Bayesian approach in the power electric systems study of reliability

Etude de la fiabilité des systèmes electriques par l'approche de Bayes

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ملخص
يهدف هذا العمل إلى تسليط الضوء على مزايا منهج النظرية الافتراضية في دراسات أنظمة الطاقة الحديثة.إن التنظيم الجديد لقطاع الطاقة الكهربائية والدرجة العالية للابتكار التكنولوجي يجعل البيانات المتعلقة بتشغيل مكونات أنظمة الطاقة غير مؤكدة ، مع ما يترتب عليه من عدم وجود معلومات من أجل تقييم فعال للموثوقية وفقا للمنهج التقليدي. يتعلق هذا الارتياب أحيانا بنماذج الموثوقية نفسها، التي تفتقر إلى التحقق من صحة تعديلات النموذج. من خلال هذه الدراسة، واستنادا على النموذج الاحتمالي العام، سنعرف نموذجا احتماليا عاما لدراسة الموثوقية. و بعد ذلك نعرّج على المنهجيات النظرية الافتراضية والتي وضعت في قائمة من المشاكل، استنادا إلى تعريف "شعاع الحالة" المناسب و شعاع وصف لكفاءات النظام بغية تعريف وحساب الموثوقية أو لتقويمها وحساب الوقت. إن الهدف من عملنا هو الحصول على نموذج نستطيع من خلاله الحصول على معلومات مفيدة "بداهة"، فيما يتعلق بنهج النظرية الافتراضية. وبذالك يسمح لنا بتحديد أدوات النماذج الاحتمالية، وهذه هي المعلومات حول عملية نماذج والضرر (الضرر التراكمي، الضرر مضاعف، وما إلى ذلك) قادرة على تعويض البيانات المذكورة أعلاه.

الكلمات المفتاحية : *موثوقية - الطاقة الكهربائية - النظرية الافتراضية - نموذج ويبل - الاحتمال.*

Résumé
Ce travail met en évidence les prérogatives et les avantages de l'approche bayésienne dans les études de fiabilité des systèmes électriques modernes. La nouvelle organisation du secteur de l'énergie électrique et le degré constant d'innovation technologique rendent les données plus incertaines liées au fonctionnement des composants des systèmes électriques, avec le manque conséquent d'information pour une estimation efficace de la fiabilité selon l'approche classique. Parfois, cette incertitude concerne les mêmes modèles de fiabilité, manquant adéquatement la validation des adaptations des modèles. Dans cet article, en se référant aux données d'incertitude, nous commençons par définir un modèle probabiliste général, pour les études de fiabilité. Par la suite, les méthodologies bayésiennes sont encadrées dans une liste de problèmes plus large basée sur la définition d'un «vecteur d'état» opportun et d'un vecteur décrivant les performances du système en vue de définir et de calculer ou d'estimer la fiabilité du système. Le but de notre travail est d'établir un modèle utile basé sur l'information «a priori», par rapport à l'approche bayésienne. Cela nous permet d'obtenir des outils pour l'identification de modèles probabilistes, des informations sur les processus usagers et les modèles de dommages (dommages cumulatifs, multiplicatifs, etc.), adapté pour compenser le manque de données.

Mots clés: *Fiabilité – Système Eélectrique – Approche Bayesienne – Model de Weibull – Probabilité.*

Abstract
This work aims to highlight prerogatives and advantages of the Bayesian approach in the reliability studies of the modern power electrical systems. The new organization of the electric energy sector and the consistent degree of technological innovation make the data more uncertain related to the operation of the electric systems components, with the consequent lack of information for an efficient estimation of the reliability according to the classical approach. Sometimes, this uncertainty concerns the same reliability models, missing adequately the validation of the model adaptations. In this paper, referring to the uncertainty of data, we define a general probabilistic model, for reliability studies. Subsequently, Bayesian methodologies are framed in an ampler problem list, based on the definition of an opportune "vector of state" and of a vector describing the system performances, aiming to the definition and the calculation or the estimation of system reliability. The purpose of our work is to establish a useful model based on information *"a priori"*, with respect to the Bayesian approach. This allows getting tools for the identification of probabilistic models, information on usury process and damage models (cumulative damage, multiplicative, etc.), suitable to compensate the lack of data.

Keywords: *Reliability - Power System - Bayes Theorem - Weibull Model - Probability.*

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1. INTRODUCTION

The revolution of the systems order owed to the liberalization of the market is a further element of push toward the renewal of the methodological tools. Nowadays, it is important to speak about reliability systems [1], without going deeply into topics of systems analysis, technological innovation, diagnostics, automation, communication and environment.

Traditionally, the implementation of the reliability methodologies aims of the assessment indices for the system availability characterization, based on knowledge of failure rates assuming the problem data [2]. However, the modern system, characterized by exchange of technology, cannot be characterized by deterministic data because they cannot condemn a reality in continuous evolution. In scientific terms, we can say that today the electrical power system is characterized by dynamism and randomness or it should be described in terms of stochastic process. In this climate of uncertainty it comes incontrovertible need to identify the methodological tools for the system indices evaluation that must serve as decision support. While the information is not known in a deterministic way, the fact remains that we know the system history. Thus the idea of providing spontaneously updated information on the basis of past experience is generated using what we called Bayesian approach. It integrates, in a natural way, the bases of probabilistic model of the reliability analyzing statistics, which is not able anymore to use some classical techniques such as assistants, students, etc. since they are based on the need to have adequate historical data.

In this paper, the absence of data (problem of uncertainty of the data) is due to both technological innovation that changes the scenario associated with it, the emphasis on new prospects in this light that are detectable for the reliability, and how it poses today as the basis for a strategy for monitoring the performance of the system, rather than a simple discipline as prevented in the past.

Reliability Studies are an increasingly important aspect in design and operation of each component, equipment or engineering system.

Appropriate specifications and standards of reliability: assessment of failure and repair rates, probability of occurrence of failures in systems are prerequisites for the design and operation of systems.

However, reports that currently reliable do not always take into account of new technological scenarios, production and market.

Reliability discipline was initially based on historical data analysis related to equipment operation to ensure a series of estimated parameter (failure rate, mean time to failure, function reliability, availability, etc.) characterizing the performance of the devices themselves.

During the applications change to all fields of engineering, the discipline has, over the years, developed methods for estimating the characteristic values of the failure rate, as other parameters of interest, developing their own models of the disciplines in statistics and probabilistic gender [3]. In particular, the theory of reliability has also consolidated methodologies for the complex systems analysis, which, starting from the characteristic parameters of the components, allows to extend the analysis reliability system as a unity. All methods require the knowledge of the characteristics of life of system components and, therefore, require a series of preliminary analysis on the behaviors of components during operation. The gradual introduction of new technology components reduces the availability of historical series on the components operation. Today, a component of a system can be considered "exceeded" even after a few years, thanks to the use of new technologies, new materials or new techniques, more and more quickly tested. For these devices, both for the apparent lack of an adequate amount of historical data, and the high reliability of the devices themselves, lack sufficient data to estimate an effective reliability of the second classical approach.

In order to take into account the adequate scarcity of information in recent years, different methods have been proposed to describe the imprecise knowledge in the reliability evaluations of electric power systems, including the sensitivity analysis [4], numerical calculations based on standard technique deviation [5], the fuzzy logic [6,7], arithmetic intersperse [8]. Promising new applications of Bayesian methods of estimation know a renewed interest for the analysis of electric power systems under conditions of uncertainty. Indeed, they offer some significant advantages compared to the classical one:

- From a theoretical point of view, they characterize, in a similar manner, all the uncertainties and unknowns of the random variables problem. We can directly describe in probabilistic terms the uncertainty concerning the interest quantities.
- From an operational point of view, they allow a more efficient (i.e. with a lower mean square

error), compared with the need to characterize the *"a priori"* distribution of the interest variables. Knowing this distribution *"a priori"* considered as a limit, because of its *"subjective"* characteristic, the procedure is assessed based on experts knowledge. Moreover, its effect on the final estimation may be limited by using Bayesian techniques which are "robust" compared to *"a priori"* hypothesis or Bayesian methods that are "empirical" [9,10];

- The updating of the estimations obtained following the acquisition of new data through the evaluation of the parameters distribution *"a posteriori"* is simple and immediate, for a given class of distributions. Indeed, another limitation of the application of Bayesian methods has always been that of computational problems, mostly related to problems of numerical integration for determining the average and confidence intervals afterwards. Today, that limit appears largely over, for the development of appropriate calculation software and powerful hardware and efficient estimation techniques Monte Carlo [11,12].

2. RELIABILITY STUDIES

2.1. Characterization of the failure rate as a random variable

As a consequence of technological developments, a review of the reliability, designed to integrate the strategies of "control" in line is required, as a useful tool for identifying "critical components" of the system, to deal with the uncertainty of the data. It is preferable to use an appropriate model of knowledge *"a priori"* of the reliability, as general as possible, (i.e. not requiring specific assumptions such as, for example - typical use - of the Exponential Model, Weibull, Gaussian, etc.) and that is capable of continuous updating of the data, although limited, will be available during the system operation. The effect of the uncertainty of the data can be described by appropriate models of its inherent randomness, developing the concepts outlined in [13-15]. These studies start from the principle that knowledge of a phenomenon is never complete, but is characterized by several inaccuracies, such as:

- Inaccuracies in the model, or even, the absence of an analytical model.
- Inaccuracies related to the parameters estimation that influences the physical system.
- Lack of appropriate tools required to the observation of the system.
- Presence of too many variables that contribute to affect the normal evolution of the system.
- Presence of difficult elements to be controlled (for example, some environmental parameters).

These elements are combined to form the 'environment' that requires a description of their uncertainty and tightness. Provided with operating conditions with appropriate random variables X_1, X_2, ... X_n and using the generic component model for proportional failure rate (PHM: Proportional Hazard Model), the failure rate function $h(t;X_1, X_2,...X_n)$ can be expressed as the product of two functions. The first function $h_0(t)$ depends only on time, whereas the second $\psi(X_1, X_2, ... X_n)$ depends only on random variable r.v. describing the operating conditions:

$$h\ (t;X_1,X_2,...,X_n)\ =\ h_0(t).\ \psi\ \ (X_1,X_2,...,X_n) \quad (1)$$

$h_0(t)$ is a deterministic function, $\psi(X_1, X_2, ... X_n)$ is a non-negative function of r.v. $X_1,X_2,... X_n$ and it is, therefore, a r.v. that will be suitable in the succession with Z:

$$Z = \psi(X_1, X_2, ..., X_n) \quad (2)$$

The failure rate function, so defined, can be considered as the estimation of the conditioned failure rate of $h(t \mid Z)$.

$$h(t,Z) = Z.h_0(t) = h(t \mid Z) \quad (3)$$

Where: $Z=1$, $Z<1$ and $Z>1$, respectively, nominal, favorable and unfavorable conditions of operation.

For every value z of Z, the conditioned reliability function is:

$$R(t|Z=z) = e^{-z\int_0^t h_0(u)du} = e^{-zH_0(t)} \quad (4)$$

Where $H_0(t)$ is the total failure rate function.

Such expression $H_0(t)$ is valid in the case in which the r.v. Z is constant in $(0,t)$, or under

environmental conditions "static."
Applying the theorem of the total probability, with $g_z(z)$ it is the pdf. of r.v. Z, the reliability $R(t)$ can be written as:

$$R(t) = \int_0^{+\infty} g_z(z).R(t \mid z)dz \tag{5}$$

If the $R(t \mid Z)$ is concerned as a r.v. function of Z, the mean value coincident as well known with the respect *"a priori"* "optimal", in the sense of the least squares of the reliability function $R(t)$ is :

$$\mu = E[R(t)] = \int_0^{+\infty} g_z(z).R(t \mid z)dz = \int_0^{+\infty} g_z(z).e^{-zH_0(t)}dz \tag{6}$$

The medium value of the reliability can be concerned as Laplace transforms of the $g_z(z)$ calculated for $s=H_0(t)$.

Assessing the probability of an event, in matter of reliability, we point out the failure of a component or a system. The failure rate is therefore the reliability indicator the most often used.

2.2. A systemic approach of the electric systems reliability

In the last paragraph, we have exposed the randomness character of the system failure rate. The method is an extension of traditional reliability methods which is based on probability distributions of failure time. It is impossible to estimate the reliability characteristics in operating conditions. In fact, traditional approaches of reliability studies are based on the central concept of failure rate, which is linked to the average characteristics of the component and cannot, by its very nature, take into account time of the dynamic performances.
It is important to match the concept of system reliability to the dynamic evolution of the system state variables that describe its performance. It is very interesting that the approach proposed in [16], where the reliability performance is defined as the conditional probability that the system performance indices are within critical limits for a given time space. The dynamic state estimation becomes the prerequisite for the prediction of reliability indices system. Moreover, we can even think of providing preventive control logic capable to ensure that, at every moment, reliability constraints are satisfied (Fig.1).
The adjustment articulation cycle involves the encoding complex procedures that require the support of advanced methodological tools.
Suppose that we have identified the vector of variables state x of the system estimated through extended Kalman filter. We can find a vector $y=g(x)$ which describes the performance of the system. It is interesting to characterize the general failure mode of k-exempt critical surface defining a multidimensional $s_k(y)=0$.

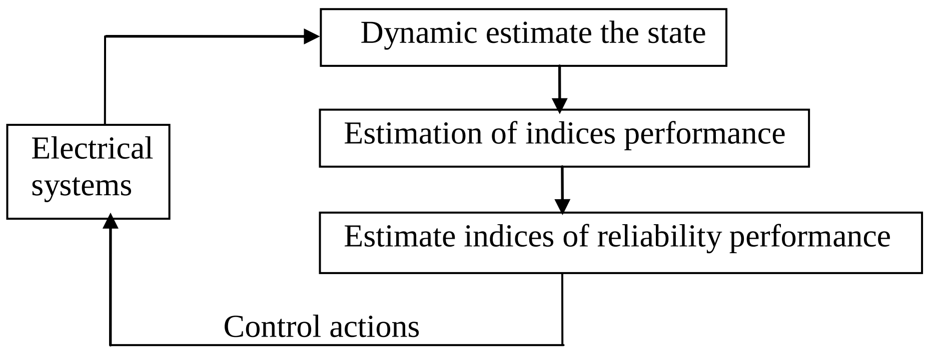


Figure 1 : Reliability performances cycle control system

It is possible to define the probability that the system introduces the way of k-exempt failure, supposing to know the probability density function $f(y)$

$$F_k(t) = \int \ldots . \int_{\Omega_k} f(y)dy \tag{7}$$

Where the subspace Ω_k is individualized by the relationship: $s_k(y)<0$.

The system reliability in presence of n failure ways is:

$$R(t) = 1 - \int_{\Omega} \ldots . \int f(y)dy \tag{8}$$

Where: $\Omega = \Omega_1 \bigcup \Omega_2 \ldots . \Omega_n$

Below, based on of the considerations set out above, we will proceed by identifying a reliability model in an appropriate way of operating conditions, updating the data according to the Bayesian approach.

2.3. Critical aspects of the models selection

In choosing the most suitable reliability model based only on a statistical approach, it is necessary to avoid serious errors in assessment of significant size such as failure rates.

In absence of data, the suitability of the models may be only evaluated on the basis of the *"a priori"* and forming a perfect part under the inference of Bayesian approach. This important theoretical aspect in operating mode describes the reliability function to evaluate efficiently the parameters of interest.
It is important to introduce a simple example, that hypothesizes - for a component datum - the availability of the only information around the mean value of duration (*m* = 45 years) and standard deviation (*s* = 15 years) [17] for cables in *HV*. Incidentally, these data exclude the hypothesis, often adopted in literature; of exponential model (that is characterized by mean value and standard deviation of peer value).
The selected models, Weibull and Log-Normal, have the same values of assigned parameters *m* and *s*. The expressions and the values of the parameters of the pdf drawn through simple inversion of the relationships that give *m* and *s* in operation of density parameters are with (*t* in years):

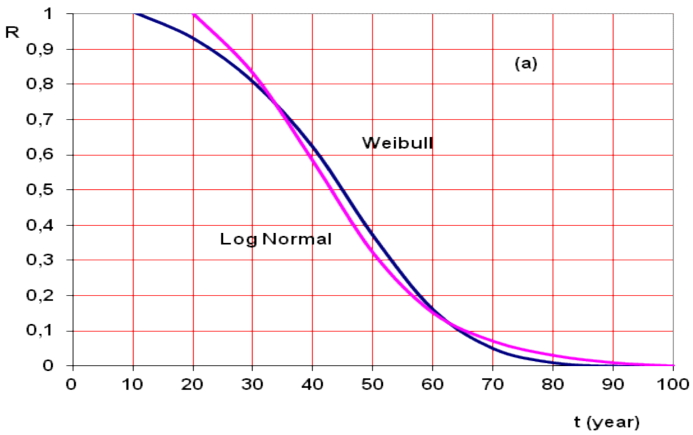
Weibull: $$f(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right] \tag{9a}$$

$(\alpha = 50.13 years; \quad \beta = 3.341)$

Log-Normal: $$f(t) = \frac{1}{\sigma t\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}(\ln(t) - \xi)^2\right] \tag{9b}$$

$(\xi = 3.754; \sigma = 0.3246)$

An examination of figure 2 below shows that, the reliability function *R(t)* of Log-Normal and Weibull are similar which is not the case for their failure rates *h(t)*. In particular, the failure rate of Log-Normal, as it is known in theory, starts on the origin $t = 0$, reaches the maximum and then decreases versus t, contrary to the increase of the Weibull failure rate.



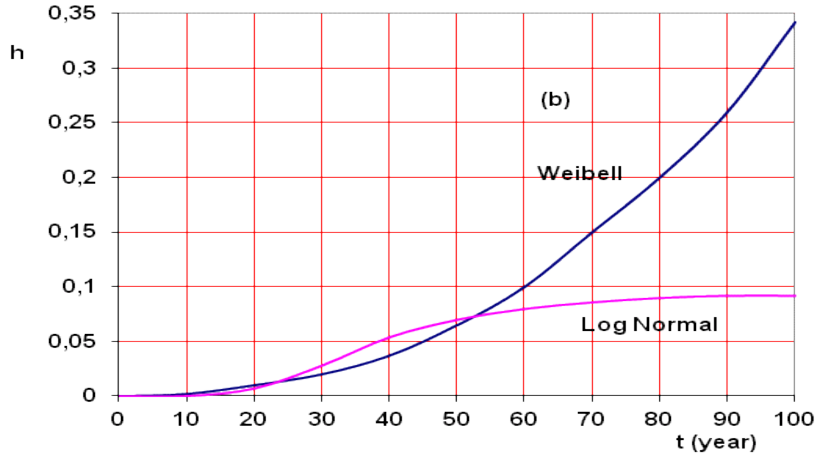


Figure 2 : Reliability function (top) and failure rate (bottom)

Therefore, the use of the Weibull model, if it supposed “true” the Log-Normal model, shows an aging of the component that in reality “does not exist “, and this would lead to excessive maintenance costs.
It is important to mention that the Weibull and Gaussian models, which are, in the electrical engineering, among the most applied, can have many theoretical limits in reality. In fact, the Gaussian model has the following inherent limitations:

- A Gaussian variable, being able to assume (if the average value is larger than the standard deviation) negative values, is not theoretically suitable for covering the duration times;
- Poor flexibility (its density can only have one form, the bell, or the "bell-shaped", with no asymmetry);
- Monotony of failure rate functions, increasing.

As it is well known, its widespread use in the field of applied statistics is derived from the famous Central Limit Theorem, which rarely finds suffrage about durations: it is difficult for the duration of the component can be described by the sum of several separate times, if not in the case of systems with redundancy "*stand by*".
The Weibull distribution is justified in reliability - for the time period - under the theory of "Extreme Values", where the value of the component is determined by the minimum durations of several elements in "series" (weakest link theory). There should also be noted that pure type Weibull distributions have the disadvantage of the monotony of the failure rate function: increasing (if the shape parameter β of the (9a) is greater than 1), constant (β=1), or decreasing (β<1). This type of property is an apparent disadvantage to the reliability components on time intervals long enough (as it is required, designed to maximize the time of maintenance in a "life extension").
The probability model selection consists of choosing an appropriate distribution for the reliability data.
The definition of an equivalent law must take into account simultaneously several factors: requirements of simplicity, criteria of adequacy with respect to the diversity of laws, validity of approximations and preservation of the Bayesian approach coherence.
The tendency of modern research turns to identifying more flexible models that use well-established experimental methods, such as the "reverse power" to the insulating function of electrical stress and, "Stress / Strength" models [18].

2.4. The "Stress-Strength" model

Over the last decade, the liberalization of the energy market, has encouraged the study of diagnosis and maintenance methods that optimize costs by ensuring the overall power system reliability. Since the isolation of the components of high-voltage plants is one of the weak points, many efforts have been directed towards improving levels of insulation reliability and, therefore, the system availability.
In this field, for the reliability theory, it is a device under stress (“stress”), linked to environmental conditions in which they operate: "last" as the effort does not exceeds the resistance (“strength”) of the device in this type of solicitation.
Introducing random variables *X*, *Y*. (*X*: "Stress"; *Y*: "Strength"),

The component reliability can be express:

$$R = P(X < Y) = P(Y - X > 0) \tag{10a}$$

As seen, this is a particular case of the use of a «performances index» of the system, defined in this case by the variable: *Z=Y-X*, resulting «critical surface» *Z*<0.
With reference to *f(y)F(y)* the pdf. (cdf.) of *Y* and with *g(x)G(x)* the pdf. (cdf.) of *X*, the the component reliability is given - in the hypothesis, generally approved in this context of statistic independence of X and Y - from:

$$R = \int_0^\infty g(x) P(X < Y \mid X = x) dx = \int_0^\infty g(x)(1 - F(x)) dx \tag{10b}$$

However, the insulators are subject to the wear that depends on operating conditions which are affected by considerable uncertainty. In what follows, we assume that:
1. The insulation ability to withstand the electrical stresses, i.e. Strength *Y* is a random variable that is assumed to be described by a biparametrical Weibull distribution (particularly suitable for insulating polymer solids), with cumulative distribution function (cdf.) type:

$$F(y) = 1 - \exp\left[-(y/\alpha)^\beta\right] \tag{11a}$$

Where α and β are respectively the scale and the form parameters, usually dependent on stress (electric field and temperature), and the age *t*.
Generally, it is reasonable, based on experimental data, to suppose that β is independent of the time, and α decreases with age, with the law of "Inverse Power"

$$\alpha = \alpha(t) = k / t^m \tag{11b}$$

2. Stress is a random event whose peak value, corresponding to *X* Stress above, it is a random variable described by a proper probability density function, dependent on the genesis and location; in [18]. It is argued that a valid hypothesis is that *X* is also generated by a Weibull law, with parameters θ and β, having fdc:

$$G(x) = 1 - \exp\left[-(x/\theta)^\beta\right] \tag{11c}$$

3. The insulation degeneration occurs when the entity of excess voltage is above to the level of resistance of the insulator: since the resistance and the stress are both random variables, the degeneration due to overvoltage property of the unloaded function risk *P* of which reliability *R* can be valued.

In order to make Bayesian estimation, it is convenient to express the Weibull distributions of variable "stress" *X* and "strength" *Y* versus the following parameter:

$$z = \left(\frac{1}{\vartheta}\right)^\beta, \; w(t) = \left(\frac{1}{\alpha}\right)^\beta \tag{12}$$

According to (11b), the *w* parameter is expressed in the following way:

$$w(t) = \left(\frac{t^m}{k}\right)^\beta = \frac{t^b}{k^\beta} \; ; b = m\beta \tag{13}$$

The function reliability assumes then the following expression

$$R(t) = \frac{1}{[1 + w(t)/z]} = \frac{z}{[z + w(t)]} \tag{14}$$

Which can be rewritten in the following format "Log-Logistic" [18]

$$R(t) = 1/\left[1 + (\lambda t)^b\right]; \; \lambda = (\theta / k)^{1/m} \tag{15}$$

In the considered model, the only unknown parameter is the transformed scale *z* parameter, related by (12) with the scale parameter θ of stress. The uncertainty of this magnitude is significant, since the extent of the over voltage depends on a set of variables inherent uncertainties (eg. topology and network status). If we adopt a Bayesian approach, the unknown quantity z should be considered as a

random variable *Z*. Thus the various reliability parameters are variables random described by appropriate distributions.
Assuming that *Z* has *"a priori"* Gamma distribution, characterized by a density function:

$$p(z) = \frac{z^{(\nu_0 - 1)}}{\delta_0^{\nu_0}\Gamma(\nu_0)}\exp(-z/\delta_0)\,,\; z > 0 \tag{16}$$

Where ν_0 and δ_0 are respectively the form and the scale parameters (the subscript 0 refers to the *"a priori"* distribution). The use of the Gamma distribution for the scale parameter of the Weibull distribution is broadly approved in literature because besides its flexibility, it is the "conjugate" of the Weibull (or the *"a posteriori"* is still has Gamma distribution) and, it is finally sufficiently strong in comparison with the respect of the parameters, reliability [9].

The *"a priori"* distributions of the quantum *p* of the life time T_P is of the type "Gamma generalized" [9], expressible quantum being such in terms of *Z* as it follows:

$$T_p = (1/\lambda)\left[P/(1-P)\right]Z^{1/b} \tag{17}$$

The reliability density *R(0 < r < 1)* is given by:

$$q(r) = q(r, \nu_0, \delta_0) = w(1-r^2)^{-1}\, p\left[(wr)/(1-r)\right] \tag{18}$$

So, once observed a sample of stress values $D(X_1, X_2, \ldots X_n)$, the updating of the distribution, by virtue of the Bayes theorem, it is carried out according to the report:

$$p(z|D) = p(z)L(D|\,z)/C \tag{19}$$

Where $L(D|\,z)$ is the data likelihood function, with *C* expressed from:

$$C = \int_0^{\infty} L(D|\,z)\,p(z)dz \tag{20}$$

The distribution *"a posteriori"* of *Z* is still a Gamma, whose new parameters are express by:

$$\nu = \nu_0 + n,\quad \delta = \delta_0/(1+U\delta_0),\quad U = U(D) = \sum_{j=1}^{n} X_j^{\gamma} \tag{21}$$

The best estimator, in terms of mean square error, the function $\tau=\tau(Z)$ is given by the average *"a posteriori":*

$$\tau^0 = \int_0^{\infty} \tau(z)\,p(z|D)dz \tag{22}$$

Examples of functions $\tau=\tau(Z)$ proper *R* and T_P, whose distributions *"a posteriori"* are described by the same form distributions using new parameters.
The choice of an *"a priori"* distribution must be justified by the quality and the quantity of the information available and according to its impact on the subsequent posterior distribution.
Of course, these distributions provide enough information to also get the desired intervals of Bayesian confidence.

3. NUMERICAL APPLICATION

The following is an exemplification of the numerical procedure with specific reference to cables for high voltage transmission blocks with XLPE. The "strength" and the operating temperature is constant and assumed, respectively 10kV/mm and 90°C. The peak value of high voltage is assumed to be described by a Weibull distribution with the form parameter β=12 and the scale parameter θ to be estimated.
To describe the variation in time of the "strength" of the insulators to high voltage, we used the relation (11b). For the calculations, considering that the parameters *m* and *k* , are respectively 0.12 and 120, with *t* in hours; these values lead to a reduction of the insulation strength that is significant in the first life period of the components and which then gradually tends to the saturation, as also noted in the literature for the XLPE.

Figure 3 shows the pdf. *f(y)* of the strength *Y* for three different aging times (5, 10 and 20 years). In presence of a few data, the Bayesian method is greatly estimated robust as it combines the limited data available to other information of various pre-existing source.

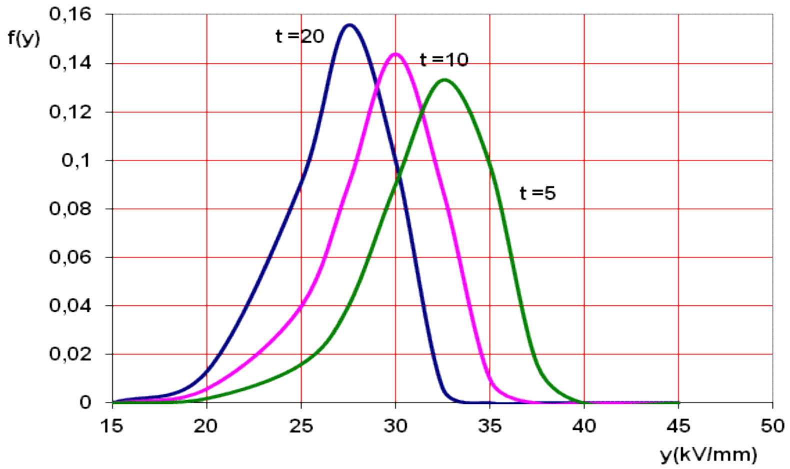


Figure 3: Density function of strength for different values of service life

In order to illustrate the method efficiency, we chose *"a priori"* distribution for *R* with a considerable variance to express considerable uncertainty associated with *R*. Before applying the Bayesian method, the reliability evaluation depends on the value of θ, equal to $\theta_0 = 16.32\sqrt{2} = 23.08 \quad kV/mm$, in order to have, then, a comparison to the estimated value. Thus, the reliability was determined using the log-logistic model with: $\lambda = 1.0809 \times 10^{-6} h^{-1}$ and $b = 1.44$.

The reliability function is showed in Figure 4 with some meaningful values reported in table 1.

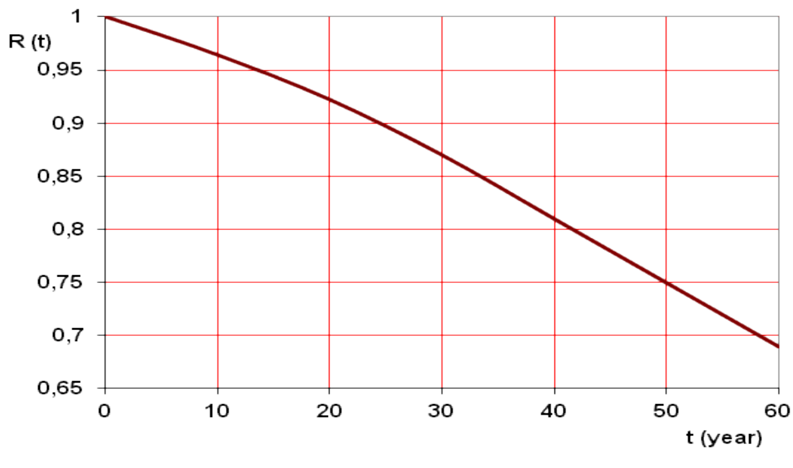


Figure 4 : Reliability function of time. The preliminary estimate log-logistic model

It is easy to determine the failure rate *h(t)* and the pdf of operating time f(t) shown in figure 5. At this point it is selected for the *"a priori"* Gamma distribution of parameters Zi, $\nu_0 = 1$ (Exponential Distribution), with $\delta_0 = 1/\theta^{\lambda}$ and $\theta = \theta_0 = 23.08\, kV/mm$.

The sample of D values of stress, $D = (23.78,\ 22.31,\ 21.77,\ 20.26,\ 24.57)$, was generated using after the Weibull distribution, whose density is given in figure 6.
According to previous reports, it is possible to determine the *"a posteriori"* and *"a priori*" distribution together in figure 7.
It can be said that the distribution of R is more concentrated: the variance of *"a priori"* and *"a*

posteriori" are equal to 0.143 and 0.018 respectively.
In order to evaluate the efficiency of the Bayesian estimation method regarding to the maximum likelihood, Monte Carlo simulations were carried out with different samples sizes.

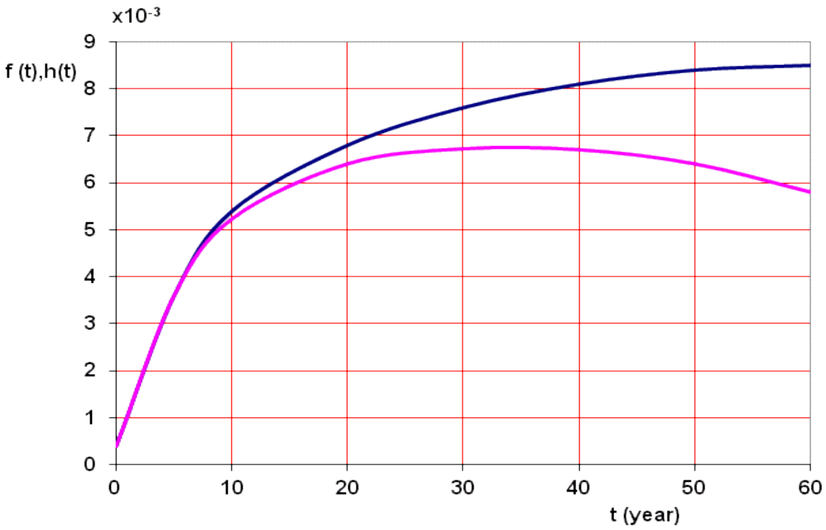


Figure 5 : Probability density functions and the failure rate time

The pdf is shown in Figure 6.

Table 1: Reliability at fixed time of the service life according to preliminary estimation (non-Bayesian)

T (years)	5	10	20	30
R	0.988	0.967	0.916	0.860

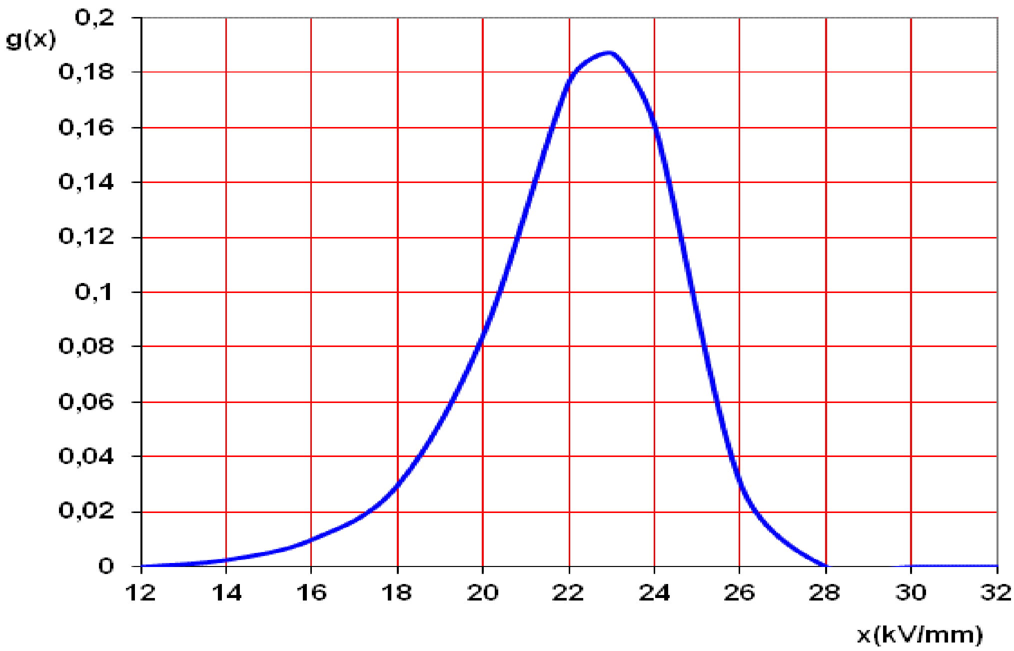


Figure 6 : Probability density function of Stress

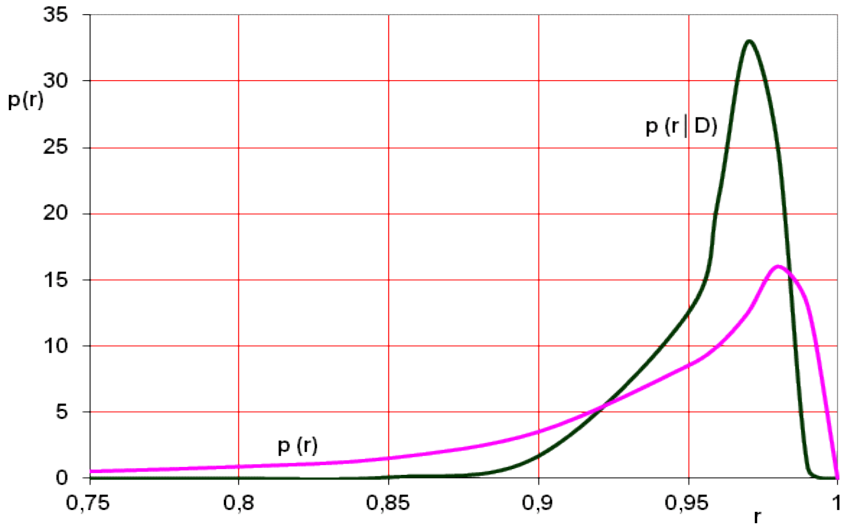


Figure 7 : Function of the *"a priori"* probability density (-) and *"a posteriori"* (-) the reliability to 10 years of service life

For more simplicity, the following table shows some significant results. In particular for limited number of samples (n=1, n=3) or mean size samples (n=10, n=30) presented in the table, the results in terms of:

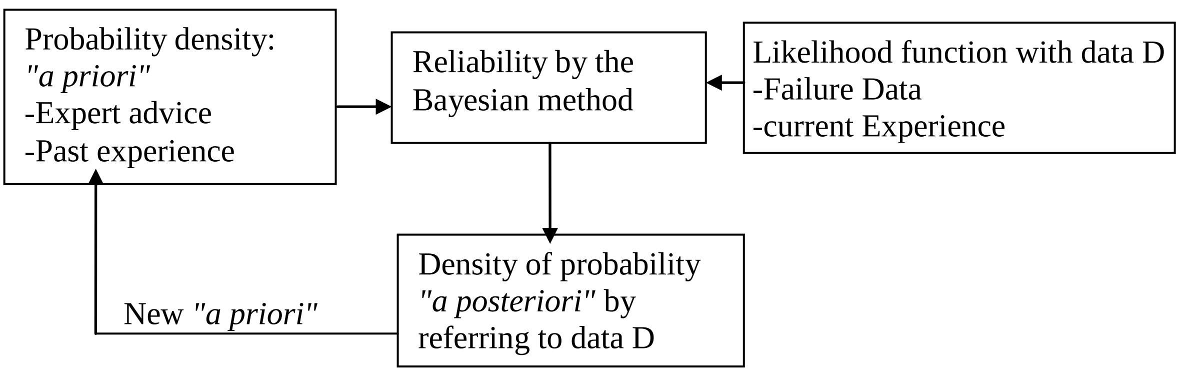
1. Root Mean Square Error of the Bayesian estimator (RMSEB).
2. Root Mean Square Error of the maximum likelihood estimator (RMSEL).
3. Bayesian estimator efficiency EFF = RMSEL / RMSEB.

Table 2: Calculated Values of RMSEB, RMSEL and EFF related to different samples sizes

Sample size	RMSEB	RMSEL	EFF
1	11.6	388	33.6
3	8.48	17.7	2.09
10	5.21	6.69	1.28
30	3.19	3.45	1.08

The results show clearly the remarkable efficiency of the Bayesian estimator than the maximum likelihood estimator, especially for a limited sample size.

Reliability study by Bayesian method



4. CONCLUSIONS

This work is devoted essentially to the Bayesian reliability studies of power systems. It has been described as a general probabilistic model that takes into account the problem of uncertainty of the data, typical systems characterized by high degree of technological innovation. In particular, it was shown that the proposed approach can play an important role in preventive strategies control of electrical systems. Therefore, they were made of the considerations on Bayesian estimation methods, where the superiority of Bayesian estimators compared to those classics is manifested even more clearly the more limited is the number of samples available.

pdf. : Probability density function
cdf.: cumulative distribution function
r.v.: random variable
HV: high voltage
XLPE: Cross Linking Polyethylene used predominantly as high voltage electrical cables insulation.

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