Scale invariance properties of rainfall in AMMA-CATCH observatory (Benin, West Africa).

Propriétés d'invariance d'échelle des précipitations de l'observatoire AMMA-CATCH (Benin, Afrique de l'Ouest).

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الملخص

أجريت هذه الدراسة على معطيات في أوقات معينة لتساقط الأمطار بين سنة 1999-2012 بثلاثين محطة Benin - AMMA-CACTH. توجد علاقة وطيدة بين اللحظات الإحصائية لبيانات هطول الأمطار ومدة تساقطها. هناك سلوك يبين ثبات المقياس بين اللحظات الإحصائية وفترات المتابعة الخاصة بهطول الأمطار، مع الأس ثبات على نطاق وتلبية عدم المساواة: 0.5 < n <1 الخصائص الإحصائية لهطول الأمطار في موقع الدراسة يبين ان ثبات المقياس بسيط. ولوحظت هذه الخصائص الزمانية من ثبات المقياس على اثنين من فترات الوقت: من 5 دقائق إلى 45 دقيقة الى 1 اليوم لبعض المحطات Koko و، Dogue، Dogue، Betrou، WEWE، Sarmanga، Pelebina، Momongou وParakou و دقائق5 إلى 1 ساعة ومن 1 ساعة إلى 1 يوم بالنسبة للمحطات الأخرى المدروسة . هذه العلاقة في ثبات المقياس التي وجدت في هذه الدراسة ذات أهمية كبيرة لأنها سوف تحل مشاكل تفكك المقياس (من صغيرة إلى كبيرة) التي كثيرا ما تصادف في الهيدرولوجيا. وهذه الأهمية الهيدرولوجية التي تنتج من هذه الدراسة ستكون موضوعا للعمل في المستقبل

الكلمات المفتاحية: *المطر السنوي الأقصى- ثبات السلما بسيط-أس السلم -البنين*

Résumé

Cette étude a été effectuée sur les données temporelles de pluies de 1999 à 2012 de trente stations du site AMMA-CACTH-Bénin. Les moments statistiques des données de pluies et leurs durées sont reliés par des fonctions puissances. Il existe un comportement d'invariance d'échelle entre les moments statistiques et les durées d'observations des pluies, avec un exposant d'invariance d'échelle vérifiant l'inégalité: $0.5 < \eta < 1$. Les propriétés statistiques des pluies sur le site d'étude suivent l'hypothèse des processus d'invariance d'échelle simple. Ces propriétés temporelles d'invariance d'échelle sont observées sur deux intervalles de temps: de 5min à 45min et de 45min à 1 jour pour certaines stations à savoir Koko, Dogue, Beterou, Wewe, Sarmanga, Pelebina, Momongou et Parakou et de 5min à 1h et de 1h à 1 jour pour les autres stations d'étude. Cette relation d'invariance d'échelle trouvée dans cette étude est d'une très grande importance car elle permettra de répondre aux problèmes de désagrégation d'échelle (passage d'une petite à une grande échelle) souvent rencontré en hydrologie. Cette importance hydrologique qui découle de cette étude sera le sujet de nos prochains travaux.

Mots clés*: pluie maximale annuelle - invariance d'échelle simple - exposant d'échelle- Bénin*

Abstract

Data from thirty stations in AMMA-CATCH observatory (northern Benin) were used during the period from 1999 to 2012. The relationship between the moments and the durations could be described by the power-form functions. It indicated the scaling behavior of the moments with durations. The properties of rainfall followed the hypothesis of simple scaling process. Furthermore, the scaling properties of the rainfall in time series were simple scaling and were composed of two different regimes for two distinct intervals. The first from 5min to 45min and from 45min to one day durations for Koko, Dogue, Beterou, Wewe, Sarmanga, Pelebina, Momongou and Parakou and finally from 5min to one hour and from one hour to one day durations for the other study stations. The scaling exponents were estimated in all the stations and checks: $0.5 <\eta< 1$. The possibility of using wide sense simple scaling in northern Benin has been demonstrated.

Keywords: *annual maximum rainfall- simple scaling- scaling exponents-Benin.*

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1. INTRODUCTION

In recent years, the need for insights into rainfall process leads to new approaches for modeling the distribution of rainfall intensities, in time and space. There is particular lack of knowledge about rainfall variability at different scales [1].The knotty problem of whether the rainfall process exhibits properties that are independent of temporal scale and the physical reasons for this remain a widely discussed topic among hydrologists, meteorologists and scientists. The possibility to characterize rainfall by scale invariance (scaling) properties has been investigated during recent years [1-3]. The early theory of simple scaling [4] has undergone substantial refinement into general and more practicable theory of multiscaling[5, 6]. It has been shown that multifractal fields may be produced by the so-called multiplicative cascade processes where some large-scale flux is concentrated into smaller spatial units [6]. It has been argued that this behavior is expected for the flux of water in the atmosphere. The scaling (generally multiscaling) structure of fields produced by a cascade process can be expressed by some statistical relationships [5-9]. These relationships may be very useful in practical rainfall applications. For example, because the scale independence, the possibility exists to derive statistical information about the rainfall process at scales smaller than the resolution scale of the available data. It constitutes an exciting way to deal with the increasing problem of inadequate rainfall data input in hydrological models. The scaling or scale-invariant models enable us to transform data from one temporal or spatial model to another one and thus help to overcome the difficulty of inadequate data.

However, before developing applicable methods for data processing, the scale invariance behavior must be carefully validated on real rainfall data and the number of such investigations is still rather low. Recent investigations have mostly dealt with either radar data (e.g. [6-8,10-13]), or rainfall time series (e.g. [1,14,15]).

In spite of recent advances in the investigation of the scaling properties of hydrological fields, very few studies from different geographical areas have been made to determine invariance properties of the meteorological observations. This is especially true for rain gauge observations since it is extremely expensive and time consuming to observe rainfall by gauge over large space and time scales. Even so, most historical data in practice have been collected by rain gauges: it would be highly beneficial if scaling parameters could be found for data over different scales of observations. In Benin the rainfall characteristics were not yet known.

Several authors showed that the knowledge of rainfall characteristics was essential for many types of hydrologic studies, specifically regarding planning, design and management of water resources system [1,6-8].

The main objective of this study was to focus on the analysis of the scale invariance of rainfall properties based on rain gauge observation in Benin and the analysis of scale invariance of rainfall data in all the OHHVO (Hydro-meteorological Observatory of upper Oueme Valley) stations in northern Benin.

2. Methodology

2.1 Study area and Rainfall Data

West Africa is a major heat source for Earth's climate system. It plays a role on a global scale in generating circulation anomalies. During the last century, important decreases due to changes in precipitation for most rivers were remarked in Benin. The current study focuses on the Upper Oueme Basin which is the network of the AMMA-CATCH-Benin observatory. It's characterized by a Sudanian climate with annual precipitation varying between 1200 to 1300 mm/year. Daily temperature ranges are about 10˚C. AMMA-CATCH Benin observatory is delimited by latitude from 9°N to 10°N and longitude from 1.5°E to 3°E (Figure 1). Oueme is the largest river draining itself over two third of Benin rivers system. It is inhabited by about 400,000 people and has at least 35 ethnic groups.

Two seasons per year exist: a dry season (November to May) and a rainy season (May to October) [16]. For the analysis in this study, thirty rain gauge stations were selected from the dataset of network of the AMMA-CATCH Benin observatory. The location of the analyzed stations is shown in Figure 1. The periods of observation in this study range from 1999 to 2012 and the name of study rain gauges are listed in Table 1. The analysis was performed on Annual Maximum Rainfall Intensity (AMRI) series for various durations: 5min, 10min, 30min, 45min (sub-hourly duration) and 1hour, 2hours, 3hours, 6hours, 12hours, 24hours (hourly).

(a)

(b)

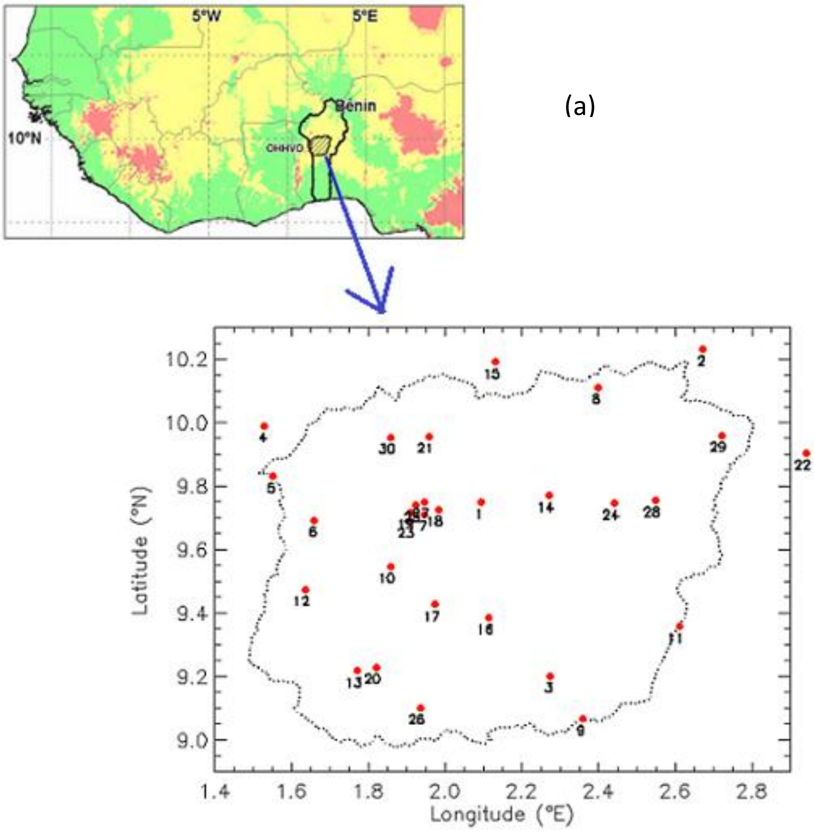

Figure1: Study area and rain gauge locations: OHHVO in Benin (Fig. **1a)** and rain gauge site in OHHVO (Fig. 1b).

Table1: List and number of recording rain gauges used in the analyses.

N°	**Station name**
1	Affon
2	Bembereke
3	Beterou
4	Birni
5	Copargo
6	Djougou
7	Donga
8	Fo-Boure
9	Koko
10	Momongou
11	Parakou
12	Pelebina
13	Sarmanga
14	Sonoumon
15	Tobre
16	Wewe
17	Adiangdia
18	Akekerou
19	Ananinga
20	Angaradebou
21	Bari
22	Biro
23	Bombone

24	Bori
25	Dapelefoun
26	Dogue
27	Gaouga-Gou
28	Gori-Bouyero
29	Ina-Ceta
30	Tebo

2.2 Simple scaling

Simple scaling has been defined in term of probability distributions and statistical moments. In a rainfall context, Gupta and Lovejoy [6] have provided the definition of scaling of probability distribution as a random function X (t) given by:

$$X(\lambda t) \overset{dist}{=} \lambda^H X(t) \qquad \textbf{(1)}$$

or in term of its fluctuations ΔX as:

$$\Delta X(\lambda t) \overset{dist}{=} \lambda^H \Delta X(t) \qquad \textbf{(2)}$$

The parameter λ is essentially a scale parameter and H is a non-integer scaling exponent. These relationships imply that the function (Equation 1) changes according to simple scale function λ^H. Gupta and Lovejoy introduced the notations of strict and wide sense simple scaling [6]. The former referred to simple scaling of probability distribution (Equation 1). In a less restrictive type, the statistical moment have a simple scaling properties, if the relationship (1) is satisfied. To specify this scaling in rainfall, authors considered a field of rainfall intensity I_1. The field averaged over a scale specified by the parameter λ was denoted I_λ. Wide sense simple scaling was defined as:

$$E\left[I_\lambda^q\right] = \lambda^{H_q} E\left[I_1^q\right] \qquad (3)$$

Where E denotes expectation (or ensemble average), q is the order of the moment and a scaling exponent corresponding to H in Equation (1). Although the scaling expressed in terms of statistical moments is not a scaling in its strict sense, it's usually considered as a working assumption due to the difficulties to be involved in evaluating Equation (1).

2.3 Multiscaling

The most common way to define multiscaling is the term of statistical moments; by substituting Hq in the Equation (3) for a function K (q), the general multiscaling form of the relationship becomes:

$$E\left[I_\lambda^q\right] \propto \lambda^{K(q)} \qquad (4)$$

With the same notation as in Equation (3), the scaling of the moments is thus no longer described by a single exponent H but by a nonlinear moment scaling function K(q), a second characteristic function of multiscaling behavior.

- In the first step, the original (temporal) field I_1 is averaged over a scale specified by λ to obtain I_λ. All values of I_λ are raise to q to obtain the field I_λ^q . The statistical moment $E\left[I_\lambda^q\right]$ is obtained as the average of I_λ^q. By performing the procedure for a fixed q and a vary λ; the accuracy of the scaling of the statistical moments can be evaluated in the log-log plot of $E\left[I_\lambda^q\right]$ versus λ. A straight lined behavior implies scaling and the slope is an estimation of K (q) for the particular q. By performing the procedure for difference value of q, different values of the K (q) function may be obtained.
- In the second step of the procedure, the K (q) function is estimated by plotting the value of K (q) as a function of q obtained in step one. The shape of the K (q) function then specifies the types of the scaling involved. If K (q) becomes straight line, the function reduces to Hq and the data exhibit simple scaling as expressed by Equation (3). If K (q) becomes nonlinear, the data are multiscaling as expressed in Equation (4). These methods are used in this study.

2.4 Scale invariance properties of rainfall in time series

Let's considerer the random variable I_d, the maximum annual value of local rainfall intensity over a duration d is defined as:

$$I_d = \max_{0 \le t \le 1year}\left[\frac{1}{d}\int_{t-d/2}^{t+d/2} X(\xi)\, d\xi\right] \qquad (5)$$

Where X (ξ) is the time continuous stochastic process representing rainfall intensity and d is the duration. It is supposed that I_d represents the Annual Maximum Rainfall Intensity (AMRI) of duration d. Here, some concepts are introduced about scaling of the probability distribution. A generic random function I_d will be denoted by simple scaling properties if it obeys to the following equation 6:

$$I_d \stackrel{dist}{=} \left(\frac{D}{d}\right)^{-H_d} I_D \qquad \textbf{(6)}$$

D is a aggregated time duration (i.e.: 2, 3... 24 hours).

The scaling ratio is defining by $\lambda_d = \frac{D}{d}$

$$I_d \stackrel{dist}{=} \lambda_d^{-H_d} I_{\lambda_d d} \qquad \textbf{(7)}$$

The Equation (7) is rewriten in terms of the moments of order q about the origin which is denoted by $[I_\lambda^q]$. The resulting expression is:

$$E[(I_d)^q] = \lambda_d^{-H_d q} E[(I_{\lambda_d} d)^q] \qquad (8)$$

If we assume that the wide sense simple scaling exists, the scale-invariant models will be enable us to transform data from one temporal to another one and thus, will help us to overcome the difficulty of inadequate. The data distribution of IDF (Intensity-Duration-Frequency) for short duration of rainfall intensity can be estimated from daily rainfall. For each year the $E[(I_d)^q]$ estimated, is calculated for fixed values of q (q=0, 1, 2, 3, 4 and 5). Then, for each q the average is calculated over the length of recorded data. The scaling properties of average singular measures are tested using Equation (8). By fitting with a line of $E[(I_d)^q]$ versus durations in log-log plot, we obtain the value of the K (q) function for fixed q.

3. RESULTS AND DISCUSSION

The temporal scale invariance of rainfall was investigated. Through the temporal analysis, information was obtained about the precipitation process. For all thirty stations, the variation of empirical moments with scale in time was obtained. And the straight-line behavior with the R^2 was around 0.98 (Table 2 and 3). Figures 2 and 3 illustrate the log-log plots of the moment versus duration for two stations: Beterou and Tebou of OHHVO among the thirty studied stations. The same behavior was obtained in all studied stations.

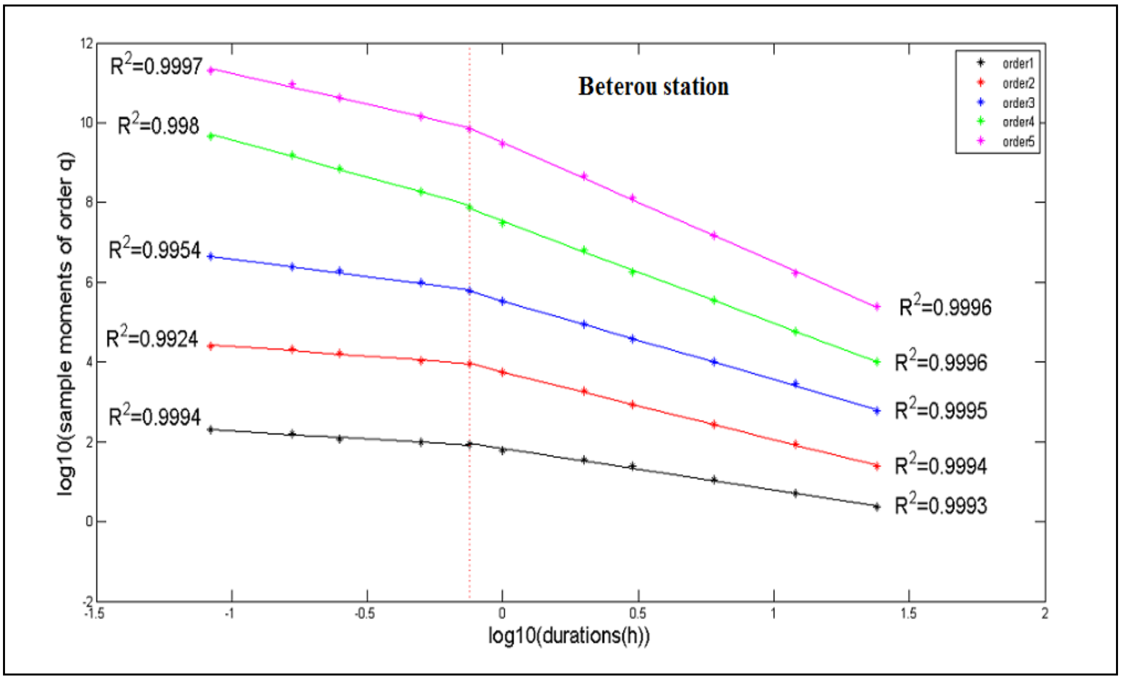

Figure 2: Sample moments of order q versus durations, for sub-daily annual maximum rainfall intensities recorded at Beterou. (5≤ d ≤45min and 45min ≤ d ≤ 24 h)

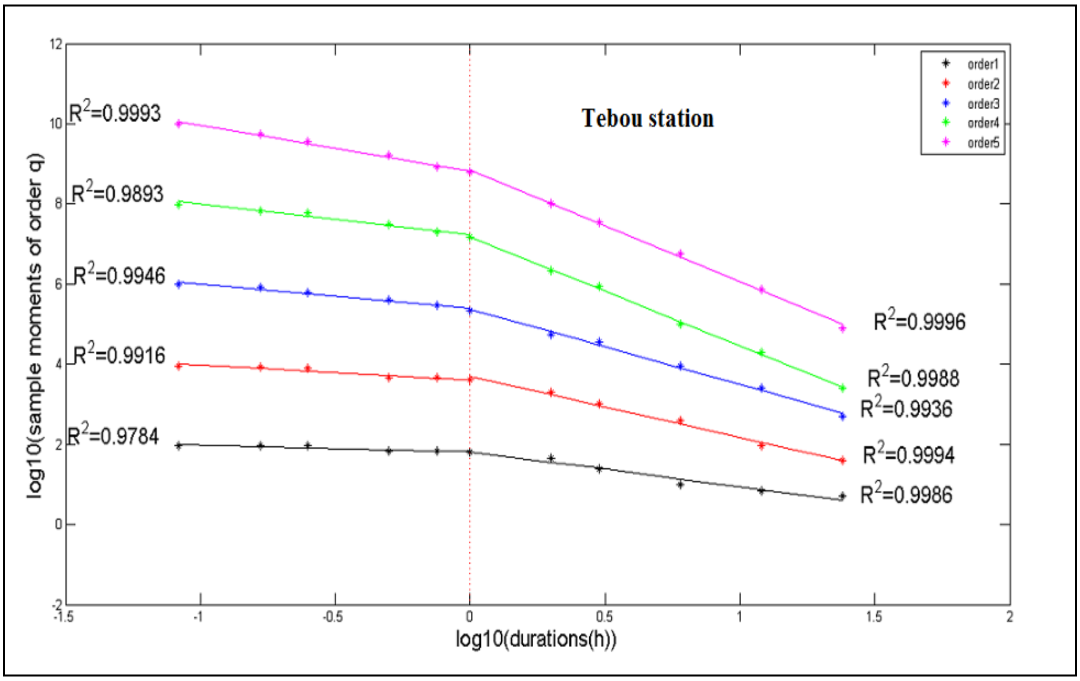

Figure 3: Sample moments of order q versus durations, for sub-daily annual maximum rainfall intensities recorded at Tebou. (5≤ d ≤1hour and 1hour ≤ d ≤ 24 h)

Observing the log-log plots of the moment versus duration in the thirty studied stations, we note that these stations can be classified in two groups. The first group involves: Koko, Dogue, Beterou, Wewe, Sarmanga, Pelebina, Momongou and Parakou. The relationships between the moments and durations in these stations are linear with two different gradients: the first regime is from 5min to 45min, and the second regime is form 45min to 1 day. The second group includes the other study station. The relationships between the moments and the durations are linear for two differents regimes: the first one is from 5min to 1 hour, and the second form 1 hour to 1 day. These results indicate that two different scaling regimes exist for AMMA-CATCH Benin observatory's rainfall. The change in gradient occurs at around 1hour (Figure 3) for the second group of stations. This number of groups had been observed previously for Canada rainfall [17], for Brazil rainfall [2] and for Japan rainfall [1]. In our study, for the first group of stations, the change in gradient occured at around 45min as observed for Korea rainfall [18]. The two break points (45min and 1hour) observed in this study are not in agreement with those found for Tunisia rainfall in northern Africa [3]. The authors showed the change in gradient at around 30 min. The break points could imply a transition in the rainfall dynamic from a steep slope (for short duration rainfall) to a milder slope (for long duration rainfalls) because of high variability of rainfall intensities in convective cells. However, there is no physical basis to the rainfall generating mechanism or the rainfall may have an artificial break related to the resolution of the measuring device at rain gauge stations [2]

To evaluate the types of scaling (simple or multiple), the relationships between the scaling exponents K (q) and the order of moment q were established for each section of the scaling regimes. The plots of K (q) versus q of every scaling relationship were made in ordinary scale for all study stations. Figures 4 and 5 show the relationships between the scaling exponents of the moments and the orders of the moments at two stations (Beterou and Tebou) of OHHVO among the thirty study stations.

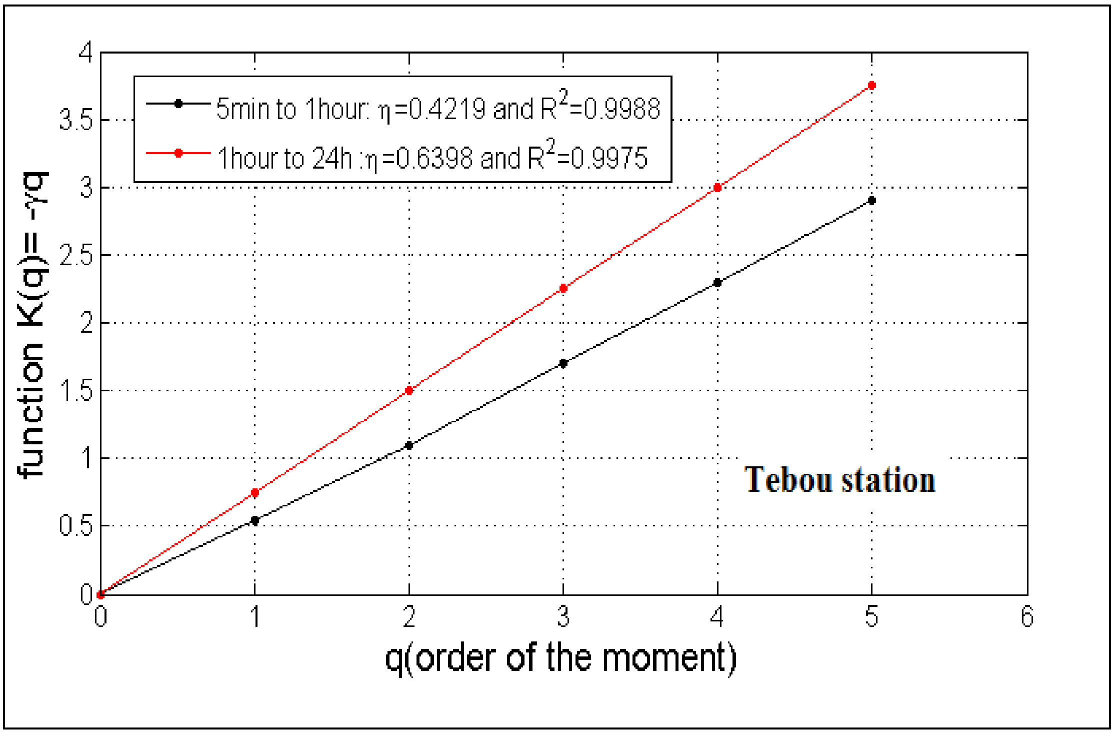

Figure 4: Relationship between K(q) and order of moment q for Tebou

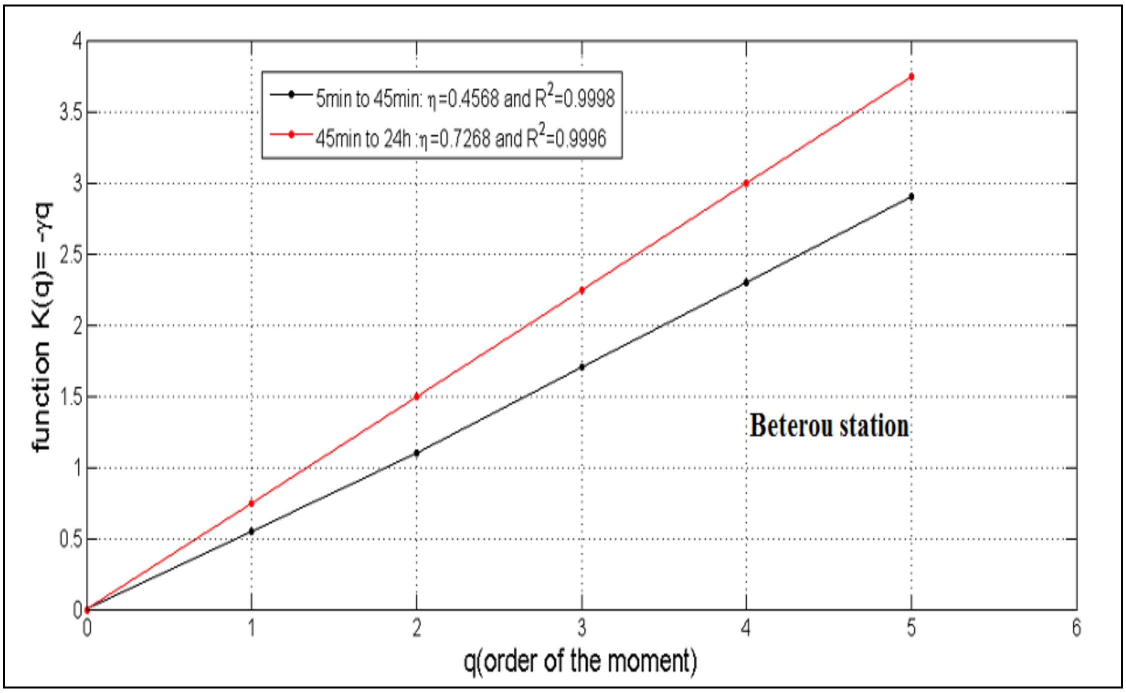

Figure 5: Relationship between K (q) and order of moment q for Beterou station

These plots and those of all the thirty studied stations showed that the relationships between K (q) and q were strongly linear as indicated by the very high values of the coefficient of determination (R^2) for the fitted linear regression lines. The values of the scaling exponents for each interval were computed and shown in the tables 2 and 3. Table 3 and table2 showed respectively the values of the scaling exponents for the second station group and those of the first station group. The values R^2 were greater than 0.98. The perfect linearity of the scaling exponents plotted against the order of the moments from data for these stations, strongly supports the simple scaling properties of the five moments versus durations. It is obvious that a linear relationship exists between scaling exponents and the moment orders. These two properties imply that the property of simple scaling in the wide sense will exist in

the stations if rainfall of all durations is included. This important property enables the derivation of the moments of the AMRI for a durations d from the AMRI for other durations D for which AMRI are available. Such a derivation is feasible using a suitable scaling regime and its scaling exponents. These results are according with those obtained for Canada rainfall [17], for Brazil rainfall [2], for Japan rainfall [1], for Korea rainfall [18] and by for Tunisia [3].

Table 2: The scaling exponent factors for the first group of stations

N0 (station number)	**Name of station**	**5min to 45min**		**45min to 24hours**	
		H	R^2	**H**	R^2
9	Koko	0.4576	0.9961	0.689	1
26	Dogue	0.4012	0.9982	0.7272	0.9992
2	Beterou	0.4568	0.9998	0.7268	0.9996
16	Wewe	0.4119	0.9995	0.6441	0.9999
13	Sarmanga	0.3887	0.9992	0.7151	0.9976
12	Pelebina	0.3353	0.9983	0.5359	0.9987
10	Momongou	0.4338	0.9999	0.6598	0.9979
11	Parakou	0.4228	1	0.7025	0.999

Table 3: The scaling exponent factors for the second group of stations

N0 (station number)	**Name of station**	**5min to 1hour**		**1hour to 24hours**	
		H	R^2	**H**	R^2
1	Affon	0.3613	1	0.7545	1
2	Bembereke	0.3492	0.9941	0.7352	0.9986
4	Birni	0.356	0.9952	0.4922	1
5	Copargo	0.5209	0.9995	0.8239	1
6	Djougou	0.5869	1	0.7929	1
7	Donga	0.5521	0.9969	0.7458	0.9993
8	Fo-Boure	0.4829	0.9978	0.6259	0.9998
14	Sonoumon	0.3820	0.9928	0.7269	0.9990
15	Tobre	0.4029	1	0.7628	0.9985
17	Adiangdia	0.4283	0.9950	0.5689	0.9968
18	Akekerou	0.3952	0.9998	0.8254	0.9956
19	Ananinga	0.4064	0.9969	0.7919	1
20	Angaradebou	0.4458	0.9929	0.8059	1
21	Bari	0.5235	1	0.7069	0.9872
22	Biro	0.4829	1	0.5642	0.9921
23	Bombone	0.5521	1	0.7239	0.9995
24	Bori	0.3670	0.9995	0.6358	0.9999
25	Dapelefoun	0.4458	0.9997	0.5739	1
27	Gaouga-Gou	0.3658	1	0.6662	0.9956
28	Gori-Bouyero	0.3489	0.9990	0.7635	1
29	Ina-Ceta	0.5369	0.9994	0.8329	1
30	Tebou	0.4219	0.9988	0.6398	0.9975

Conclusion

In this paper, the time scale invariance properties of statistical moments of the Annual Maximum Rainfall Intensity (AMRI) rainfalls were investigated in AMMA-CATCH Benin observatory (Northern Benin). The main results of study can be summarized as follows. The analysis of the relationships between the moments and the durations showed that these relationships can be described by power-form functions, indicating the scaling behavior of these moments with durations. It's shown that the properties of rainfall follow the hypothesis of simple scaling process. Furthermore, the scaling properties of the rainfall in time series are simple scaling and composed of two different regimes for two distinct intervals. The first interval is from 5min to 45min and from 45min to 1 day durations at Koko, Dogue, Beterou, Wewe, Sarmanga, Pelebina, Momongou and Parakou. The second interval is from 5min to 1hour and from 1hour to 1 day durations for the other study stations. Results of this study are of significant practical importance in AMMA-CATCH Benin observatory (Northern Benin) because statistical rainfall inferences can be made with the use of a simple scaling in time. Furthermore, daily data are more widely available from standard rain gauge measurements, but data for shorter durations are often not available for the required site. Results of this study are also of significant practical importance because statistical rainfall inferences can be made from a higher aggregation model (i.e. observed daily data) to a finer resolution model (i.e. less than 1 h or 45min that might not have been observed). The findings from this study can be utilized in our future study to estimate the sub hourly IDF.

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