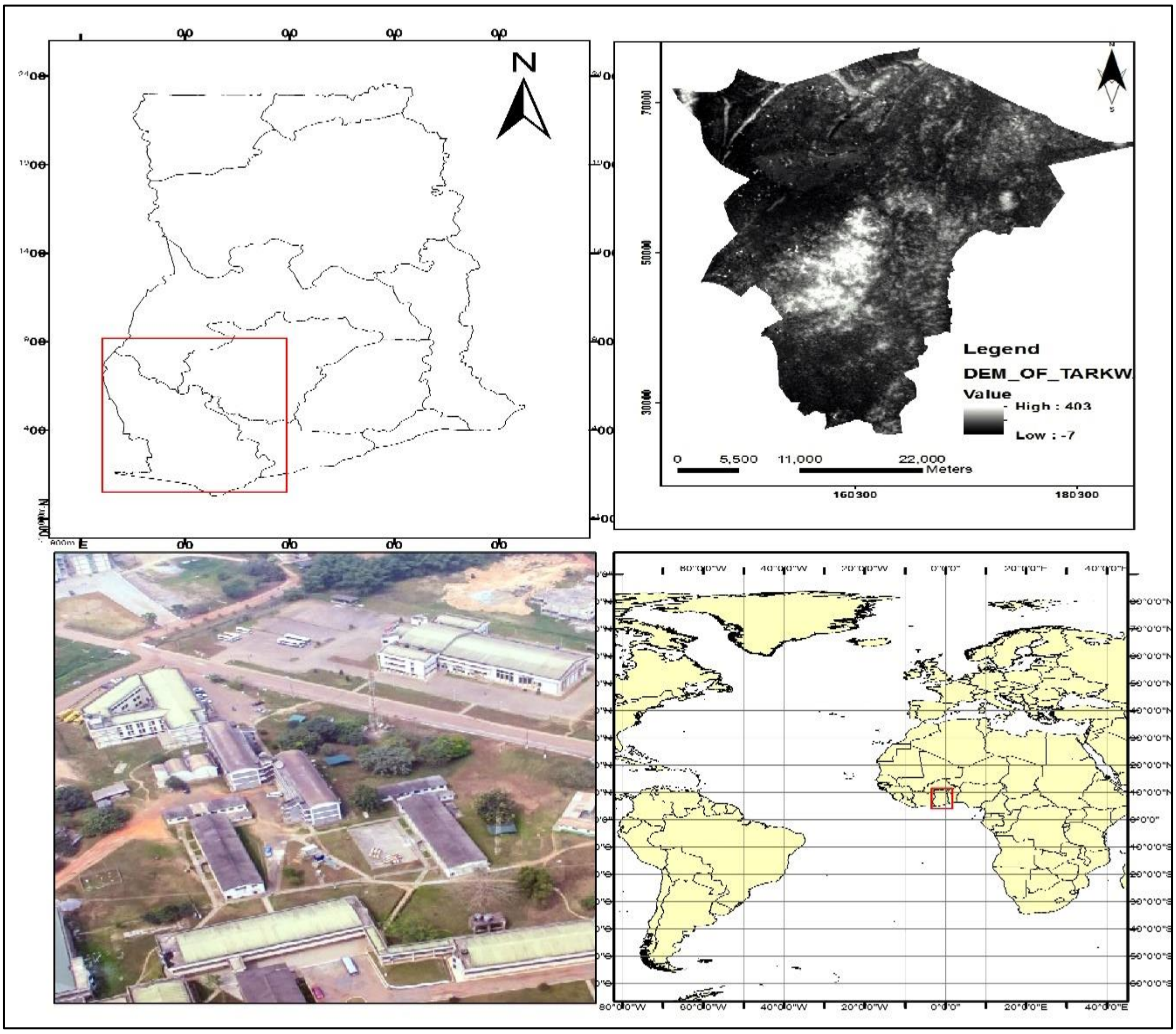

Figure 1. Study Area

In this study, a total of 12 DGPS data collected by field measurements in UMaT, Tarkwa, Ghana, situated in West Africa, were used in the OLS and TLS model formulation. It is well acknowledged that, one of the contributing factors affecting the estimation accuracy of models is related to the quality of datasets used in model-building (Dreiseitl and

Ohno-Machado, 2002; Ismail *et al*., 2012). Therefore, to ensure that the obtained field data from the GPS receivers are reliable and accurate, several factors such as checking of overhead obstruction, obstruction, observation period, observation principles and techniques as suggested by many researchers (Yakubu and Kumi-Boateng, 2011; Ziggah *et a*l., 2016) were performed on the field. In addition, all potential issues relating to GPS survey work were also considered.

3. Methods

3.1 Ordinary Least Square (OLS) and Total Least Square (TLS)

Least Square method is a statistical technique that is capable of determining the line of best fit of a model and seeks to find the minimum sum of the squares of residuals. This method is extensively used in regression analysis and estimation (Miller, 2006). Considering a system of equations in the form as denoted by Equation 1 to be solved by least squares:

$$BX \approx L \tag{1}$$

Where $B \in R^{m \times n}, X \in R^{n \times d}, L \in R^{m \times d}$, and $m \geq n$ (Annan *et al*., 2016a; Schaffrin, 2006). *B* is the design matrix, *X* is the matrix of the unknown parameters, and *L* is the observation matrix.

The solution of the unknown parameters matrix X by OLS approach can be achieved as denoted by Equation 2:

$$X = \left[B^T B\right]^{-1} \left[B^T L\right] \tag{2}$$

The corresponding error vector V can be achieved by using Equation 3 as denoted by:

$$V = BX - L \tag{3}$$

On the other hand, solution of unknowns parameters $\hat{X}$ by TLS approach is obtained as denoted by Equation 4:

$$L + V_L = \left[B + V_B\right]\hat{X} rank(B) = m < n \tag{4}$$

Where V_L is the error vector of observations and V_B is the error matrix of the data matrix, the assumption that both have independently and identically distributed rows with zero mean and equal variance (Akyilmaz, 2007).

Golub and Van Loan, (1980), invented TLS to rectify the inefficiency associated with the OLS. Thus, accounting for perturbations in data matrix and observation matrix (Annan *et al*., 2016b). TLS is a mathematical algorithm that yields a unique solution in analytical form in terms of the Singular Value Decomposition (SVD) of the data matrix (Markovsky and Van Huffel, 2007). According to Golub and Van Loan, (1980) and Okwuashi and Eyoh, (2012a), the TLS algorithm is an iterative process which looks to minimize the errors in Equation 5 as denoted by:

$$\min\left\|[B,L]-[\hat{B},\hat{L}]\right\|_F, [\hat{B},\hat{L}] \in R^{n(m+1)} \tag{5}$$

The optimization process goes on until a minimizing $[\hat{B},\hat{L}]$ is obtained, any $\widehat{X}$ that satisfies $\hat{B}\hat{X}=\hat{L}$ is the TLS solution (Annan *et al*., 2016a). In order to obtain the solution of $B\hat{X}=L$, we write the functional relation as denoted by Equation 6:

$$[B,L][X^T,-1]^T \approx 0 \tag{6}$$

The TLS problem can be solved using the Singular Value Decomposition (SVD) (Markovsky and Van Huffel, 2007; Ge and Wu, 2012). The SVD of the augmented matrix $[B,L]$ is required to determine whether or not it is rank deficient. Matrix $[B,L]$ can be represented by SVD as denoted by Equation 7 as:

$$[B,L]=USV^T \tag{7}$$

Where U = real valued m x n orthonormal matrix, $UU^T = I_m$, V = real value n x n orthonormal matrix, $VV^T = I_n$, S = m x n matrix with diagonals being singular values, off-diagonals are zeros. The rank of matrix $[B,L]$ is m + 1, and must be reduced to m using the Eckart-Young Mirsky theorem (Annan *et al*., 2016a). The TLS solution after the rank reduction is given by Equation 8 denoted as:

$$[\hat{X}^T,-1]=\frac{1}{V_{m+1,m+1}}V_{m+1} \tag{8}$$

If $V_{m+1,m+1} \neq 0$, then $BX = L = -1/(V_{m+1,m+1}).B[V_{1,m+1},\cdots,V_{m,m+1}]^T$ belongs to the column space of $\hat{B}$, hence X solves the basic TLS problem (Okwuashi and Eyoh, 2012a). The corresponding TLS correction is achieved by using Equation 9 as denoted by:

$$\left[\Delta\hat{B},\Delta\hat{L}\right] = [B,L] - \left[\hat{B} - \hat{L}\right] \qquad (9)$$

3.2 Models Performance Evaluation

In order to determine the accuracies of the models used, the various statistical indicators were employed to determine the working efficiency of the models. Hence, to make an unprejudiced valuation of the models, statistical indicators such as Root Mean Square (RMSE), Mean Biased Error (MBE), and Mean Absolute Error (MAE), Horizontal Position error (HE), and Standard Deviation were used. Their individual mathematical languages are given by Equation 10 to Equation 14 respectively denoted by:

$$RMSE = \sqrt{\sum \frac{E^2}{n}} \qquad (10)$$

where n is the number of observation points and E^2 is the square of the error. The MBE was calculated using the formula below:

$$MBE = \sum \frac{E}{N} \qquad (11)$$

where E is the error and n is the number of observation points. The MAE was calculated using the formula:

$$MAE = \sum \frac{|E|}{N} \qquad (12)$$

where $|E|$ is the absolute error and n is the number of observation points.

$$HE = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2} \qquad 13$$

Where X_2 and Y_2 are the existing coordinates, and X_1 and Y_1 are the observed coordinates.

$$SD = \sqrt{\sum \frac{(x - \bar{x})^2}{n-1}} \qquad 14$$

The Standard Deviation (SD) measures how closely the data are clustered around the mean. *n-1* is the degree of freedom.

4. Results and Discussions

Table 1 shows the existing coordinates and the observed DGPS data. From Table 1, it was observed that, there is a difference between the existing data and the measured data. Table 2 and Table 3 shows the results obtained by the OLS and TLS models. The residual graphs for the OLS and TLS models are represented by Figure 2 and Figure 3. Figure 4 is the horizontal shift error graph obtained by the OLS and TLS models. From the graph, it was observed that the performance of the OLS model was encouraging as compared to the TLS.

Table 1. Existing Coordinates and DGPS field Data (Units in metres)

ACTUAL	ACTUAL	DGPS	DGPS	RESIDUALS	
X	**Y**	**X**	**Y**	**ΔX**	**ΔY**
163244.2300	69656.2000	163244.2300	69656.2000	0.0000	0.0000
163216.6680	69604.4470	163216.5376	69604.3286	0.1304	0.1184
163363.5750	69380.3820	163363.1177	69380.1969	0.4573	0.1851
163463.0150	69520.9960	163462.4604	69520.7708	0.5546	0.2252
163505.7080	69583.2260	163505.2266	69582.9690	0.4814	0.2570
163509.1960	69598.9950	163509.0747	69599.0941	0.1213	-0.0991
163517.3220	69634.4160	163515.8068	69635.0266	1.5152	-0.6106
163531.1310	69786.7810	163529.5562	69787.1048	1.5748	-0.3238
163497.2520	69817.9670	163494.9389	69818.4032	2.3131	-0.4362
163423.3670	69777.2060	163422.5800	69776.7900	0.7870	0.4160
163394.6160	69753.3440	163392.7060	69753.3482	1.9100	-0.0042
163359.0890	69687.6660	163351.3071	69688.4810	7.7819	-0.8150

Table 2. Results obtained by the OLS models (Units in metres)

ACTUAL	ACTUAL	ADJUSTED DGPS OLS		RESIDUALS	
X	**Y**	**X**	**Y**	**ΔX**	**ΔY**
163244.2300	69656.2000	163245.8392	69656.6958	-1.6092	-0.4958
163216.6680	69604.4470	163217.9890	69604.9549	-1.3210	-0.5079
163363.5750	69380.3820	163363.7027	69380.4783	-0.1277	-0.0963
163463.0150	69520.9960	163463.4563	69520.6158	-0.4413	0.3802
163505.7080	69583.2260	163506.4051	69582.6250	-0.6971	0.6010
163509.1960	69598.9950	163510.3055	69598.7258	-1.1095	0.2692

163517.3220	69634.4160	163517.1555	69634.6104	0.1664	-0.1944
163531.1310	69786.7810	163531.4144	69786.5363	-0.2834	0.2447
163497.2520	69817.9670	163496.9279	69817.9311	0.3241	0.0359
163423.3670	69777.2060	163424.4771	69776.5937	-1.1101	0.6123
163394.6160	69753.3440	163394.5438	69753.2700	0.0722	0.0740
163359.0890	69687.6660	163352.9523	69688.5891	6.1367	-0.9231

Table 3. Results obtained by the TLS model (Units in metres)

ACTUAL	**ACTUAL**	**ADJUSTED DGPS TLS**		**RESIDUALS**	
X	**Y**	**X**	**Y**	**ΔX**	**ΔY**
163244.2300	69656.2000	163247.7070	69664.4334	-3.4770	-8.2334
163216.6680	69604.4470	163217.7791	69614.4181	-1.1111	-9.9711
163363.5750	69380.3820	163352.0532	69385.4190	11.5218	-5.0370
163463.0150	69520.9960	163457.2129	69519.7871	5.8021	1.2089
163505.7080	69583.2260	163502.5647	69579.2970	3.1433	3.9290
163509.1960	69598.9950	163507.1551	69595.0755	2.0409	3.9195
163517.3220	69634.4160	163515.5595	69630.3250	1.7625	4.0910
163531.1310	69786.7810	163536.5335	69780.2261	-5.4025	6.5549
163497.2520	69817.9670	163503.7753	69812.8890	-6.5233	5.0780
163423.3670	69777.2060	163430.1214	69775.1919	-6.7544	2.0141
163394.6160	69753.3440	163399.4096	69753.4291	-4.7936	-0.0851
163359.0890	69687.6660	163355.2823	69691.2106	3.8067	-3.5446

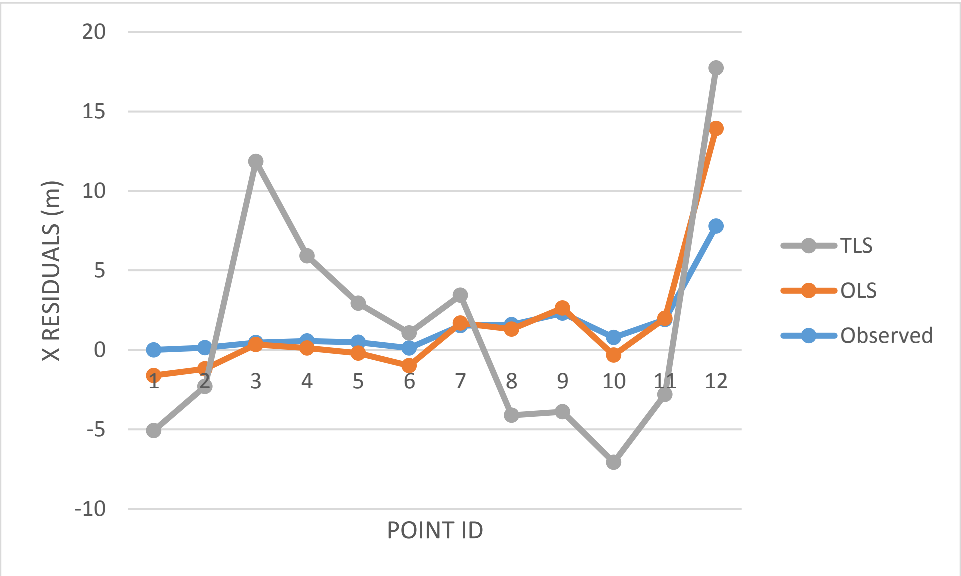

Figure 2 Residuals plot in the Eastings

Figure 2 show the variations of residuals in eastings when the adjusted coordinates produced by the TLS and OLS methods respectively were subtracted from the actual existing coordinates. It was observed that the OLS method gave better estimates of the unknowns as compared to the TLS. This could possibly be that perturbation exists in the design matrix and the observation matrix formed from the coordinates obtained from the study area. Therefore, the OLS was preferred to that of the TLS for the study area.

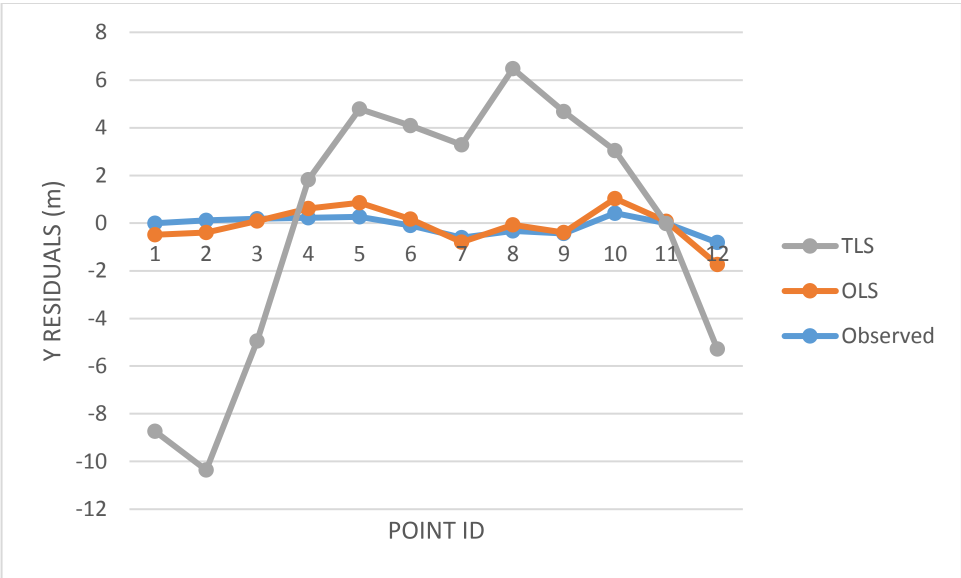

Figure 3 Residuals plot in the Northings

Similarly, Figure 3 show the variations of residuals in northings when the adjusted coordinates produced by the TLS and OLS methods respectively were subtracted from the actual existing coordinates. It was observed that the OLS method gave better estimates of the unknowns as compared to the TLS. This could possibly be that perturbation exists in the design matrix and the observation matrix formed from the coordinates obtained from the study area. Therefore, the OLS was preferred to that of the TLS for the study area.

The horizontal shifts of the positions for all the twelve points for both OLS and TLS methods were computed and the results is shown in Table 4. Figure 4 show the plot of the horizontal shift of the OLS and TLS models.

Table 4 Horizontal Displacement of Points (Units in metres)

POINT ID	OBSERVED	OLS	TLS
1	0.000000	1.683847	8.937472
2	0.176133	1.415275	10.03282
3	0.493341	0.159941	12.57471
4	0.598578	0.582493	5.926703
5	0.545706	0.920407	5.031637
6	0.156635	1.141691	4.419022
7	1.633604	0.255891	4.454513
8	1.607744	0.374424	8.494335
9	2.353870	0.326082	8.266772
10	0.890183	1.267767	7.048299
11	1.910005	0.103387	4.794335
12	7.824461	6.205739	5.201457

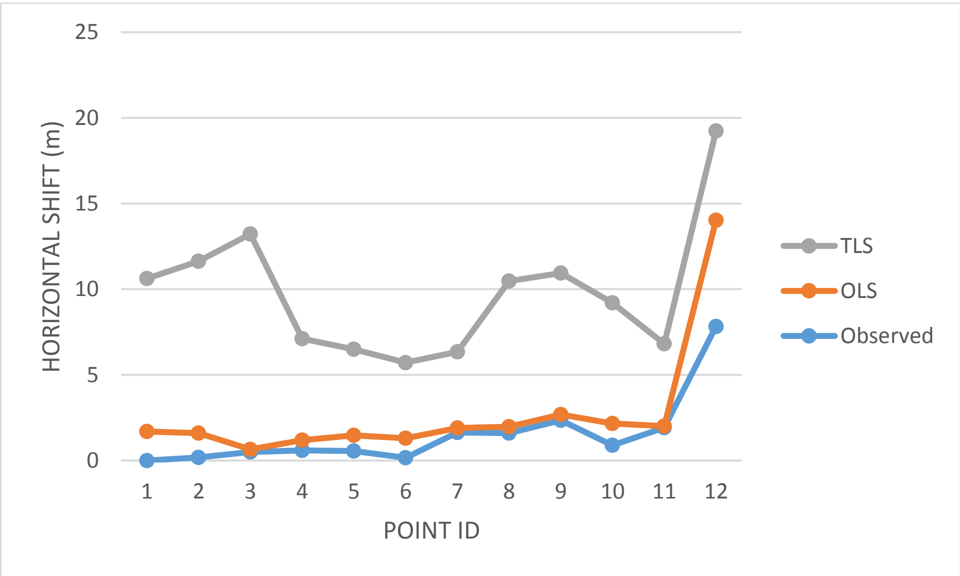

Figure 4 Horizontal Position error plot by the two models

Figure 4 represents the degree of horizontal positional accuracy in both OLS and TLS methods for the points obtained in the study area. A critical look at Tables 4 above revealed that the OLS method produced marginally better results than the TLS method. Therefore, the OLS method was preferred to that of the TLS for the study area.

. In order to further access the statistical validity of the two models applied, the Mean Horizontal Position Error (MHE), Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Bias Error (MBE), and Standard Deviation (SD) were applied as the performance criteria index. The performance criteria index (PCI) values attained for computing the M, RMSE, MAE, MBE are shown in Table 5, Table 6 and Table 7 below.

Table 5. Statistical Model Validation of the observed data (Units in metres)

PCI	M	MSE	MAE	MBE	RMSE	SD
X	1.468917	6.310989	1.468917	1.468917	2.512168	2.128578
Y	-0.090600	0.140009	-0.090600	-0.090600	0.374178	0.379187
HE	1.515855	6.450998	1.515855	1.515855	2.539881	2.128554

Table 6 Statistical Model Validation of the OLS model (Units in metres)

PCI	M	MSE	MAE	MBE	RMSE	SD
X	8.333E-06	3.781019	8.33E-06	8.33E-06	1.944484	2.030947
Y	-1.66E-05	0.201895	-1.66E-05	-1.66E-05	0.449327	0.469307
HE	1.203079	3.982913	1.203079	1.203079	1.995724	1.663134

Table 7. Statistical Model Validation of the TLS model (Units in metres)

PCI	M	MSE	MAE	MBE	RMSE	SD
X	0.001283	29.31036	0.001283	0.001283	5.413904	5.654638
Y	-0.00632	27.24676	-0.00632	-0.00632	5.219843	5.451944
HE	7.098507	56.55712	7.098507	7.098507	7.520447	2.594045

From Table 5 to Table 7 above, it can be seen that the OLS gave a better result as compare to the TLS. The performance criteria indices of M, MSE, MAE, MBE, RMSE, and SD of OLS were lower than that of TLS, showing that OLS has a better performance for the Study area than TLS in this study.

5. Conclusions and Recommendation

The conclusions made from this study is that, both the OLS and TLS methods have been utilized to adjust DGPS data. The performance of the two models developed were then compared based on statistical indicators of M, MSE, MAE, MBE, RMSE, and SD. The performance and efficiency of each adjustment techniques was assessed using an existing dataset that was not used to form the models. It was realized that, the OLS and TLS produced different results. This signifies that in terms of adjustments using the two models, there will be no identical results due to their working efficiencies and advantages over each other. However, it is recommended by the authors based on the results achieved in this study that, OLS is the proposed method for adjusting DGPS data for the study area, since the OLS method gave marginally better results than the TLS method. Therefore, the applicability of using least squares techniques to adjust DGPS data have been achieved in this study.

Based on the results and conclusions present in this study, it can be confidently and uncertainly say that, OLS method should be adapted in adjusting DGPS networks since it produced marginally better results than the TLS method. In addition, more research works should be conducted on other least squares techniques that was not adopted in this study such as partial least squares, generalized least squares, least squares collocation and many others and compared the outcomes to what was achieved in this study. This will further enhance the assessment of the least squares in selecting the most optimal among them for adjusting survey field data. This will study will therefore create opportunity for geospatial professional in geoscientific communities to realize the significance of least squares regressions models in solving some of the problems in mathematical and satellite geodesy.

Acknowledgement

The authors are very grateful and would like to thank the anonymous reviewers for their helpful comments to make this piece of knowledge a better material for academic purpose, and also to thank all geospatial professionals across the globe for their contributions and their impartation of knowledge related to this work and in all fields of geoscientific community. Special thanks go to the staff members of the Department of Geomatic Engineering, University of Mines and Technology, Tarkwa, Ghana, for providing us with the required knowledge and instruments used for the collection of data that was used to undertake this study.

References

Acar, M., Ozuledemir, M. T., Akyilmaz, O., Celik, R. N., and Ayan, T. (2006), "Deformation analysis with Total Least Squares", *Natural Hazards and Earth System Sciences*, Vol. 6, pp. 663-669.

Akyilmaz, O. (2007), "Total Least Squares Solution of Coordinate Transformation", *Survey Review*, Vol. 39, No. 303, pp. 68-80. http://doi.org/10.1179/003962607X165005

Amiri-Simkooel, A., and Jazaeri, S. (2012), "Weighted Total Least Squares Formulated by Standard Least Squares Theory", *Journal of Geodetic Science*, pp. 1-2.

Annan, R. F., Ziggah, Y. Y., Ayer, J., and Odutola, C. A. (2016a), "Accuracy Assessment of heights obtained from Total station and level instrument using Total Least Squares and Ordinary Least Squares Methods", *Journal of Geomatics and Planning*, Vol. 3, No. 2, pp. 87-92.

Annan, R. F., Ziggah, Y. Y., Ayer, J., Odutola, C. A. (2016b), "A Hybridized Centroid Technique for 3D Molodensky-Badekas Coordinate Transformation in the Ghana Reference Network using Total Least Squares Approach", *South African Journal of Geomatics*, Vol. 5, No. 3, pp. 269-284.

Ansah, E. (2016), "DGPS Networks Adjustments using Least Squares Collocation", *Unpublished BSc Project Work*, University of Mines and Technology, Tarkwa, Ghana, 57pp.

Dawod, G. M., Mirza, N. M., and Al-Ghamdi, A. K. (2011), "TS 8 Simple Precise Coordinate Transformations for Geomatics Applications in Makkah Metropolitan Area, Saudi-Arabia", *Bridging the Gap Between Cultures FIG Working Week*, 2010, Marrakech, Morocco, pp. 18-22.

Dreiseitl, S., and Ohno-Machado, L. (2002), "Logistic Regression and Artificial Neural Network Classification Models: A Methodology Review", *J Biomed Int*, Vol. 35, No. 5-6, pp. 352-359.

Effah, S. F. (2015), "A Comparative Study of Ordinary Least Squares and Total Least Squares in Predicting 2D Cartesian Coordinates-A Case Study", *Unpublished Thesis*, University of Mines and Technology, Tarkwa, Ghana, 30 pp.

Forson, K. I. (2006), "Design of distribution network for University of Mines and Technology", *Unpublished BSc Project Report,* University of Mines and Technology, Tarkwa, Ghana, 10pp.

Ge, X., and Wu, J. (2012), "A New Regularized Solution to Ill-Posed Problem in Coordinate Transformation", *International Journal of Geosciences,* Vol. 3, pp. 14-20.

Ghilani, D. C. (2010), "Adjustment Computations, Spatial Data Analysis", Fifth Edition, Wiley & Sons, INC. Hoboken, New Jersey, USA, 674 pp.

Ghilani, D. C., and Wolf, P. R. (2012), "Elementary Surveying, An Introduction to Geomatics. Thirteen Edition", Pearson Education Inc., Upper Saddle River, New Jersey 07458, USA, 983 pp.

Golub, G. H., and Van Loan, C. F. (1980), "An analysis of the Total Least Squares problem", *SIAM Journal on Numerical Analysis,* Vol. 17, No. 6, pp. 883-893.

Ismail, S., Shabri, A., and Samsudin, R. (2012), "A Hybrid Model of Self-Organizing Maps and Least Square Support Vector Machine for River Flow Forecasting", *Hydrol Earth Syst Sci*, Vol. 16, pp. 4417-4433.

Lenda, G. (2008), "Application of Least Squares Method for Approximation the Surface Engineering Structures", *Journal of Geomatic and Environmental Engineering,* Vol. 2, No. 1, pp. 50-56.

Markovsky, I., and Van Huffel, S. (2007), "Overview of Total Least Square Methods", *Signal Processing,* Vol. 87, No. 10, pp. 2283-2302.

Makovsky, I., Sima, D. M., and Huffel, S. V. (2009), "Generalization of the Total Least Squares Problem, *Advanced Reviewed Article*, pp. 1-2.

Miller, S. J. (2006), "Methods of Least Squares", Statistics Theory, *Cornell University*, USA, Vol. 3, pp. 1-2.

Moritz, H. (1972), "Advanced least squares method Report", No. 75, Dept. of Geodetic Science, OSU.

Nievergelt, Y. (1994), "Total Least Squares: State -of-the-Art Regression in Numerical Analysis", *Society of Industrial and Applied Mathematics*, Vol. 36, No. 2, pp. 258-264.

Odutola, C. A., Beiping, W., and Ziggah, Y. Y. (2013), "Testing Simple Regression Model for Coordinate Transformation by Comparing its Predictive Result for Two Regions", *Academic Research International*, SAVAP International Publishers, Vol. 4, No. 6, pp. 540-549.

Okwuashi, O., and Eyoh, A. (2012a), "Application of total least squares to a linear surveying network", *Journal of science and Arts*, Vol. 4, No. 21, pp. 401-404.

Okwuashi, O., and Eyoh (2012b), "3D Coordinate transformation using total least squares", *Academic Research International*, Vol. 3, No. 1, pp. 399-405.

Okwuashi, O. (2014), "Adjustment Computation and Statistical Methods in Surveying", A Manual in the Department of Geoinformatics & Surveying, Faculty of Environmental Studies, University of Eyoh, Nigeria.

Schaffrin, B. (2006), "A note on Constrained Total Least Square estimation", Linear Algebra and Its Application, Vol. 417, pp. 245-258.

Seidu, M. (2004), "GIS as a Tool in Water Monitoring for Public Health and Safety Management", *Unpublished BSc Report*, University of Mines and Technology (UMaT), Tarkwa, Ghana, 6pp.

Yakubu, I., and Kumi-Boateng, B. (2011), "Control Position Fix using Single Frequency Global Positioning System Receiver Techniques – A Case Study", *Res J Environ Earth Sci*, Vol. 3, No. 1, pp. 32-37.

Yang, T. (1997), "Total Least Squares Filter for Robot Localization", *Digital Signal Processing Proceedings, 13th International Conference,* Santorini, pp. 1-2.

Ziggah Y. Y. (2012), "Regression Models for 2-Dimensional Cartesian Coordinates Prediction: A Case Study at University of Mines and Technology (UMaT)", *International Journal of Computer Science and Engineering Survey (ISCSES)*, Vol. 3, No. 6, 62pp.

Ziggah, Y. Y., Youjian, H., Yu, X., and Basommi, L. P. (2016b), "Capability of Artificial Neural Network for Forward Conversion of Geodetic Coordinates (φ, λ, h) to Cartesian Coordinates (X, Y, Z)", *Math Geosci*, Vol. 48, pp. 687-721.