South African Journal of Geomatics, Vol. 10. No. 2, August 2021

Comparative accuracy assessment of the Bowring, Chord and Power series models for direct and indirect determination of geodetic coordinates

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DOI: http://dx.doi.org/10.4314/sajg.v10i2.9

Abstract

The computation of geodetic coordinates is the basis of geodetic surveying and foundation to modern techniques for geodetic network analyses and design of integrated survey schemes for monitoring and detecting structural deformations. The positional accuracy achievable by Direct and Indirect models of geodetic position determination depends on the varying lengths, azimuths and latitude of the first point of the network of stations. Existing knowledge gaps preclude a comprehensive understanding of the relative accuracies of these methods. Therefore, the aim of this study is to determine the achievable accuracies of three models (Bowring, Chord and Power Series) for direct and indirect position determination vis-a-vis the network configuration. The data comprised of 33 controls in the D-Chain geodetic network located in North-Central Nigeria, with a range of network of lines between 15.530km and 113.254km. Various attributes of the network such as azimuth, angle, distance, and coordinates were computed to a high accuracy and precision using a program written in the Matlab software environment. The results of the direct and indirect computation were summarised using descriptive statistics. Also, the accuracies of the computed coordinates were assessed by comparisons with the provisional (initial) coordinates of the controls. In the analysis of coordinate differences, the positional root mean square error (RMSE) for each of the three models in decreasing order of accuracies are: 4.572639341″ (Chord), 4.601685022″ (Power Series) and 4.601701034″ (Bowring). The positional mean absolute deviation (MAD) for the three models in decreasing order of accuracies are 3.788841258″ (Chord), 3.813184934″ (Power Series) and 3.813198679″ (Bowring) and this agrees with the RMSE trend for the network. This study has shown that the D-chain network configuration favours the use of Chord model for position determination based on the adopted configuration.

Keywords: *Geodetic coordinates, Direct problem, Indirect problem, Nigerian horizontal geodetic network, D-chain.*

1. Introduction

Geodesy (the science of the measurement and mapping of the earth's surface) is essentially an application of mathematics (Omogunloye *et al.,* 2012). It makes use of coordinates and associated reference systems (Jekeli, 2006). Over the years, the precision of geodetic measurements has increased by several orders of magnitude and geodesy has proven immensely valuable for both scientific and commercial applications (NAS, 2010; Nwilo *et al.,* 2016; Omogunloye *et al.,* 2018). According to Fajemirokun (2006), geodesy takes care of the following scientific tasks: determination of the size and shape of the earth; establishment and maintenance of national and global three-dimensional geodetic networks; determination of earth's surface displacements; measurement and representation of geodynamic phenomena; and earth's external gravity field determination. A reference ellipsoid is generally considered as the best approximation to the size and shape of the earth. Therefore, it is used as the surface upon which to perform terrestrial geodetic computations (Krakiwsky and Thomson, 1974; Omogunloye *et al.,* 2017).

There are two essential problems in the computation of coordinates, directions, and distances on a given ellipsoid – the Direct problem and the Inverse/Indirect problem (Rapp, 1991; Jekeli, 2006). The Direct and Inverse problems on the ellipsoid are necessary geodetic operations and can be related to the equivalent operations of plane surveying; radiations (computing coordinates of points given bearings and distances radiating from a point of known coordinates) and joins (computing bearings and distances between points having known coordinates) (Omar, 2017). In plane surveying, the coordinates are 2-Dimensional (2D) rectangular coordinates, usually designated East and North and the reference surface is a plane, either a local horizontal plane or a map projection plane (Deakin and Hunter, 2010). However, in geodesy, the coordinates are geographic coordinates in mode of latitude (φ), longitude (λ) and elevation (h), and the reference surface in form of a spheroid surface is usually projected on a spherical surface or ellipsoid. The Direct geodetic problem is the calculation of geodetic coordinates—the latitudes and longitudes of several points lying on the geoid—by the coordinates of another point and the length and azimuth of the geodetic line connecting these points. The Indirect geodetic problem is the determination of the length and azimuth of the geodetic line between two points by the geodetic coordinates of these two points on the geoid (Omar, 2017). Various applications depend on different methods of calculating the Direct and Indirect problem of geodetic coordinates. The need to find the best fit methods of calculating these attributes is very paramount in solving geodetic problems and also finding the best methods that fit the short, medium and long-distance measurement of geodetic application (Lenart, 2013).

Different approaches have been put forward for solving these problems and are generally classified in terms of "short", "medium", and "long" line formulae. Each method includes distinctive approximations which tend to restrict the interstation distance over which some formulae are useful for a given accuracy (Krakiwsky and Thomson, 1974). According to Sjöberg (2009), the Direct and Indirect geodetic problems on the geodesics are still very relevant in the application of satellites launching and landing, law of the sea, military surveys and in the determination of the best optimum route for aircraft and ship navigational routes. There are different models used for direct and indirect geodetic position determination, for example, the Puissant method, Gauss Mid-Latitude method, Legendre method, Chord method, Power series method and the Bowring method. Each model has its associated approximations in terms of series truncation or geometric approximation in its derivation. Specific accuracy estimates are obtainable if a series of test lines are computed with the most accurate sets of formulae as opposed to the results from an approximate method. Such computations have been done by Gupta (1972) for a number of methods and by Badi (1983) for Bowring method. Accuracy of any model is often expressed in the context of the desired position computation accuracy. For example, 1 arc second of accuracy corresponds to 30m length on the surface of an ellipsoid. This implies that, given a set of latitudes and longitudes, one would expect that any distance computed from them should be correct to 1mm, which implies that latitude (φ) and longitude (λ) should be given to an accuracy of the order of 0.00001 arc of seconds. Usually there may be cases where less accuracy would suffice depending on the purpose for which the result is to be applied.

The Power series method was one of the earliest methods used in solving the Direct problem. It was developed using the principle of Maclaurin series in solving the problem on the ellipsoid (Jordan and Eggert, 1962). The solution to the indirect problem of the Power series is not as straightforward as the direct solution. The Bowring method uses a conformal projection of the ellipsoid on a sphere also known as Gaussian projection of the second kind. In the adoption of this method by Bowring (1981), the scale factor is taken to be at the starting point of the line. Also, the first and second derivatives of scale factor with respect to latitude are set to zero. The geodesic from ellipsoid is then projected to the corresponding line on the sphere where spherical trigonometry can be applied (Rapp, 1991; Omogunloye *et al.,* 2016). The Bowring method has accuracies of 1mm or 2mm for both direct and indirect problems for line lengths up to 120km order and 3 or 4mm for line lengths of 150km (Jekeli, 2006). The Chord method of direct computation is an outcome of three-dimensional (3-D) geodetic coordinate system; it is based on the principle of Molodensky formulas (Vincenty, 1986). Both King (1971) and Hradilek (1976) also advance the development of the Chord methods with vector notation and discovered that the methods are varied in some places. The accuracy of the Chord method can be extended to sub-millimetre precision (Vincenty, 1986; Omogunloye *et al.,* 2016). It has previously been shown that the Power series method of the fifth order derivative at latitude of 10° was able to achieve an accuracy of 0.00001 arc seconds with a 100km length as opposed to its achieving same accuracy at a latitude of 70° but for a 60km range of length. However, the Bowring method is favoured at higher latitudes of 70° with same accuracy but for a 70km length of line.

According to Gupta (1972), there is sensitivity to the results obtainable in most cases depending on the varying lengths, azimuths and latitude of the first point of the network. It is usually sufficient to know the maximum distance at the poorest azimuths and latitudes, for which a specific direct model yields a given accuracy. Existing knowledge gaps preclude a comprehensive understanding of the relative accuracies of these methods. The main thrust of this study is to show that the accuracy of the direct solution models must be based on accuracy criteria and not on the basis that a given formula is accurate for specific range or length of lines. Therefore, this research work considered three methods out of the various models developed by the geodesists as a solution to the direct and indirect problem. A program was written to facilitate the computation processes involved in the particular models adopted, and their accuracies were compared.

2. Materials and Methods

2.1. Description of D-Chain geodetic network

The D-Chain geodetic network cuts across Kwara, Niger, Kebbi and Kaduna States in Nigeria, West Africa. The network lies within the boundary extent of latitudes 08°30’ - 11°30’N and longitudes 04°00’ - 06°30’E (Moka et al., 2007). The coordinates of the network consist of 33 coordinate points and 42 triangulation formations. The longest line in the triangulation is D14 to D21 and the shortest line is D2 to D6. Figure 1 presents a map of Nigeria showing the D-Chain horizontal geodetic triangulation network. Nigeria has a total area of 923,768 km^2 (CIA, 2016) and shares boundaries with Benin Republic to the west, Cameroon to the east, Niger to the north and Chad to the north-east. The country’s coastline spans a length of approximately 853km facing the Atlantic Ocean (Ibe, 1988; Nwilo, 1995; Nwilo and Badejo, 2006). According to Nwilo (2013), early developments in Nigeria during colonial times provided the impetus for the establishment of survey controls. This was later followed by the establishment of framework controls using methods such as traversing, triangulation, trilateration, geodetic levelling and trigonometric levelling (Nwilo et al., 2016). Nigeria dropped the Clarke 1858 projection in 1926 and adopted the modified Clarke 1880 Transverse Mercator projection in the same year (Adalemo, 1990; Adewola, 1990; Nwilo et al., 2016). In 1975, the Nigeria Transverse Mercator (NTM) was replaced by the Universal Transverse Mercator (UTM) which was introduced in Nigeria by the Federal Surveys Department (Uzodinma and Ezenwere, 1993).

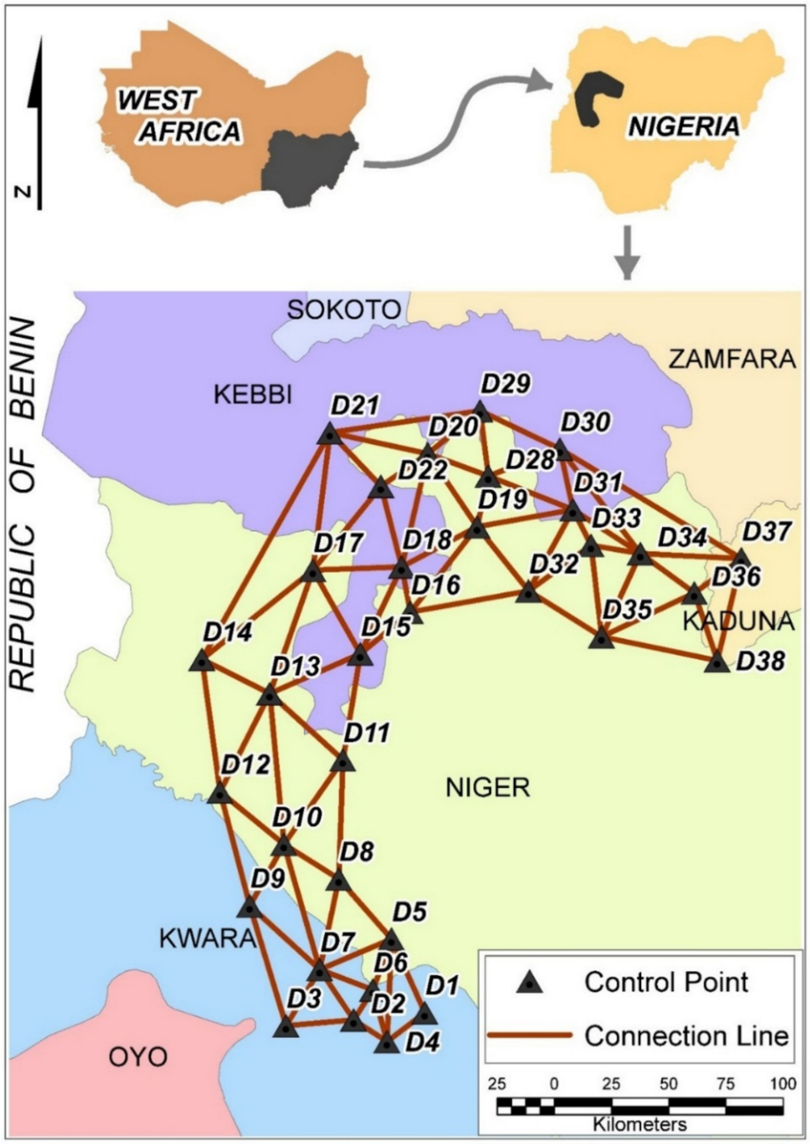

Figure 1: Map of Nigeria showing the D-Chain horizontal geodetic triangulation network.

2.2. Data acquisition

The initial (provisional) geodetic coordinates of D-Chain used in this study were derived from an earlier work done by Omogunloye (2010) on the optimal simultaneous adjustment for all stations within the Nigerian horizontal geodetic network. However, the original coordinates of D-chain which were adjusted by Omogunloye (2010) were obtained from the work of Field (1977). The triangulation stations were observed by terrestrial measurements of angles and distances using triangulation and trilateration methods. The observations took place between the late 1940s and early 1960s. The L40 triangulation station at Minna which represents the origin of the horizontal geodetic network of Nigeria, was selected as the origin. The geodetic coordinates of L40 were derived after taking the mean of astronomical values projected through four arms of the Nigerian triangulation network and the astronomically derived coordinates of L40 (Fajemirokun and Nwilo, 1990). The projection used for the datum is the modified Clarke 1880 (Onabajo, 2006). Information on the geodetic parameters of Minna datum has already been provided in other studies (e.g. Uzodinma and Ezenwere, 1993; Uzodinma and Ehigiator-Irughe, 2013).

2.3. Computation of station coordinates

Whereas the Chord model computations were based on coordinate conversion between the geodetic curvilinear coordinates (φ, λ, h) and geocentric (X, Y, Z), the Power Series and Bowring models computation were based on the assumption that the geodesic on the surface of the ellipsoid is a function of the geodetic latitude (φ), longitude (λ) and azimuth (α) along the geodetic line. Bowring model derives its equations to calculate the direct problem for geodesic for lines up to 150km length. The method uses a conformal projection of the ellipsoid on a sphere called Gaussian projection of the second order. The local Minna datum was adopted as the reference system in the computations. The coordinates of station D1 ($\boldsymbol{\varphi} =$ 9.019°N, $\boldsymbol{\lambda} =$ 5.0011°E) were selected as the initial coordinates and the coding for the computation of the other stations coordinates was executed using the azimuths and distances in the Matlab R2016a software environment. The program procedurals were coded in A Matlab file (.m), written in Matlab Editor and run in the command window. Matlab is a robust programming language developed by MathWorks used for simple and complex matrix operations, data plotting and with support for algorithm implementation. The Power series, Bowring and Chord models were used to compute the respective azimuths, distances, latitudes and longitudes between all stations in the triangulation network.

2.4. Accuracy assessment

To check the relative accuracies of the Bowring, Chord and Power Series models for direct and indirect determination of geodetic coordinates, the computed coordinates were compared with the initial coordinates. In the accuracy assessment, the coordinate differences were computed using Microsoft Excel 2010 while parameters such as standard deviation - SD (σ), standard error of the mean (SEM), mean absolute deviation (MAD) and root mean square error (RMSE) were computed using the Statistical Package for the Social Sciences (SPSS) version 16. The formulae for SD and SEM are given in equations 1 and 2.

$$\text{SD}\ (\sigma) = \sqrt{\frac{\sum_{i=1}^{n}(x-\bar{x})^2}{n-1}} \qquad [1]$$

Where x= computed coordinate, $\bar{x}$ = initial coordinate, and n is the number of stations. The SEM is defined as the standard deviation of the sampling distribution of the mean and its formula is given by Okolie et al. (2020).

$$\text{SEM}\ (\sigma_m) = \frac{\sigma}{n} \qquad [2]$$

The RMSE represents the sample standard deviation of the differences between predicted values and observed values. RMSE is a measure of accuracy, to compare forecasting errors of different models for a particular data and not between datasets, as it is scale-dependent (Hyndman and Koehler, 2006). The MAD and RMSE have been widely used by scientists and researchers (Chai and Draxler, 2014), to measure model performance.

$$MAD = \frac{1}{n}\sum_{i=1}^{n}|e_i| \quad [3]$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2} \quad [4]$$

Where:

e_i = the coordinate difference for the ith station.

n = the total number of stations.

Finally, to test for the relationship between the initial and computed coordinates, a paired samples *t*-test was run. The paired-samples *t*-test is used for comparing the mean of two matched groups or cases (Ross and Wilson, 2017).

3. Results and Discussion

3.1. Computed coordinates, azimuths and distances

Table 1 shows the coordinates of the stations computed by the Bowring, Chord and Power series models. In the table, φ_B, φ_C and φ_P represent the latitudes computed by the Bowring, Chord and Power series models respectively while $\boldsymbol{\lambda_B}$, $\boldsymbol{\lambda_C}$ and $\boldsymbol{\lambda_P}$represent the longitudes computed by the Bowring, Chord and Power series models respectively. Table 2 shows the minimum and maximum azimuths and distances computed from the three models for short, medium and long lines respectively. Short lines have lengths that are less than 30km, medium lines have lengths between 30km and 70km while long lines are longer than 70km.

Table 1: The computed station coordinates, using the three models.

S/N	Stn ID	φ_B (°)	λ_B (°)	φ_C (°)	λ_C (°)	φ_P (°)	λ_P (°)
1	D1	9.018938	5.001059	9.018939	5.001060	9.018938	5.001059
2	D2	8.989992	4.721305	8.989992	4.721305	8.989992	4.721305
3	D3	8.967094	4.455227	8.967094	4.455227	8.967094	4.455227
4	D4	8.904959	4.851949	8.904959	4.851949	8.904959	4.851949
5	D4	8.904898	4.851909	8.904898	4.851909	8.904898	4.851909
6	D5	9.309090	4.869869	9.309091	4.869869	9.309090	4.869869
7	D6	9.108511	4.796679	9.108511	4.796679	9.108511	4.796679
8	D7	9.188371	4.587739	9.188372	4.587739	9.188371	4.587739

9	D8	9.545191	4.662513	9.545193	4.662513	9.545191	4.662513
10	D9	9.439902	4.312542	9.439903	4.312542	9.439902	4.312542
11	D10	9.681235	4.446131	9.681237	4.446131	9.681235	4.446131
12	D11	10.012564	4.678493	10.012569	4.678494	10.012564	4.678493
13	D12	9.887759	4.194782	9.887761	4.194781	9.887759	4.194782
14	D13	10.274416	4.391233	10.274420	4.391234	10.274416	4.391233
15	D14	10.409589	4.124270	10.409593	4.124269	10.409589	4.124270
16	D15	10.430527	4.747349	10.430532	4.747350	10.430527	4.747349
17	D16	10.594174	4.943278	10.594185	4.943279	10.594174	4.943278
18	D17	10.759915	4.560754	10.759921	4.560755	10.759915	4.560754
19	D18	10.772292	4.908999	10.772303	4.909000	10.772292	4.908999
20	D19	10.930317	5.205523	10.930331	5.205525	10.930317	5.205523
21	D20	11.217807	5.011818	11.217822	5.011819	11.217807	5.011818
22	D21	11.301521	4.627439	11.301532	4.627439	11.301521	4.627439
23	D22	11.089734	4.827288	11.089745	4.827289	11.089734	4.827288
24	D28	11.130130	5.249952	11.130144	5.249954	11.130130	5.249952
25	D29	11.386648	5.217224	11.386662	5.217226	11.386648	5.217224
26	D30	11.235968	5.530653	11.235982	5.530655	11.235968	5.530653
27	D31	10.997259	5.582770	10.997271	5.582772	10.997259	5.582770
28	D32	10.677084	5.407489	10.677096	5.407490	10.677084	5.407489
29	D33	10.857138	5.653435	10.857150	5.653437	10.857138	5.653435
30	D34	10.824474	5.847809	10.824486	5.847811	10.824474	5.847809
31	D35	10.496434	5.695291	10.496446	5.695292	10.496434	5.695291
32	D36	10.669488	6.059495	10.669499	6.059497	10.669488	6.059495
33	D38	10.402949	6.149815	10.402961	6.149817	10.402949	6.149815

Table 2: Minimum and maximum azimuths and distances computed from the three models.

Parameter	**Method**	**Short lines**		**Medium lines**		**Long lines**	
		Min	**Max**	**Min**	**Max**	**Min**	**Max**
Forward azimuth (°)	Bowring	12.489167	352.744870	1.961084	354.706274	6.907450	351.061896
	Chord	12.352719	352.828290	1.944812	354.754798	6.835666	351.143359
	Power series	12.344301	352.821840	1.941995	354.764438	6.838382	351.122463
Backward azimuth (°)	Bowring	99.505906	265.079494	92.864832	261.740401	121.138494	209.361605
	Chord	99.626213	265.081426	92.947882	261.715153	121.530961	209.163679
	Power series	99.608530	265.060965	92.912269	261.658941	121.481126	209.150989
Distance $(\boldsymbol{m})$	Bowring	15.582128	29.640445	30.631440	69.649687	73.678322	113.739419
	Chord	15.505701	29.367792	30.470897	69.374019	73.454641	112.993657
	Power series	15.505016	29.368658	30.473273	69.373389	73.454225	113.030382

3.2. Accuracy assessment of the coordinate differences

Table 3 presents the differences between the initial and computed coordinates while Table 4 presents the minimum and maximum differences between the initial coordinates (φ_I, λ_I) and the coordinates computed from the three models. Of the 33 stations in the network, the Chord model yielded the lowest differences in the coordinates ($\varphi_I - \varphi_C$ = 0.03244392″; $\lambda_I - \lambda_C$ =-0.24850728″) while the Bowring model yielded the highest differences in the coordinates ($\varphi_I - \varphi_B$= 8.14182516″; $\lambda_I - \lambda_B$ = 3.77085240″).

Table 5 shows the mean, SEM and SD of the coordinate differences. When compared with the initial coordinates, the latitudes determined by the Chord model yielded the lowest SEM and SD ($SEM_{\varphi_I - \varphi_C}$ = 0.38911572″; $SD_{\varphi_I - \varphi_C}$= 2.30203908″) while the latitudes determined by the Bowring model yielded the highest SEM and SD ($SEM_{\varphi_I - \varphi_B}$= 0.39200364″; $SD_{\varphi_I - \varphi_B}$ = 2.31912396″). Similarly, the longitudes determined by the Chord model yielded the lowest SEM and SD ($SEM_{\lambda_I - \lambda_C}$= 0.21179160″; $SD_{\lambda_I - \lambda_C}$ = 1.25297604″) while the longitudes determined by the Bowring model yielded very high SEM and SD ($SEM_{\lambda_I - \lambda_B}$= 0.21214512″; $SD_{\lambda_I - \lambda_B}$ = 1.25506800″).

Table 3: Differences between the initial and computed coordinates.

S/N	Stn ID	$\varphi_I - \varphi_B$ (″)	$\lambda_I - \lambda_B$ (″)	$\varphi_I - \varphi_C$ (″)	$\lambda_I - \lambda_C$ (″)	$\varphi_I - \varphi_P$ (″)	$\lambda_I - \lambda_P$ (″)
1	D1	0.22179183	0.14581204	0.22029820	0.14532323	0.22179077	0.14581203
2	D2	0.38898421	-0.01944871	0.38736726	-0.01952666	0.38898302	-0.01944873
3	D3	0.74159337	-0.09787879	0.74005149	-0.09710962	0.74159218	-0.09787880
4	D4	0.14601655	-0.17795069	0.14620050	-0.17771655	0.14601658	-0.17795069
5	D4	0.36780901	-0.03222851	0.36649933	-0.03248259	0.36780799	-0.03222852
6	D5	0.03528216	-0.24861099	0.03244398	-0.24850742	0.03528056	-0.24861098
7	D6	0.32198485	0.07476844	0.32026102	0.07462398	0.32198362	0.07476843
8	D7	1.18486355	1.29952524	1.17964986	1.30013936	1.18486002	1.29952510
9	D8	1.11288722	1.03204160	1.10550650	1.03221204	1.11288256	1.03204147
10	D9	0.71442835	0.56816043	0.70761665	0.57048046	0.71442445	0.56816031
11	D10	1.31500000	1.32694714	1.30721404	1.32775075	1.31499527	1.32694701
12	D11	4.08833620	1.46643147	4.07298745	1.46247667	4.08832760	1.46643132
13	D12	1.58768257	1.86538911	1.57890028	1.86738095	1.58767760	1.86538897
14	D13	3.54407068	0.59946316	3.52691871	0.59744767	3.54406168	0.59946306
15	D14	1.48039232	2.26748990	1.46499870	2.27035485	1.48038378	2.26748968
16	D15	2.78157729	1.26284170	2.76335139	1.25842393	2.78156821	1.26284158
17	D16	4.41485643	2.95838791	4.37524303	2.95531954	4.41483398	2.95838739
18	D17	3.99344799	2.45430733	3.95858187	2.45337455	3.99342850	2.45430679
19	D18	4.34879245	2.88337410	4.30891181	2.88036031	4.34876988	2.88337359
20	D19	4.61707147	3.15555197	4.56781547	3.15020733	4.61704340	3.15555153
21	D20	4.29314426	2.45543540	4.24231567	2.45115495	4.29311568	2.45543501
22	D21	3.88606596	2.02054038	3.84381293	2.01870573	3.88604250	2.02053992
23	D22	4.19855916	2.56208236	4.15712183	2.55946972	4.19853590	2.56208188

24	D28	4.57133900	3.05170487	4.52170131	3.04626992	4.57131074	3.05170443
25	D29	4.50618172	3.15276745	4.45576117	3.14743563	4.50615306	3.15276700
26	D30	5.15452674	3.77085235	5.10493108	3.76379754	5.15449814	3.77085187
27	D31	5.54642627	2.26873724	5.50324501	2.26243075	5.54640044	2.26873697
28	D32	5.45591109	2.92036185	5.41467347	2.91511769	5.45588608	2.92036155
29	D33	5.62183034	2.39464047	5.57880607	2.38824660	5.62180457	2.39464020
30	D34	5.85294439	2.48782159	5.80997780	2.48106712	5.85291862	2.48782131
31	D35	7.43747554	3.63350663	7.39504793	3.63013640	7.43745021	3.63350630
32	D36	6.16437420	2.89645939	6.12189221	2.88903092	6.16434851	2.89645908
33	D38	6.30245593	3.18537445	6.26093711	3.17761602	6.30243065	3.18537412

Table 4: The minimum and maximum coordinate differences.

	Range ($''$)	**Min.** ($''$)	**Max.** ($''$)	**Sum** ($''$)
$\varphi_I - \varphi_B$	8.10654300	0.03528216	8.14182516	117.0060404
$\varphi_I - \varphi_C$	8.06568480	0.03244392	8.09812872	116.0849038
$\varphi_I - \varphi_P$	8.10651924	0.03528072	8.14179960	117.0055087
$\lambda_I - \lambda_B$	4.01946336	-0.24861096	3.77085240	63.04806108
$\lambda_I - \lambda_C$	4.01230512	-0.24850728	3.76379748	62.95515804
$\lambda_I - \lambda_P$	4.01946300	-0.24861096	3.77085204	63.04805244

Table 5: Mean, SEM and SD of the coordinate differences.

	Mean ($''$)	**SEM** ($''$)	**SD** ($''$)
$\varphi_I - \varphi_B$	3.34302984	0.39200364	2.31912396
$\varphi_I - \varphi_C$	3.31671168	0.38911572	2.30203908
$\varphi_I - \varphi_P$	3.34301436	0.39200184	2.31911388
$\lambda_I - \lambda_B$	1.80137304	0.21214512	1.25506800
$\lambda_I - \lambda_C$	1.79871876	0.21179160	1.25297604
$\lambda_I - \lambda_P$	1.80137304	0.21214512	1.25506764

Table 6 shows the MAD and RMSE of the coordinate differences. The latitudes determined by the chord model yielded the lowest MAD and RMSE ($MAD_{\varphi_I-\varphi_C}$ = 3.31671168$''$; $RMSE_{\varphi_I-\varphi_C}$= 4.01852556$''$) while the latitudes determined by the Bowring model yielded the highest MAD and RMSE ($MAD_{\varphi_I-\varphi_B}$= 3.34302984$''$; $RMSE_{\varphi_I-\varphi_B}4.04975520''$). Similarly, the longitudes determined by the Chord model yielded the lowest MAD and RMSE ($MAD_{\lambda_I-\lambda_C}$= 1.83159540$''$; $RMSE_{\lambda_I-\lambda_C}$ = 2.18185308$''$) while the longitudes determined by the Bowring model yielded the highest MAD and RMSE ($MAD_{\lambda_I-\lambda_B}$= 1.83429432$''$; $RMSE_{\lambda_I-\lambda_B}$ = 2.18520828$''$).

Table 6: MAD and RMSE of the coordinate differences.

	MAD ($''$)	RMSE ($''$)
$\varphi_I - \varphi_B$	3.34302984	4.04975520
$\varphi_I - \varphi_C$	3.31671168	4.01852556
$\varphi_I - \varphi_P$	3.34301436	4.04973720
$\lambda_I - \lambda_B$	1.83429432	2.18520828
$\lambda_I - \lambda_C$	1.83159540	2.18185308
$\lambda_I - \lambda_P$	1.83429396	2.18520792

3.3. Relationship between the initial and computed coordinates

Tables 7 and 8 show the results of the paired samples *t*-test for the latitude and longitude coordinates respectively. The test for the latitudes shows significant differences in the scores for φ_I and φ_B (t = 8.528; p = 0.000), φ_I and φ_C (t = 8.524; p = 0.000), and φ_I and φ_P (t = 8.528; p = 0.000). Similarly, the test for the longitudes shows that there are significant differences in the scores for $\boldsymbol{\lambda}_I$ and $\boldsymbol{\lambda}_B$ (t = 8.491; p = 0.000), $\boldsymbol{\lambda}_I$ and $\boldsymbol{\lambda}_C$ (t = 8.493; p = 0.000), and $\boldsymbol{\lambda}_I$ and $\boldsymbol{\lambda}_P$ (t = 8.491; p = 0.000). This shows significant variability between the initial coordinates and the coordinates computed by the three models.

Table 7: Paired Samples *t*-test, Latitude.

	Paired Differences (°)					**t**	**df**	**Sig. (2-tailed)**
	Mean	**SD**	**SE**	**95% Confidence Interval of the Difference**				
				Lower	**Upper**			
φ_I *and* φ_B	0.00092862	0.00064420	0.00010889	0.00070732	0.00114991	8.528	32	0.000
φ_I *and* φ_C	0.00092131	0.00063946	0.00010809	0.00070165	0.00114097	8.524	32	0.000
φ_I *and* φ_P	0.00092862	0.00064420	0.00010889	0.00070733	0.00114991	8.528	32	0.000

Table 8: Paired Samples *t*-test, Longitude

	Paired Differences (°)					**t**	**df**	**Sig. (2-tailed)**
	Mean	**SD**	**SE**	**95% Confidence Interval of the Difference**				
				Lower	**Upper**			
$\boldsymbol{\lambda}_I$ **and** $\boldsymbol{\lambda}_B$	0.00050038	0.00034863	0.00005893	0.00038062	0.00062014	8.491	32	0.000
λ_I *and* λ_C	0.00049964	0.00034805	0.00005883	0.00038009	0.00061920	8.493	32	0.000
λ_I *and* λ_P	0.00050038	0.00034863	0.00005893	0.00038062	0.00062014	8.491	32	0.000

4. Conclusion and Recommendations

In the analysis of coordinate differences, the positional RMSE for each of the three models in decreasing order of accuracies are: 4.572639341$''$ (Chord), 4.601685022$''$ (Power Series) and 4.601701034$''$ (Bowring). The positional MAD for the three models in decreasing order of accuracies are 3.788841258″ (Chord), 3.813184934″ (Power Series) and 3.813198679″ (Bowring) and this agrees with the RMSE trend for the network. The SD, SEM, MAD and RMSE of the differences between the initial coordinates and the coordinates determined by the Chord model were consistently the lowest in all cases. Thus, the Chord model is proposed as the most accurate of the three methods for position determination of the network based on the adopted configuration. This study has shown that for relatively lower latitude network configuration like the D-Chain, the Chord method has produced a better positional accuracy of network of stations than the Power Series and the Bowring's Models. It is recommended that similar studies be conducted for other sub-networks of the Nigerian horizontal geodetic network to determine the most suitable models for position determination.

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