MACORVIAN CHARACTERISTICS OF THE NIGERIAN STOCK MARKET

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ABSTRACT

Stock Market prediction has been one of the active research areas that have enjoyed attention in the fields of actuarial science and quantitative finance. This article investigates the Markovian characteristics of the Nigeria Stock Market using weekly data on All Share Index (ASI) market, 30-Index and five sub-sectors of Nigerian stock exchange from October 4, 2013 to September 30, 2016. The Chapman-Kolmogorov’s principles of handling transition probabilities and limiting distributions methods were employed for predicting future market behaviour. The findings suggest that compounded returns of the indices for the sectors and the market are highly volatile. The long-run distribution forecasts established that the market converged to stationarity after six weeks, while industrial sector has the shortest stationarity step period of five weeks. It is also observed that it will take about 31 weeks for the market and the 30-Index to reach the best return state, while about 78 weeks period is required to revert to the worst performing state. Further analysis findings of the mean return time suggest that it will take only about two weeks period for the indices returns of the market and the sectors under study to transit to the average state irrespective of the current state. Generally, the findings established the volatile nature of the market and its rapid tendency for deterioration. Finally, it is important to note that the 30-Index and ASI exhibit similar Markovian characteristics. It is pertinent to ensure strict compliance of the 30-Index stocks to the regulatory risk management frameworks for the robustness and sustainability of the market.

Key Words: Markov Process, Nigeria Stock Market, All Share Index, limiting distribution, Mean return and First Passage time, Prediction

INTRODUCTION

Stock Market prediction has been one of the active research areas that have enjoyed attention of researchers given the evident interest of many corporations. This market is a network that provides platform several major economic transactions in the world at a dynamic rate based on market equilibrium. The stock markets in the recent past have become an integral part of the global economy. Any fluctuation in this market influences our personal and corporate financial lives, and the economic health of a country. The stock market has always been one of the most popular investments due to its
high returns (Kuo, Lee & Lee, 1996; Hassan & Nath, 2005). However, there is always some risk to investment in the Stock market due to its unpredictable behaviour. So, an ‘intelligent’ prediction model for stock market forecasting would be highly desirable and would of wider interest. Reliable prediction of stock prices could offer enormous profit opportunities in reward and proactive risk management decisions. This quest has prompted researchers, in both industry and academia to find a way past the problems like volatility, seasonality and dependence on time, economies and rest of the market. Recently, techniques of Artificial Intelligence and Machine Learning - like Artificial Neural Networks, Fuzzy Logic and Support Vector Machines, have been widely used to solve these problems (Gupta & Dhingra, 2012).

The use of Markov processes in finance and economics is not new. Hamilton (1990) applies Goldfeld and Quandt’s (1973) Markov switching regression in characterizing growth dynamics within an autoregressive process, and observes that an economy switches between two distinct phases of fast and slow growths in a manner governed by the outcome of a Markov process. Nefti (1984) applies a second-order Markov process to US employment data and finds that the US economy transits between two states (rising and falling states) with respect to unemployment rates. Kim and Nelson (1998) and Kim, et al. (1998) also apply regime-switching models to stock returns from the US data. Chu, Santoni and Liu (1996) adopt a two-step approach to underpin stock return behaviour. First, they model stock return as a Markov switching process, and then estimate a volatility equation, given different return regimes derived in the first stage. Several theoretical and empirical studies were carried out in an effort to model and predict the market volatility patterns (Ibiwoye & Adeleke, 2008; Hamadu & Ibiwoye, 2010; Hamadu, 2014a; Hamadu, 2014b; Fama, 1965). Earlier, Kendal (1953) suggested that stock prices follow a simple random and changes in price are independent as well as the gains and losses. This study examines the Markovian characteristics of the Nigerian equity market. Markov processes are used to model systems with limited memory. They are used in many areas including actuarial science, financial engineering and modelling, resource management, communication systems, transportation networks and decision systems amongst others. The short and long terms behaviours of the market returns will be investigated. Also, the mean first return time and the mean first passage time will be computed as a consequence of the limiting distributions. For comparative analysis of daily behaviour of stock returns of developed markets, see (Ko & Lee, 1991; Li, 2006). The remainder of the article is organised as follows: Section 2 provides the overview of the Nigerian capital market, data and methods are given in Section 3, analysis of results and discussion are presented in Section 4, while Section 5 concludes the article.

OVERVIEW OF THE NIGERIAN CAPITAL MARKET

The Nigerian Capital Market is a channel of mobilizing long-term funds by providing mechanism for private and public savings through financial instruments (equities, debentures, bonds and stocks) with major components consisting of the Security and Exchange Commission (SEC) and the Nigerian Stock Exchange (NSE). Founded in 1960, the NSE is the second largest market in sub-Saharan Africa with fully automated exchange that provides the listing and trading services as well as electronic Clearing, Settlement and Delivery (CSD) services through Central Securities Clearing System (CSCS). The exchange keeps on evolving as a competitive market and meeting the needs of investors. It operates fair, orderly and transparent markets with over 200 listed equities and 258 listed securities, and had attracted the best of African enterprises as well as the local and global investors (NSE, 2013). The market has become an integral part of the global economy such that any shock in the market has contagious consequences. Moreover, the Nigeria’s capital market has enjoyed a decade of unprecedented growth. The market capitalization increased by over 90.0% from 2003 to 2008. However, from a peak in March 2008, the market capitalization went declined spirally by about 46% in 2009 (SEC Report, 2009). The convergence of global economy makes all countries and all markets sensible to the happenings in other countries (the contagious effect). The 2008 global financial
meltdown originated from the United States of America (USA) had varying degree of impacts on different capital markets in various countries. This situation is compounded with the continuous volatility in the global oil price which in theory adversely and significantly affecting capital markets (Njiforti, 2015; Asaolu & Ilo, 2016). Nigeria recently experienced economic recession as a consequence of the 2014-2016 global oil price downturn. In view of these, the various SEC reports came with several recommendations to reposition the Market as a world class institution. The main recommendations are; the development of an enforcement framework to prevent market manipulation, and the establishment of principles for risk management for capital market operators.

DATA AND METHODS

Data

The data for the study are weekly closing indices for the Nigerian stock market. The indices are: all share Index (NSE ASI), the selected drivers of the market volume (NSE 30); the NSE Banking, the NSE Insurance, the NSE Consumer Goods, NSE Oil/Gas, and NSE Industrial. The period covered from 4th October, 2013 to 30th September, 2016. Details of the data can be accessed from https://www.cscsnigeriaplc.com/home/dailypricelist1. To obtain the appropriate discretized classes for the market and other sub-sector indices, the computed compounded returns were partitioned in line with Tukey’s five-number summaries (Mosteller& Tukey, 1977) to derive the 5-state Markov process. The states are denoted from A to E, representing the worst and best returns, with state C being the average return.

Methods

This study uses the Markovian characteristics of the stochastic processes of the market and stocks returns. The methodology adopted the Chapman Kolmogorov’s principles of handling transition matrices and limiting distributions. The returns are assumed to be a first-order Markov process \( X(t) \) for any \( t_0 < t_1 < \ldots < t_n \) the conditional cumulative distribution function of \( X(t_n) \) for a sample values of \( X(t_0), X(t_1), \ldots, X(t_{n-1}) \) depends only on \( X(t_{n-1}) \). Which can be written as:

\[
P[X(t_n) \leq x_n | X(t_{n-1}) = x_{n-1}, X(t_{n-2}) = x_{n-2}, \ldots, X(t_0) = x_0] = P[X(t_n) \leq x_n | X(t_{n-1}) = x_{n-1}]
\]

(1)

Thus, given the present state of the process, the future state is independent of the past. In the case of the strong Markov property, for all \( t \), the process \( \{X(t+s) - X(t) | s \geq 0\} \) has the same distribution as the process \( \{X(s) | s \geq 0\} \) and is independent of \( \{X(s) | 0 \leq s \leq t\} \). Thus, when the state of process is known at time \( t \), the governing probability of the future change of state of the process will be determined as if the process started at time \( t \), independently of the history of the process up to time \( t \) (Ibe, 2009), while the time \( t \) is a constant. The strong Markov property allows for the replacement of the fixed time \( t \) with a nonconstant random time (losifescu, 1980; Norris, 1997; and Stirzaker, 2005).

Discrete – Time Markov Chains

The present study considers the discretized –time process \( \{X_t, t = 0, 1, 2, \ldots\} \) for all \( i, j, n, m \) given

\[
P[X_t = j | X_{t-1} = i, X_{t-2} = n, \ldots, X_0 = m] = P[X_t = j | X_{t-1} = i] = P_{ij}
\]

where \( P_{ij} \) is the state transition probability, which is the conditional probability that the process will be in a state \( j \) at time \( t \) immediately after the next transition, given that it is in a state \( i \) at time \( t - 1 \). The Markov chain that strictly follows this rule is a non-homogeneous Markov chain. However, our study considers homogeneous Markov chains in which

\[
P_{ij} = P_{ji}
\]

implying that
\[ P[X_t = j|X_{t-1} = i, X_{t-2} = n, \ldots, X_0 = m] = P[X_t = j|X_{t-1} = i] = P_{ij} \] (2)

where 0 \leq P_{ij} \leq 1, and \( \sum_j P_{ij} = 1, i = 1, 2, \ldots, n \).

The \( P_{ij} \), are usually displayed as a square matrix \( P \), where \( P_{ij} \) is the entry in the \( i \)th row and \( j \)th column:

\[
P = \begin{bmatrix}
P_{11} & P_{12} & \cdots & P_{1n} \\
P_{21} & P_{22} & \cdots & P_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
P_{n1} & P_{n2} & \cdots & P_{nn}
\end{bmatrix}
\]

**The Limiting distribution**

Let \( p_{ij}(n) \) denote the conditional probability that the process will be in a state \( j \) after exactly \( n \) steps, given that the current state is \( i \). That is,

\[
p_{ij}(0) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}
\]

\( p_{ij}(1) = p_{ij} \)

Given the Chapman–Kolmogorov equations, which provide a generalization of the \( n \)–Step transition Probability, if we define \( P[X(0) = i] \) as the probability that it is in the state before it makes the first transitions, then the set \( \{P[X(0) = i]\} \) defines the initial condition for the process, and for an \( n \)–state process,

\[
\sum_{i=1}^{N} P[X(0) = i] = 1
\]

Let \( P[X(n) = j] \) denote the probability that it is in state \( j \) at the end of the first \( n \) transitions, then for the \( n \)– state process,

\[
P[X(n) = j] = \sum_{i=1}^{N} P[X(0) = i] p_{ij}(n)
\] (3)

For the class of Markov chains, it can be shown that as \( n \to \infty \) the \( n \)–step transition probability \( p_{ij}(n) \) does not depend on \( i \), which means that \( P[X(n) = j] \) approaches a constant as \( n \to \infty \) for the class of Markov chains. That is, the constant is independent of the initial conditions. Thus, for the class of Markov chains in which the limit exists, we define the limiting – state probabilities as follows:

\[
\lim_{n \to \infty} P[X(n) = j] = \pi_j, j = 1, 2, \ldots, n
\]

Since the \( n \)– step transition probability can be written as

\[
p_{ij}(n) = \sum_k p_{ik}(n-1) p_{kj}
\]

Then, if the limiting – state probabilities exist and do not depend on the initial state, we have that

\[
\lim_{n \to \infty} p_{ij}(n) = \pi_j = \lim_{n \to \infty} \sum_k p_{ik}(n-1) p_{kj}
\]

\[
\sum_k \pi_k p_{kj}
\] (4)

If we define the limiting – state probability vector \( \pi = [\pi_1, \pi_2, \ldots, \pi_N] \), then we have that
\pi_j = \sum_k \pi_k p_{kj} \\
\pi = \pi P \\
1 = \sum_j \pi_j

**First Passage and Recurrence Times**

The probability of first passage time from state \(i\) to state \(j\) in \(n\) transitions, \(g_{ij}(n)\), is the conditional probability that given the process is presently in state \(i\), the first time it will reach state \(j\) occurs in exactly \(n\) transitions. The probability of first passage from state \(j\), is \(g_{ij} = \sum_{n=1}^{\infty} g_{ij}(n)\)

Thus, \(g_{ij}\) is the conditional probability that the process will ever enter state \(j\), given that it was initially in state \(i\). Obviously \(g_{ij}(1) = p_{ij}\) and a recursive method of computing \(g_{ij}(n)\) is

\[ g_{ij}(n) = \sum_{l \neq j} p_{il} g_{lj}(n-1) \]

When \(i = j\), the first passage time becomes the *recurrence time* for state \(i\). The relationship between the \(n\)–step transition probability \(p_{ij}(n)\) and the first passage time probability \(g_{ij}(n)\) can be expressed as:

\[ p_{ij}(n) = \sum_{m=1}^{n} g_{ij}(m)p_{jj}(n-m) \quad n = 1,2, \ldots \]

\[ = \sum_{m=1}^{n} g_{ij}(m)p_{jj}(n-m) + g_{ij}(n) \quad n = 2,3, \ldots \]

This follows from the fact that \(p_{jj}(0) = 1\), leading to

\[ g_{ij}(n) = \begin{cases} 
0 & \text{if } n = 0 \\
p_{ij}(n) & \text{if } n = 1 \\
p_{ij}(n) - \sum_{m=1}^{n-1} g_{ij}(m)p_{jj}(n-m) & \text{if } n = 2,3, \ldots 
\end{cases} \]

If we define \(\mu_{ij}\) as the mean first passage time from state \(i\) to state \(j\), then it can be shown that (Taha, 2007)

\[ \mu_{ij} = \sum_{n=1}^{\infty} n g_{ij}(n) = 1 + \sum_{k \neq j} p_{ik} \mu_{kj} \]

From this equation, the mean first passage time from state \(i\) to \(j\) is the holding time in state \(i\) plus the mean first passage time from state \(k\) to state \(j\), \(k \neq j\), given that the next state the process visits when it leaves state \(i\) is state \(k\). Similarly, the mean recurrence time is given by

\[ \mu_{ii} = \sum_{n=1}^{\infty} n g_{ii}(n) = 1 + \sum_{k \neq i} p_{ik} \mu_{ki} \]

**RESULTS AND DISCUSSION**

The analyses of results are presented in Tables 1 – 3. Table 1 gives the descriptive statistics of the ASI and sectoral stocks indices returns, the transition matrices, limiting distributions and first passage times are presented in Table 2, while the mean returns times are detailed in Table 3. Graphical representation of the volatility patterns of the indices returns are displayed in Figure 1.
Table 1: Descriptive Statistics of Returns

<table>
<thead>
<tr>
<th>Statistics</th>
<th>ASI</th>
<th>30 INDEX</th>
<th>BANKING</th>
<th>INSURANCE</th>
<th>GOODS</th>
<th>OIL&amp;GAS</th>
<th>INDUSTRIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>0.1690</td>
<td>0.1791</td>
<td>0.2397</td>
<td>0.0651</td>
<td>0.1514</td>
<td>0.1642</td>
<td>0.1423</td>
</tr>
<tr>
<td>(Quartile)75%</td>
<td>0.0125</td>
<td>0.0125</td>
<td>0.0211</td>
<td>0.0127</td>
<td>0.0133</td>
<td>0.0304</td>
<td>0.0162</td>
</tr>
<tr>
<td>(Median)50%</td>
<td>0.0000</td>
<td>-0.0002</td>
<td>-0.0019</td>
<td>-0.0001</td>
<td>-0.0015</td>
<td>0.0039</td>
<td>-0.0006</td>
</tr>
<tr>
<td>(Quartile)25%</td>
<td>-0.0149</td>
<td>-0.0164</td>
<td>-0.0262</td>
<td>-0.0123</td>
<td>-0.0195</td>
<td>-0.0315</td>
<td>-0.0162</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.1303</td>
<td>-0.1352</td>
<td>-0.1867</td>
<td>-0.0964</td>
<td>-0.1265</td>
<td>-0.1472</td>
<td>-0.1988</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0011</td>
<td>-0.0013</td>
<td>-0.0010</td>
<td>-0.0001</td>
<td>-0.0014</td>
<td>0.0051</td>
<td>-0.0004</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0355</td>
<td>0.0371</td>
<td>0.0506</td>
<td>0.0214</td>
<td>0.0367</td>
<td>0.0519</td>
<td>0.0393</td>
</tr>
<tr>
<td>N</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.3445</td>
<td>0.2807</td>
<td>0.5148</td>
<td>-0.3806</td>
<td>0.0890</td>
<td>0.5159</td>
<td>-0.6477</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>6.2183</td>
<td>5.6672</td>
<td>3.9753</td>
<td>2.3679</td>
<td>3.3087</td>
<td>1.0466</td>
<td>7.1664</td>
</tr>
<tr>
<td>CV</td>
<td>-3313.4010</td>
<td>-2757.5900</td>
<td>-4859.8350</td>
<td>-15913.8900</td>
<td>-2568.0000</td>
<td>1015.5692</td>
<td>-10091.6500</td>
</tr>
</tbody>
</table>

The descriptive of the compounded returns is displayed in Table 1. It is observed that only oil & gas sector have a positive mean return, while the means return of the market, 30 Index and the four other sectors are negative for the period under study. Also, the coefficients of variation (CV) are in thousands indicating the volatile and stochastic nature of the market and various sectors. This can easily be observed from very noisy patterns exhibited in Figure 1. The coefficients of skewness are quite moderate with only insurance and industrial sectors being negative. With a kurtosis of 6.2183, the market is highly peaked. The same observation can be made for the 30 index and the industrial sector, whereas, the oil & gas is having a kurtosis of 1.0466. In general, the oil & gas sector is more stable in price returns. As expected, all the minimum returns are negative, while the maximums are positive as evidenced in Figure 1. It is worthy of note that in general, the 30-Index returns exhibit similar descriptive patterns as the main ASI.

Figure 1
Table 2: Transition Matrices, Limiting Distributions and First Passage Times for Returns

<table>
<thead>
<tr>
<th>Sub-sector</th>
<th>Transition Matrices</th>
<th>Limiting Distributions</th>
<th>First Passage Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALL SHARE INDEX</td>
<td>$P_{ij}$</td>
<td>$\mu_{ij}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 INDEX</td>
<td>$P_{ij}$</td>
<td>$\mu_{ij}$</td>
<td></td>
</tr>
<tr>
<td>BANKING SUB-SECTOR</td>
<td>$P_{ij}$</td>
<td>$\mu_{ij}$</td>
<td></td>
</tr>
<tr>
<td>INSURANCE SUB-SECTOR</td>
<td>$P_{ij}$</td>
<td>$\mu_{ij}$</td>
<td></td>
</tr>
</tbody>
</table>

Where $P_{ij}$ represents the transition matrix, $\mu_{ij}$ represents the limiting distribution, and $\mu_{ij}$ represents the first passage times.
Table 2 presents the transition matrices, stationarity probabilities and first passage time probabilities of the market, 30 index and five other sectors of the stock exchange. The limiting distribution of the market was achieved after six steps; that of the 30 index, Banking and Oil & Gas were attained after 9 steps each. On the otherhand, the stationarity for Insurance, Consumer Goods and Industrial sectors were reached after seven, ten and five weeks respectively. The mean first passage time, \( \mu_{ij} \), gives the expected number of weeks needed to reach state \( j \) from state \( i \) for the first time. From the results of All Share Index, it takes 29.3 weeks for the market return to transit from A to B for the first time, from A to C, it takes 30.7 weeks. While the first passage time from A to D and E respectively are 31.1 and 31.0 weeks respectively. However, it takes over 76 weeks to transit from E to any of the other states. Results for 30 Index and other sectors are also displayed in the same table.

Table 3: Mean Return Time

<table>
<thead>
<tr>
<th>Industry</th>
<th>Transition States</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>ASI</td>
<td>31.27714</td>
</tr>
<tr>
<td>30 Index</td>
<td>31.1999</td>
</tr>
<tr>
<td>Banking</td>
<td>51.83323</td>
</tr>
<tr>
<td>Insurance</td>
<td>156.108</td>
</tr>
<tr>
<td>Oil &amp; Gas</td>
<td>38.81489</td>
</tr>
<tr>
<td>Industrial</td>
<td>77.08231</td>
</tr>
</tbody>
</table>

The mean return time results are displayed in Table 3. The mean first return time or the mean recurrence time is the expected number of transitions before the system return to a given state for the first time. For the All Share Index, depending on the current state of the stock, it will take approximately 31 weeks for the stock to return to state A, about .5 weeks to return to state B, 1.6 weeks to state C, 19.6 weeks to return to state D, while it will require about 78 weeks to return to its highest state.

CONCLUSION

This study examined the long-run characteristics of the Nigeria Stock Market. Markov processes are stochastic systems with limited memory that have found applications in many areas including actuarial science, financial engineering and modelling, resource management, communication systems, transportation networks and decision systems amongst others. The data used in the study are weekly closing stock indices for the market, 30-Index and five sectors listed on the Nigeria Stock Exchange. Analyses of the descriptive statistics and the Markovian predictions provided useful results. In general, the coefficient of variation of all the indices are far above the threshold which is an indication that the sectors and the market are highly volatile. The limiting distribution forecasts results established that the market converged to stationarity after 6 weeks, while industrial sector has the shortest stationarity period of five weeks. More so, it takes about 31 weeks for the market and the 30-Index to reach the best return state, while it takes about 78 weeks period to return the worst performing state. Another major finding is that it takes about only 17 and 20 weeks for Insurance sector and Oil &Gas to revert to the worst performing state from the best performing state. It is interesting to note that it takes about two weeks period for the indices returns of the market and the sectors under study to transit to the average state (state C) irrespective of the current state. In addition, the ASI and 30-Index takes about 78 weeks to attain the best return state while it takes shorter period of 31 weeks to revert to the worst return state. This demonstrate the instability and risky nature of the market for the period under study. The tendency of the market to deteriorate is potent and glaring. Finally, it is important to note that the findings established that the 30-Index is the major driver of the market as they exhibited similar Markovian characteristics. These findings will provide informed investment decisions to investors and other market participants. Strict adherence to the regulatory risk management frameworks should be ensured by Security and Exchange Commission to minimize selective information asymmetric and market manipulation. This will ensure the robustness and sustainability of the market.

REFERENCES


