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Full Length Research Article

APPLICATION OF SELF-EXCITING THRESHOLD AUTOREGRESSIVE MODEL ON EXCHANGE RATE IN NIGERIA: A COMPARATIVE APPROACH

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ABSTRACT
Exchange rates and many other financial time series data exhibit structural breaks and volatility. Nonlinearity test and a structural break test were used to detect the nonlinearity and the break date in NGN/EUR. The study revealed that nonlinearity and threshold nonlinearity exist in the exchange rate series. The results showed that the SETAR model can explain abrupt changes in NGN/EUR. The identified structural break date coincides with identifiable economic and political shocks. Given the evidence of structural break in the series, we applied unit root test and find that NGN/EUR is stationary which indicate that the ADF unit root tests are bias towards non-rejection of non-stationarity. In modelling the exchange rates data set, two SETAR models were generated, that is SETAR(2;5,2) model without dummy variable was used as a benchmark, while, dummy variable was added so as to address the identified structural break which generated SETAR(3;5,3). The diagnostic tests revealed that, the SETAR model is adequate for forecasting (i.e. both models are free from serial correlation and heteroscedasticity). The forecast results showed that the SETAR(3;5,3) model with the inclusion of dummy variable performs better than that of the SETAR(2;5,2) model without dummy variable.

Keywords: Self-Exciting Threshold Autoregressive (SETAR) Model, Nonlinearity, Unit Root, Dummy Variable, Exchange Rate

1. INTRODUCTION

It is apparent that financial market series exhibits asymmetries, structural instability, jumps etc. and linear time series models are restricting in capturing these features. Although linear models still takes its lead role in applied research and academic, researches have shown that linear models typically allow certain characteristics of financial and economic datasets inexplicable. According to Teräsvirta (2017) asymmetry, irregularities, jumps etc. has been the main reason for nonlinearity in a data set. For example, economies that rely heavily on export always ensure that, the numerical value of their currencies as a macroeconomic variable is stable against other foreign currencies and this is of great significance particularly if the mutual-dependency among the economies is concerned. In terms of political economy, the targeted level of the currencies has dynamic importance in the economy that have the characteristics of export-driven growth Emrah (2017), hence, the threshold model is a breakthrough in univariate time series.

So, since financial and economic structures are characterized by fluctuations, it is important to note that nonlinear time series methods maybe vital to describe the empirical data especially were the assumption of linearity fails. Presently, threshold principle and threshold time series models are efficaciously applied to numerous real-life problems in economic and finance sector. Moving forward, it seems that the threshold philosophy will continue to make meaningful contributions to financial and economic time series analysis (Cathy et al., 2011). Exchange rate increases more abruptly during recession better than they do during recoveries. These reactions cause asymmetric instabilities and jumps in the structure of the time series dataset, hence nonlinear methods capture these types of dynamics more accurately than linear methods. Most importantly, if the data generating procedure are characterized by structural instability and are not captured in the unit root test description or specification, the result obtained afterward maybe biased in non-rejection of the non-stationary hypothesis (Perron, 2018).

The implication of such a result indicate that any shock whether policy change/inducement, exchange rate volatility, economic recession, demand or supply will certainly have negative impact to the variable in the long run. In-turn, these features may affect precision in forecasting. Based on these principles, nonlinear models have grown massive consideration over three decades. One specific model that often appears in economic literature is the threshold autoregressive (TAR) model introduced by (Tong, 1990). The threshold autoregressive (TAR) model uses a threshold value relative to a state-determining variable (S_t) to determine regime, note that, in various economic settings the empirical presence of a threshold appears to be vital. TAR model has been a valuable instrument in nonlinear time series modelling (Tong, 1990). For instance, Chan (2009) provides a worthwhile method for researchers who are fascinated with the principle and practice of nonlinear time series analysis, while Li (2009) investigates the threshold methods in volatility modelling. The idea of nonlinearity is also extended to the threshold unit-root test as well as the threshold co-integration (Kapetanios *et al.,* 2003). Another distinctive case of the TAR model is the switching autoregressive model, as originally proposed in Tong and Lim (1980), which was later formalized by Hamilton (1989), and then applied by McCulloch and Tsay (1994). This model uses a random latent (unobservable) indicator as the threshold variable. The mixture autoregressive models of Wong and Li (2000a) and Wong *et al.,* (2009) also fall into this category. Through the threshold method, it is possible to approximate nonlinearity by employing the piecewise linear system which is formed by a regime dependent linear model (i.e. TAR models uses the threshold space to extend linear estimation in the dataset). There are several parametric nonlinear time series methods within the

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threshold principle. In this study, the piecewise class linear threshold autoregressive (TAR) models (i.e. Self-exciting Threshold Autoregressive model) are of interest. One significant characteristic of the SETAR model is that the state-determining variable is generated endogenously. According to Emrah (2017) the SETAR model, which is one of the TAR Group modeling, shows a better performance than many other linear and non-linear modeling in a study conducted on EUR/USD, EUR/TRY and USD/TRY Parities.

Generally, in several fields of research linear time series models are widely used; especially, given that many climates, economic and financial data analyst assume that virtually all time series are linear. Nonlinear models are not influenced by structural breaks or dynamic behavior of time series data, that is, nonlinear time series models can adequately accommodate either structural instability or regimes than the class of linear models. Some of the features or properties of nonlinear models include stable and unstable limit cycles, asymmetries, time irreversibility, bimodality, volatility clustering, and so on. In other, to capture and describe these features, several nonlinear time series models were designed and adopted as a generalization of linear structures. Unit root and structural break are narrowly linked and researchers ought to know that conservative unit root tests are influenced towards a false unit root null when the data are trend stationary with structural break (Perron, 2018). Vogelsang and Perron (1998) states that, if the unit root tests estimates provide evidence that either additive or innovative outliers is significant in the time series then, the results obtained from Augmented Dickey Fuller (ADF) unit tests or any other conventional unit roots test are uncertain, by implication the model excluding structural breaks is mis-specified. Also, addressing structural breaks have been a major concern in applying nonlinear models, as structural break points or jumps can be termed as outliers and omitting these points can further reduce the degree of freedom and sample points; noting that, dummy variables are a common technique of resolving structural breaks since it doesn't reduce the degree of freedom nor omit the outlier. However, dummy variables are vital policy action components on models which are used to interpret qualitative effects on economy. It is on this basis, this study empirically examine this arguments and facts against the predictive power of SETAR model with dummy variable over SETAR without dummy variable as well as establishing the fact that, in the wake of structural instability on exchange rates of the Naira (NGN) to European Euro (EUR) in the event that nonlinearity is established in time series dataset conventional unit root tests might be misleading. The following statistical principle will be carried out; descriptive statistics will be employed to evaluate the minimum and maximum exchange rates of NGN/EUR, also to ascertain the data skewness, nonlinear tests will be performed to investigate whether the series is nonlinear using the following methods; BDS, Tsay quadratic, keenan's one-degree and Likelihood ratio tests. Furthermore, parameter estimation for the respective model will be conducted utilizing the nonlinear least squares method. Furthermore, two model diagnostic test will be performed to ensure that the models are free from positive serial correlation and heteroscedasticity using the Breusch-Godfrey Serial Correlation LM Test and Autoregressive Conditional Heteroscedasticity LM Test, respectively and finally, a one-step head forecast is carried-out using the best model.

2.0 MATERIALS AND METHODS

2.1 Data Source and Description

The time series dataset employed in this study were exchange rates of the Naira to the European Euro (EUR). These exchange rates dataset here mentioned characterized Nigerian foreign reserves. The exchange rates dataset for NGN/EUR are obtained as secondary data from CBN website, www.cbn.gov.ng/rates/exrate.asp. The exchange rates are to be interpreted as the amount of NGN in one EUR. The data comprises of monthly frequency ranging from January, 2004 to March, 2019, with a total of 183 observations. The monthly data set is utilised since structural breaks/shifts/change can be detected more evidently when low observations are given. The postulation of the likelihood of two or more regimes allows the use of the threshold model. Another postulation of the SETAR model is that the variations between regimes happens endogenously and are discrete. In this study, a non-linearity test, breakpoint unit root test and a structural break test are employed to validate the decision to use the threshold model and these proponents are also a motivating factor for adopting the SETAR model to investigate the behavior of exchange rate in Nigeria. For this reason, structural break test and breakpoint unit root test as proposed by Vogelsang and Perron (1998), as well as the Bai and Perron (1998) method was used in obtaining the threshold values, threshold variable or delay parameter, multiple threshold test, estimation of threshold autoregressive coefficient and most importantly, in addressing structural breaks using dummy variable. The Bai and Perron (1998) paper on multiple structural change models give an insight into computation and analysis of threshold model estimation, multiple threshold test, consistency of estimates in-terms of the break dates, and the break dates confidence intervals.

2.2 BDS Test for Nonlinearity

The nonlinearity test developed by Brock, Dechert and Scheinkman is also a reliable and general test for nonlinearity. Initially the test is intended to test for the null hypothesis of independent and identical distribution (*iid*) so as to identify a non-random chaotic process. Using this test, the null hypothesis claims the time series is linearly dependent (i.e. the dataset is *iid*). Employing the BDS test supports scholars to investigate whether the data set under study follows a nonlinear process.

Brock et al. (1996) statistic is define as:

$$\mathrm{U}_{\mathrm{p},\epsilon} = \sqrt{\mathrm{T}}\frac{\aleph_{\mathrm{p},\epsilon} - \aleph_{1,\epsilon}^{\mathrm{p}}}{\mathrm{S}_{\mathrm{p},\epsilon}} \qquad (1)$$

where $\mathrm{S}_{\mathrm{p},\epsilon}$ is the standard deviation of $\sqrt{\mathrm{T}}\ \aleph_{\mathrm{p},\epsilon} - \aleph_{1,\epsilon}^{\mathrm{p}}$ and can be estimated consistently as highlighted by (Brock et al., 1996).

2.3 Keenan's One-Degree Test for Nonlinearity

The Keenan (1985) test examines the quadratic nonlinearity, thereby providing confirmation on threshold nonlinearity. It suggests the linearity test against Second-Level Volterra Expansion.

According to Keenan (1985), testing for nonlinearity requires three steps which are stated below:

(i) Y_t is regress on $\{1, Y_{t-1}, Y_{t-2}, \dots, Y_{t-p}\}$ where P is a determined lag order. Find the fitted values $\widehat{Y_t}$ and projected residuals $\widehat{e_t}$ for $t = p+1, \dots, n$ then the sum of squared residuals is calculated;

(ii) $\hat{Y}_t^2$ is regress on $\{1, Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}\}$ and find the projected residuals $\hat{\epsilon}_t$ for $t = p+1, \ldots, n$; and
(iii) $\hat{e}_t$ is regress on $\hat{\epsilon}_t$ with no intercept for $t = p+1, \ldots, n$. Regressing $\hat{e}_t$ on $\hat{\epsilon}_t$ will give raise to $\hat{\eta} = \hat{\eta}_o\sqrt{\sum_{t=p+1}^{n} \hat{\epsilon}_t^2}$ $\hat{\eta}_o$ is the regression coefficient.

Thus, Keenan's (1985) test statistic:

$$\hat{F} = \frac{\hat{\eta}^2(n-2p-2)}{\hat{e}_t^2 - \hat{\eta}^2} \quad (2)$$

2.4 Tsay's Test for Nonlinearity

Based on Keenan's test, Tsay's (1986) extended Keenan's work to be more robust so as to accommodate more wide-range nonlinear terms.

The procedure for conducting Tsay (1986) test is stated below:
(i) Regress Y_t on X_t in a linear form, $Y_t = X_t\theta + e_t$;
(ii) Compose the vector P_t where, $P_t = Vech(X_tX_t)$. Since $Vech(.)$ denotes the half-stacking vector operator P_t consists of a $p(p+1)/2$ elements vector containing the elements of lower triangular part of X_tX_t. Therefore, for each observation of Y_t, there corresponds a vector P_t whose elements are the unique cross products of the last *p* observations of *y*, in other words, $Y_{t-i}Y_{t-j}$ for $i, j = 1,2, \ldots, p$ where $j \geq i$;
(iii) Regress this vector P_t on the explanatory variables $X_t: P_t = Y_t = X_t\lambda + \varepsilon_t$, and save the estimate residuals $\hat{\varepsilon}$; and
(iv) Now, regress the estimated residuals $\hat{e}_t$ on $\hat{\varepsilon}_t: \hat{e}_t = \hat{\varepsilon}_t\delta + V_t$ and save the estimated residuals $\hat{V}_t$.

Test the null hypothesis $H_o: \delta = 0$ using the Tsay (1986) test statistic:

$$\frac{\hat{e}'\hat{\varepsilon}(\hat{\varepsilon}'\hat{\varepsilon})^{-1}(\hat{\varepsilon}'\hat{e})/m}{\hat{V}'\hat{V}/(n-p-m-1)} \quad (3)$$

Where, also, *m* is the number of the coefficients estimated in step 'iii', such that $m = p(p+1)/2$.

2.5 Likelihood Ratio Test for Threshold Nonlinearity

The likelihood ratio test is utilized in testing for threshold nonlinearity helps to handle the weakness of Keenan's and Tsay's quadratic test in detecting threshold nonlinearity (Tong, 1990).

Accordingly, the stated hypothesis is given below:
(i) The null hypothesis: The Time series follows an autoregressive process of order *p*; and
(ii) The alternative hypothesis: The time series follows a two-regime threshold autoregressive model of order *p* with constant variance.

Thus, Tong (1990) likelihood ratio test statistic is given as:

$$T_n = (n-p)log\left\{\frac{\hat{\sigma}^2(H_0)}{\hat{\sigma}^2(H_I)}\right\} \quad (4)$$

Where *n* − *p* represents the sample size, $\hat{\sigma}^2(H_0)$ is the maximum likelihood estimator of the noise variance from the AR(*p*) process and $\hat{\sigma}^2(H_I)$from the TAR process with the threshold examined over some finite interval.

2.6 Augmented Dickey-Fuller (ADF) Test

The ADF test is centered on t-statistic and probability method. The DF tabulated critical values are selected at significance level of 1%, 5% and 10%, respectively. In investigating the presence of unit root, the calculated t-statistic with its corresponding probability values, which is indicated as statistic ADF test, is compared to the critical values. Hence, if the probability value of the test statistic value is greater than the level of significance, the null hypothesis cannot be rejected which implies that the series is non-stationary series.

2.7 Structural Break and Breakpoint Unit Root Test

According to Perron (2018), unit roots and structural change are almost associated, so scholars ought to know that the conventional unit root tests are predisposed toward an erroneous unit root null if the data are trend stationary with a structural break.

This method proposed by Vogelsang and Perron (1998) offers two models these includes; An additive outliers (AO) model, which captures a sudden/rapid change in the mean of a dataset; and an innovational outliers (IO) model, which allows for a gradual shift in the mean of the dataset. In this study, we use the innovative outlier (IO) model, where a dummy for a break in the level is allowed along with a dummy for a break in the trend at an unknown period of time. Under the innovative outlier (IO) model, the break is modelled as evolving more slowly over time on Nigeria average monthly exchange rates of the Naira to EUR.

The IO (Zivot and Andrews, 1992):

$$y_t = \hat{\mu} + \hat{\beta}_t + \hat{\gamma}DT_t(\hat{\lambda}) + \hat{\theta}DU_t(\hat{\lambda}) + \hat{\alpha}y_{t-1} + \sum_{i=1}^{k}\hat{C}_i\Delta y_{t-i} + \hat{e}_t \quad (5)$$

where $DU = 1$ if $t > T_b$, and 0 otherwise
$DU = t - T_b$ if $t > T_b$, and 0 otherwise
T_b = Break date
k = Number of lags

2.8 Non-Linear Model Specification

The self-exciting threshold autoregressive model is a special case of the TAR model where the threshold variable $S_{t-d}$is substituted by the past values of the series y, that is when $S_{t-d} = Y_{t-d}$; *d* is the delay parameter, prompting the deviations between two or more different regimes. In other words, distinct from the TAR model, where the threshold variable is exogenous, the threshold variable of a SETAR model is endogenous. That is, when the discontinuities (regimes) result from internal changes, the relevant model class is the self-exciting threshold autoregressive (SETAR) process. But if the discontinuities are linked to an external process, it defines the class of threshold autoregressive (TAR) models. TAR model assumes that regime which occurs at time*'t'* can be determined by an observable variable S_t proportional to a threshold value. Thus, if S_t is equivalent to the dependent variable, say Y_t, in an autoregressive regression, the model is stated to as self-exciting threshold autoregressive model.

Tong (1992), A two regime SETAR model is presented in Equations (6) below;

SETAR model

$$Y_t = \begin{cases} \emptyset_{10} + \emptyset_{11}Y_{t-1} + \cdots + \emptyset_{1p}Y_{t-p} + \varepsilon_{1t} & if\ Y_{t-d} < \gamma \\ \emptyset_{20} + \emptyset_{21}Y_{t-1} + \cdots + \emptyset_{2p}Y_{t-p} + \varepsilon_{2t} & if\ Y_{t-d} \geq \gamma \end{cases}$$

Where *d* is the delay parameter, influencing the changes between two different regimes and the $\emptyset's$– represent the autoregressive parameters. The delay parameter is a positive integer.

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3.0 RESULTS

3.1 Data Description

This section shows the descriptive statistics of the dataset of Nigeria monthly exchange rate of the Naira to European Euro (i.e. NGN/EURO €1.00) from January, 2004 to March, 2019 as presented in Table 1. Also, Figure 1 shows the histogram of Nigeria monthly exchange rate of the Naira to the European Euro (i.e. NGN/EURO €1.00) from January, 2004 to March, 2019.

Table 1: Descriptive Statistics of Nigerian Monthly Exchange Rate of the NGN/EUR €1.00 from January, 2004 to March, 2019

Variable	Mean	Median	Max	Min	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	P-value
NGN/EUR	219.76	204.30	377.84	144.97	64.85	1.3166	3.376	53.90	0.0000

It is observed from Table 1 that the monthly exchange rate of the Naira per unit of Euro for the period under study ranges from 144.97 to 377.84 with an average of 219.764 and a standard deviation of 64.8537.

Apart from the first moment statistics of the series, Figure 1 shows clearly the histogram and also the descriptive nature of the dataset. The results of other statistics are also evident from Figure 1; the result shows that the standard deviation is very high indicating high level of fluctuations in the series. The statistic for skewness shows that the variable is positively skewed, implying that the distribution has long right tail, this further signify that the series is non-symmetric. While, the kurtosis value obtained for Nigeria monthly exchange rate (NGN/EURO €1.00) is 3.3760 which exceeds 3, this result shows that the normal curve is peaked (i.e. leptokurtic). The Jarque-Bera test statistic obtained for the series is 53.9045 with a p-value of 0.000000, this result indicates that the null hypothesis of normal distribution for Nigeria monthly exchange rate (NGN/EURO €1.00) is rejected, signifying that the time series data set exhibit nonlinearity. By implication, structural break/changes/or shifts occurs in Nigeria monthly exchange rate (NGN/EURO €1.00).

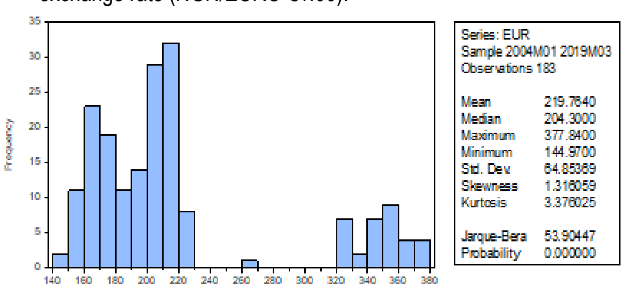

Figure 1: Histogram and Descriptive Statistics of Nigeria Monthly Exchange Rate of the Naira (NGN/EUR €1.00) from January, 2004 to March, 2019

3.2 Nonlinearity Test

Before fitting a SETAR model, it is paramount to investigate the presence of nonlinearity in the dataset first as well as testing for threshold-type nonlinearity. So, in this study four different non-linearity tests were employed namely Brock, Dechert and Scheinkman (BDS) nonlinearity test, Keneen's one-degree test for nonlinearity, Tsay's test for quadratic nonlinearity and Likelihood ratio test for threshold nonlinearity.

Table 2: Brock, Dechert and Scheinkman (BDS) Test for Nonlinearity

Dimension	BDS Statistic	Std. Error	Z-Statistic	Prob
2	0.1960	0.0088	22.3039	0.0000*
3	0.3298	0.0141	23.4215	0.0000*
4	0.4219	0.0169	24.9299	0.0000*
5	0.4857	0.0178	27.2809	0.0000*
6	0.5303	0.0173	30.5905	0.0000*

* denotes the significant of BDS Statistic at each dimension at 5% significance levels.

The BDS test as shown in Table 2 is employed to identify serial dependence in the series. Adopting this test supports this study to conclude whether or not the dataset is non-linear. Since, the P-values of each dimension are less than 5% level of significance, we reject the null hypothesis of *iid* and conclude that the exchange rate series (i.e. NGN/EUR) is nonlinear. Consequently, our findings reveals non-linearity in the series. Still, Table 3.0 below also present other nonlinear tests.

Table 3: Keenan, Tsay and Likelihood Ratio Nonlinearity Test for NGN/EUR

Test	Test Statistic	P-Value	Remark
Keenan's One-Degree Test for Nonlinearity	8.5742	0.0000	There is nonlinearity
Tsay's Test for Nonlinearity	3.9681	0.0023	There is nonlinearity
Likelihood Ratio Test for Threshold Nonlinearity	33.9582	0.0000	There is threshold nonlinearity

* denotes the significant at 5% significance levels.

The results obtained in Table 3 above are in line with the results using BDS test as shown in Table 2. As shown in Table 3 in descending order, both Keenan and Tsay's tests for nonlinearity are designed to evaluate quadratic nonlinearity, the null hypothesis is usually the time series y_t follows a linear process, while the alternative hypothesis is that y_t follows a SETAR(j) model with j > 1.

Table 3 above, shows that the Keenan test statistic is 8.5742 with p-value of 0.0000 which is highly significant at 5%, Tsay test statistic is 3.968085 with a p-value of 0.0023 which is also significant. By implication, the null hypothesis is rejected with conclusion that Nigeria monthly exchange rate of the naira to the euro follows a nonlinear process. It is important to note that, Keenan's and Tsay's quadratic nonlinear tests are inadequate in deciding the threshold non-linearity hypothesis. So, the likelihood ratio test is more preferable for testing threshold nonlinearity. In the likelihood ratio test for threshold nonlinearity, the null hypothesis assumes that the time series follows an AR(*p*) model, while the alternative hypothesis specifies that the time series follows a two-regime threshold autoregressive (TAR) model of order *p* and with constant noise variance. The result for the likelihood ratio test shows that the test statistic is 33.9582 with p-value of 0.0000 indicating that the result is highly significant at 5% level of significance. The result in Table 3, further confirms that Nigeria monthly exchange rate of the Naira to the European Euro is highly nonlinear. Therefore, the null hypothesis is rejected with conclusion that the test shows the existence of threshold nonlinearity in the NGN/EUR exchange rate series. Thus, a nonlinear model Self-exciting threshold autoregressive (SETAR) model is an adequate model for fitting NGN/EUR exchange rates.

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3.3 Augmented Dickey-Fuller Unit Root Test

It is vital to investigate for unit root before applying the time series method; that is, to ensure that the time series dataset is stationary. The Augmented Dickey-Fuller test was employed to ascertain whether the data under consideration is non-stationary or stationary. The time series plot is also employed to give a graphical view of Naira exchange rate with Euro.

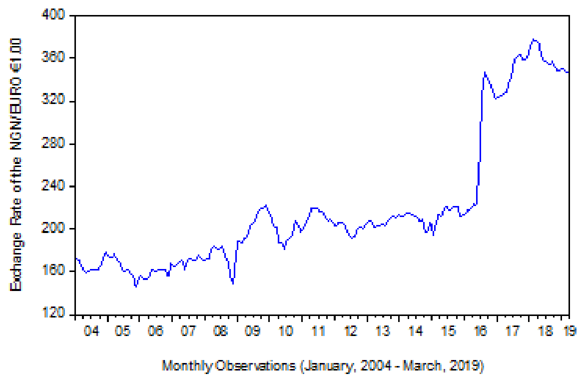

Figure 2: Time Plot of Nigeria Monthly Exchange Rate of the Naira (NGN/EURO €1.00) from January, 2004 to March, 2019

As displayed in Fig. 2 above, the Nigeria monthly exchange rate of the Naira to Euro (i.e. NGN/EURO €1.00) shows an upward trend during the period January, 2004 to March, 2019. Apart from the sharp increase in exchange rate, fluctuations in the rates within the years have also been seen. Based on the time series plot presented in Figure 2, it can be seen that there is obvious structural break and jump in the series which suggests nonlinearity. Since, fairly good estimates of parameters of time series are obtained if the series is stationary, a stationarity test using Augmented Dickey Fuller (ADF) was conducted on monthly exchange rate of the Naira – Euro so as to ascertain these claims and the results of these tests are presented in Table 4.

Table 4: Augmented Dickey Fuller (ADF)

Variable	Option	ADF	
		Test Statistic	**P–Value**
NGN/EUR	Intercept	-0.5898	0.8686
	Intercept and Trend	-2.3271	0.4163
ΔNGN/EUR	Intercept	-9.1070	0.0000*
	Intercept and Trend	-9.0467	0.0000*
	Asymptotic Critical Values		
1%	Intercept	-3.4666	
5%		-2.8774	
10%		-2.5753	
1%	Intercept and Trend	-4.0096	
5%		-3.4348	
10%		-3.1414	

* denotes the significant of ADF test statistic at 1%, 5% and 10% significance levels. Δ is the first difference operator

As shown in Table 4 using the ADF test, the test statistics for intercept only is -0.5898 with a corresponding P-value of 0.8686 which is greater than 0.05 level of significance. While, the test statistic for intercept and trend is -2.3271 with a corresponding P-value of 0.4163 also greater than 0.05 level of significance. This result indicates that Nigeria monthly exchange rate of Naira-Euro was non-stationary at all levels since their corresponding P-values of the test statistics are greater than 5% level of significance. By implication, we can conclude that the time series dataset has the presence of unit root.

3.4 Unit Root Test in the Presence of Structural Break

Vogelsang and Perron (1998) points out that structural change and unit roots are closely related, and researchers should bear in mind that conventional unit root tests are biased towards a false unit root null when the data are trend stationary with a structural break.

Table 5: Breakpoint Unit Root Test

Variable	Structural Break Result			ADF	
	Break Date	**Break Type**	**Trend Specification**	**Test Statistic**	**P–Value**
NGN/EUR	2016 M05	Innovative Outlier	Intercept	-5.6579	0.01*
			Intercept and Trend	-9.2939	0.01*
		Asymptotic Critical Values			
1%	Intercept			-4.9491	
5%				-4.4436	
10%				-4.1936	
1%	Intercept and Trend			-5.3476	
5%				-4.8598	
10%				-4.6073	

* denotes the significant of ADF test statistic at 1%, 5% and 10% significance levels.

Based on the results obtained in Table 5, the structural break point test reveals that only one significant break date in the exchange rate of NGN/EUR exists, which is evident in Figure 3 and, the break date identified is 2016M05. The structural break dates results obtained seem to point to asymmetric transmission of economic and political related shocks. In addition, another domestic shock is the new policies as a result of the 2015 general elections (i.e. new administration) which might be the explanation for the break date in May, 2016. Note that, the structural break date identified in this study is not independent of the sample period or of the data frequency.

As shown in Table 5 using the innovative outlier break type for breakpoint unit root test, the test statistic obtained for the intercept level is given as -5.6579 with a corresponding P-value of 0.01, which is less than 0.05 level of significance. While, the test statistic obtained for the intercept and trend level is -9.2939 with a P-value of 0.01 which is also less than 0.05 level of significance. Since, both levels P-values (i.e. 0.01) are less than 0.05 level of significance we reject the null hypothesis of unit root and conclude that, Naira-Euro exchange rate has no unit root (i.e. the series is stationary).

The results as shown in Table 5 follows the methodology described in Zivot and Andrews (1992) that treats the breakpoint as endogenous and specify only a single break point which was later extended by Vogelsang and Perron (1998) that uses a multiple breakpoint test in identifying structural break in a series. As described in Vogelsang and Perron (1998), we use the innovative outlier (IO) model to test for unit root in the presence of structural break. In the IO model, a dummy for a break in the level is allowed along with a dummy for a break in the trend at an unknown period of time. Also, under the innovative outlier (IO) model, the break is modelled as evolving more slowly over time on Nigeria average monthly exchange rate of the Naira (Naira per unit of Euro).

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Our results showed that the ADF unit root tests in Table 4 are biased towards the non-rejection of non-stationarity in the series. While, Table 5 shows that NGN/EUR exchange rates follows an *I(0)* process. This finding that the exchange rates of Naira to Euro is an *I(0)* process is in line with the existing literature (e.g. Zivot and Andrews, 1992). The result rejected unit root in the presence of one break. Furthermore, the results in Table 5 indicate that it is imperative to account for structural break in the time series for monthly exchange rate of Naira to European Euro. And most importantly, examine the presence of unit root more carefully if the series under consideration exhibit structural break.

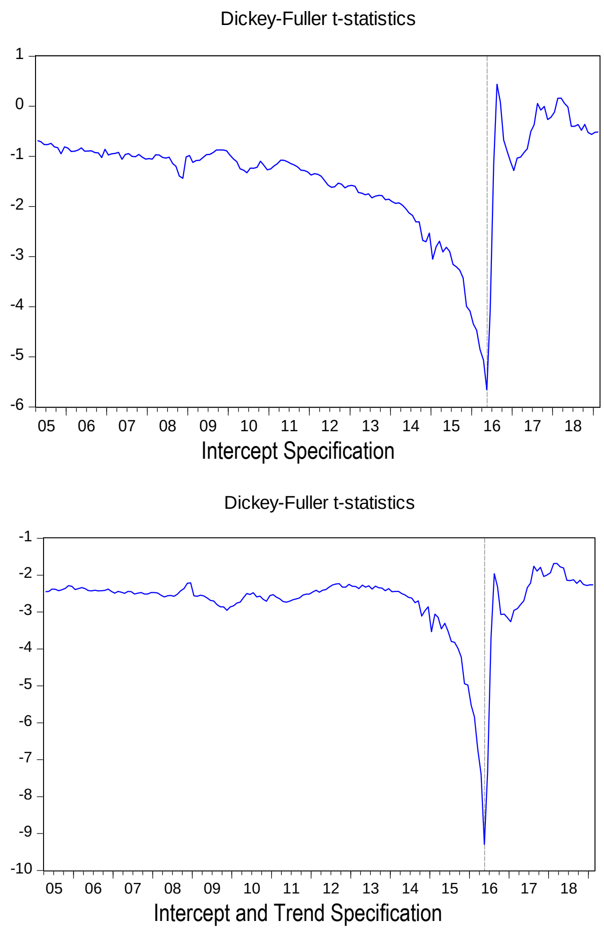

Figure 3: Graphical Plot of Structural Break Date at Different Levels

The graphs above shows the visual selection of the structural break date which is May, 2016 as shown in Table 5 above.

3.5 Estimation of Self-Exciting Threshold Autoregressive (SETAR) Model

Two SETAR models were considered in this section, that is, a SETAR model without the application of dummy variable and another SETAR model were estimated taking into consideration dummy variable so as to address the presence structural break in the model.

Two SETAR models were generated for exchange rates of Naira to Euro, and these are SETAR model without dummy variable which is considered as the bench mark model for this study. While, a SETAR model with the inclusion of a dummy variable is also considered in this study. The inclusion of the dummy variable is to help solve the issue of the identified structural break in the series, one key importance of the dummy variable is that it can be used to determine the importance of policy actions on models and are often used to account for qualitative effects (Ziyodullo, 2016). The description and summary of the threshold specification are given for both models. The multiple threshold tests are employed in this study to evaluate the threshold variable, threshold value and specify the number of regimes corresponding to each threshold variable; the SSR was used in selecting the threshold variable and regimes because it measures the overall difference between the variables under consideration.

3.5.1 Estimation of SETAR model without dummy variable

The multiple threshold test is employed in this study to evaluate threshold variable, threshold value and specify the number of regimes as shown in Table 6, while, the threshold test and parameter estimation are shown in Table 7 and 8, respectively. Figure 4 shows the line plot of the selected threshold variable and Figure 5 shows the graphical plot of SETAR(2;5,2) model.

Table 6: Regime Selection using SSR and Identification of Threshold Variable (Delay Parameter) and Threshold Value

Threshold Variable		Sum of Squared Residuals (SSR)		Regimes
NGN/EUR(-2)		6835.8396		2
NGN/EUR(-3)		7163.0740		2
NGN/EUR(-1)		7345.5131		2
NGN/EUR(-4)		8231.8010		2
NGN/EUR(-5)		8340.5723		2
		Summary		
Threshold Variable	Estimated No. of Thresholds	Threshold Data Value	Adjacent Data Value	Threshold Value used
NGN/EUR(-2)	1	222.85	221.45	222.84999

Method: Bai – Perron tests of $L+1\ vs. L$ sequentially determined thresholds. Maximum number of thresholds: 5

Based on the result obtained in Table 6 for SETAR model without dummy variables, it can been seen that, the selected threshold variable for the model is NGN/EUR(-2) since it has the lowest SSR coefficient of 6835.8396 when compared to others and it specify the SETAR model without dummy variables to a two (2) regime process. The detailed information on the threshold values includes the actual data value corresponding to the break in this case is 222.85, the actual data value for the next highest data value here is 221.45, and the estimated value use for representation purposes and parameter estimation is 222.84999. It is important to note that, any value between the lower adjacent data value and the threshold data value would produce the same observed fit.

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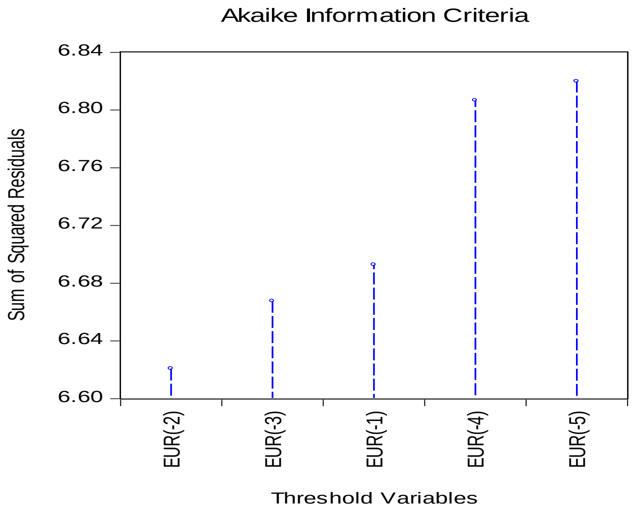

Figure 4: Line Plot of Sum of Squared Residuals on Selection of Threshold Variable

The criteria graph for the threshold variable selection as shown in Figure 4 above further provide visual evidence on the threshold variable selected. In Figure 4, the threshold variable whose model has the lowest SSR is clearly visible on the left-side of the graph, indicating NGN/EUR(-2) as the most preferred threshold variable because of its minimal SSR.

Table 7: Threshold Test of Nigeria Monthly Exchange Rate of NGN/EUR

Threshold Test	F – statistic	Scaled F – statistic	Critical Values**	Sequential F – Statistic Determined Thresholds
0 vs 1*	11.4859	68.91555	20.08	1
1 vs 2	2.5891	15.53488	22.11	
		Break Dates		
Threshold Values		Sequential		Repartition
1		222.84999		222.84999

Threshold test options: Trimming 0.15, Max. thresholds 5, * Significant at the 0.05 level
** Bai – Perron (Econometric Journal) Critical Value

Table 7 above shows the threshold test; the Scaled F-statistic for 0 vs 1 threshold obtained is 68.9156 which is greater than F-tabulated value or critical value of 20.08 at 0.05 level of significance. Since, 68.9156 > 20.08 we conclude that, 1 threshold value compared to no threshold value (i.e. zero) is significant at 5%. For 1 vs 2 threshold compared, the Scaled F-statistic obtained is 15.5349 which is less than the F-tabulated value of 22.11 at 0.05 level of significance. Thus, 2 threshold value compared to 1 threshold value is not significant at 5%. This implies that, the maximum number of threshold values estimated is one (1).

Table 8: Parameter Estimates of the SETAR Model without Dummy Variable

Dependent Variable: EXCHANGE RATE (NGN/EUR)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
	NGN/EUR (-2) < 222.84999 -- 143 obs			
Constant	6.736934	4.870557	1.383196	0.1685
NGN/EUR (-1)	1.047456	0.097618	10.73018	0.0000*
NGN/EUR (-2)	-0.110176	0.141801	-0.776976	0.4383
NGN/EUR (-3)	0.095947	0.141361	0.678736	0.4983
NGN/EUR (-4)	-0.114658	0.141115	-0.812511	0.4177
NGN/EUR (-5)	0.048455	0.097144	0.498792	0.6186
	222.84999 <= NGN/EUR (-2) -- 35 obs			
Constant	63.63540	10.39042	6.124430	0.0000*
NGN/EUR (-1)	1.517463	0.109101	13.90874	0.0000*
NGN/EUR (-2)	-1.126470	0.203255	-5.542162	0.0000*
NGN/EUR (-3)	0.602426	0.230347	2.615303	0.0097*
NGN/EUR (-4)	-0.323850	0.204800	-1.581300	0.1157
NGN/EUR (-5)	0.150951	0.101403	1.488624	0.1385
R-squared	0.990888	Mean dependent var.		221.2897
Adjusted R-squared	0.990284	S.D. dependent var.		65.10289
S.E. of regression	6.417146	Akaike info criterion		6.620860
Sum squared resid	6835.840	Schwarz criterion		6.835362
Log likelihood	-577.2565	Hannan-Quinn criter.		6.707846
F-statistic	1641.050	Durbin-Watson stat		2.017776
Prob(F-statistic)	0.000000*			

Trimming 0.15, Max. thresholds 5, * Significant at the 0.05 level

Thus, the fitted SETAR model without dummy variables for Naira-Euro Exchange rate series is SETAR(2;5,2) and its predictive equation is given below:

$$Y_t = \begin{cases} 6.7369 + 1.0475Y_{t-1} - 0.1102Y_{t-2} + 0.0959Y_{t-3} - 0.1147Y_{t-4} + 0.0485Y_{t-5} \;\; if\; Y_{t-2} < 222.85, \\ 63.6354 + 1.5175Y_{t-1} - 1.1265Y_{t-2} + 0.6024Y_{t-3} - 0.3238Y_{t-4} + 0.1510Y_{t-5} \;\; if\; Y_{t-2} \geq 222.85 \end{cases} \quad (7)$$

The result of the SETAR model without dummy variables is presented in Table 8 above. The table contains the parameter estimates of the model for lags less than the threshold value and lags greater than or equal to the threshold value. The result shows that, the first and the second regime comprises of 143 and 35 observations, respectively. In the first regime, only the coefficient of the first lag [i.e. NGN/EUR (-1)] of the dependent variable is statistically significant since its corresponding P-value is less than 0.05 level of significance while, other lags are not significant. This is indicated by the P-value of the coefficients which is less than 0.05 level of significance for the coefficient of NGN/EUR(-1) but greater than 0.05 level of significance for other regressors.

However, in the second regime, only the first, second and third lags are statistically significant since their P-values are less than 0.05 level of significance while, the other regressors are statistically insignificant because their corresponding P-values are greater than 0.05 level of significance. The R-squared shows that about 99.09% variability in exchange rate is explained by changes in its lags while the F-statistics (1641.05) with probability value 0.0000 indicates that the model or regressors are jointly significant. Also, since the Durbin-Watson statistic (i.e. 2.0178) is greater than 2, it shows that the model is free from serial correlation or autocorrelation (i.e. there is no evidence of

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autocorrelation). So, the estimates are valid for statistical explanation and policy inferences. Thus, the SETAR model established without dummy variable is given as SETAR(2;5,2).

3.5.2 Estimation of SETAR model with dummy variable

Table 9 shows the summary results of the threshold variable, threshold value and the selected regime using the multiple threshold test. Also, Table 10 and 11 shows the threshold test and estimated parameters, respectively. Figure 5 shows the line plot of the selected threshold variable and Figure 7 shows the graphical plot of SETAR(3;5,3) model.

Table 9: Regime Selection using SSR and Identification of Threshold Variable (Delay Parameter) and Threshold Value

Threshold Variable	Sum of Squared Residuals (SSR)		Regimes	
NGN/EUR(-3)	6074.342096		3	
NGN/EUR(-2)	6500.150886		2	
NGN/EUR(-1)	7001.254884		2	
NGN/EUR(-4)	8064.915834		2	
NGN/EUR(-5)	8302.899620		2	
		Summary		
Threshold Variable	Estimated No. of Thresholds	Threshold Data Value	Adjacent Data Value	Threshold Value used
NGN/EUR(-3)	2	211.24	211.17	211.23999
		223.46	223.07	223.45999

Method: Bai – Perron tests of $L+1\ vs. L$ sequentially determined thresholds. Maximum number of thresholds: 5

For SETAR model with dummy variable, the dummy variable is included to address the case of structural break present in the study variable (i.e. the dummy variable is used to capture changes or shifts in the model been estimated). The multiple threshold test is employed in this study to evaluate the threshold variable, threshold value and specify the number of regimes corresponding to each threshold variable as presented in Table 9. From the result presented in Table 9 above, it shows that the selected threshold variable for the model is NGN/EUR(-3) since it has the lowest SSR coefficient of 6074.3421 when compared to others and it specify the SETAR model to a three (3) regime process after the inclusion of a dummy variable to address the problem of structural breaks.

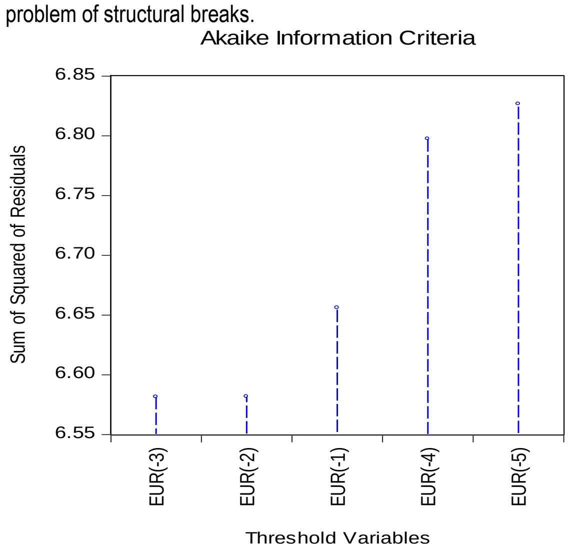

Figure 5: Line Plot of Sum of Squared Residuals on Selection of Threshold Variable

Furthermore, the criteria graph for the threshold variable selection as shown in Figure 5 above gives graphical view on the preferred threshold variable which is evident on the left-side of the graph, indicating NGN/EUR(-3) as the most preferred threshold variable because of its minimal SSR.

Table 10: Threshold Test of Nigeria Monthly Exchange Rate of NGN/EUR

Threshold Test	F – statistic	Scaled F – statistic	Critical Values**	Sequential F – Statistic Determined Thresholds
0 vs 1*	9.850810	59.10486	20.08	2
1 vs 2*	4.566214	27.39728	22.11	
2 vs 3	2.558496	15.35098	23.04	
		Break Dates		
Threshold Values		Sequential		Repartition
1		223.45999		211.23999
2		211.23999		223.45999

Threshold test options: Trimming 0.15, Max. thresholds 5, * Significant at the 0.05 level
** Bai – Perron (Econometric Journal) Critical Values

Table 10 above shows the threshold test; the Scaled F-statistic for 0 vs 1 threshold obtained is 59.1049 which is greater than F-tabulated value of 20.08 at 0.05 level of significance. Since, 59.1049 > 20.08 we conclude that, the result obtained indicates that, 1 threshold value compared to no threshold value (i.e. zero) is significant at 5%. For 1 vs 2 threshold compared, the Scaled F-statistic obtained is 27.3973 which is greater than the F-tabulated value of 22.11 at 0.05 level of significance. Thus, 2 threshold value compared to 1 threshold value is significant at 5%. While, For 2 vs 3 threshold compared, the Scaled F-statistic obtained is 15.3510 which is less than the F-tabulated value of 23.04 at 0.05 level of significance. Thus, 3 threshold value compared to 2 threshold value is not significant at 5%. This implies that, the maximum number of threshold values estimated is two (2).

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Table 11: Parameter Estimates of the SETAR Model with Dummy Variable

Dependent Variable: EXCHANGE RATE (NGN/EUR)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
	NGN/EUR (-3) < 211.23999 -- 111 obs			
Constant	34.71215	11.96031	2.902279	0.0042*
NGN/EUR (-1)	0.886425	0.112921	7.849955	0.0000*
NGN/EUR (-2)	-0.162526	0.144637	-1.123683	0.2628
NGN/EUR (-3)	0.187128	0.146084	1.280959	0.2021
NGN/EUR (-4)	-0.132775	0.145655	-0.911573	0.3634
NGN/EUR (-5)	0.014407	0.103887	0.138675	0.8899
	211.23999 <= NGN/EUR (-3) < 223.45999 -- 35 obs			
Constant	-84.03813	69.60368	-1.207380	0.2291
NGN/EUR (-1)	1.002878	0.162463	6.172971	0.0000*
NGN/EUR (-2)	0.755197	0.400806	1.884195	0.0614
NGN/EUR (-3)	-0.541206	0.420028	-1.288498	0.1994
NGN/EUR (-4)	0.311441	0.396219	0.786032	0.4330
NGN/EUR (-5)	-0.182859	0.262781	-0.695860	0.4875
	223.45999 <= NGN/EUR (-3) -- 32 obs			
Constant	80.87274	20.74418	3.898574	0.0001*
NGN/EUR (-1)	1.666632	0.198585	8.392534	0.0000*
NGN/EUR (-2)	-1.170583	0.242867	-4.819855	0.0000*
NGN/EUR (-3)	0.260474	0.241323	1.079358	0.2821
NGN/EUR (-4)	-0.007916	0.217723	-0.036358	0.9710
NGN/EUR (-5)	0.022550	0.106694	0.211350	0.8329
	Non-Threshold Variables			
Dummy Variable	8.976811	2.737832	3.278803	0.0013*
R-squared	0.991903	Mean dependent var.		221.2897
Adjusted R-squared	0.990986	S.D. dependent var.		65.10289
S.E. of regression	6.180891	Akaike info criterion		6.581406
Sum squared resid	6074.342	Schwarz criterion		6.921034
Log likelihood	**-566.7451**	**Hannan-Quinn criter.**		**6.719134**
F-statistic	1082.102	Durbin-Watson stat		1.961450
Prob(F-statistic)	0.000000*			

Trimming 0.15, Max. thresholds 5, * Significant at the 0.05 level.

Thus, the fitted SETAR model with dummy variables for Naira-Euro Exchange rate series is SETAR(3;5,3) and its predictive equation is given below:

$$Y_t = \begin{cases} 34.71 + 0.89Y_{t-1} - 0.16Y_{t-2} + 0.19Y_{t-3} - 0.13Y_{t-4} + 0.01Y_{t-5} + 8.98D_t \ \ if\ Y_{t-3} < 211.24 \\ -84.04 + 1.00Y_{t-1} + 0.76Y_{t-2} - 0.54Y_{t-3} + 0.31Y_{t-4} - 0.18Y_{t-5} + 8.98D_t\ if\ 211.24 \le Y_{t-3} < 223.46 \\ 80.87 + 1.67Y_{t-1} - 1.17Y_{t-2} + 0.26Y_{t-3} - 0.01Y_{t-4} + 0.02Y_{t-5} + 8.98D_t\ if\ Y_{t-3} \ge 223.46 \end{cases} \quad (8)$$

The result of the SETAR model with dummy variable is presented in Table 11 above. The table contains three (3) regime parameter estimates of the model and also, the estimate of the dummy variable included as a non-threshold variable. The addition of dummy variable in this estimate/or model is to capture changes/or shifts and solve the problem of structural breaks. The result shows that, two (2) threshold values were obtained, these include; 211.23999 and 223.45999, there-by prompting the generation of three (3) regimes for the series under study. The first second and third regime comprises of 111, 35 and 32 observations, respectively. In the first regime, the process contains estimates of lags less than the threshold value (211.23999); In the second

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regime, the process contains estimates of lags that are greater or equal to the threshold value (211.23999) but less than the threshold value (223.45999) (i.e. the lag estimates are within the two threshold values generated in the series). While in the third regime, the process contains lags that are greater than or equal to the threshold value (223.45999).

The first and second regime showed that, only the coefficient of the first lag [i.e. NGN/EUR (-1)] of the dependent variable is statistically significant, since, the corresponding P-values of NGN/EUR (-1) in both regimes are less than 0.05 level of significance, while other lags are not significant. While, in the third regime, both the first and second lags [i.e. NGN/EUR (-1) and (-2)] are statistically significant since their corresponding P-values are less than 0.05 level of significance, while the other regressors are statistically insignificant. The coefficient of the dummy variable obtained is 8.9768 with a corresponding P-value of 0.0013 which is less than 5% level of significance. This result shows that, the dummy variable included to solve the presence of structural breaks is significant at 0.05 level of significance. The R-squared shows that about 99.19% variability in exchange rate is explained by changes in its lags. While, the F-statistics which shows the general significance of the model is given as (1082.102) with P-value of 0.0000 indicating that the model or regressors are jointly significant. Also, since the Durbin-Watson statistic (i.e. 1.9615) is closer to 2, it shows that the model is free from serial correlation or autocorrelation (i.e. there is no evidence of autocorrelation). So, the estimates are valid for statistical explanation and policy inferences. Thus, the SETAR model established with the inclusion of the dummy variable is given as SETAR(3;5,3).

3.6 Model Diagnostic Test on SETAR Model with and without Dummy Variable

Diagnostic tests were applied to the models to confirm that they are suitable and adequate for forecast. So, to authenticate the suitability and adequacy of SETAR(2;5,2) and SETAR(3;5,3) models, a serial correlation test is performed utilizing the Breusch-Godfrey Serial Correlation LM Test. While, heteroscedasticity is tested utilizing the autoregressive conditional heteroscedasticity (*ARCH)* test. The results of the aforementioned diagnostic tests for NGN/EUR are presented in Tables 12 and 13, respectively.

Table 12: Breusch-Godfrey Serial Correlation LM Test on Monthly Exchange Rate of NGN/EUR

Models	F-Statistic	Prob.	Chi-Square Statistic	Prob.
SETAR(2;5,2) without Dummy Variable	0.934608	0.6084	62.63754	0.4179
SETAR(3;5,3) with Dummy Variable	0.638684	0.9699	50.63408	0.8254

* Significant at the 0.05 level.

Using the Breusch-Godfrey correlation test as shown in Table 12 above, the result obtained showed that, the null hypothesis of no serial correlation is not rejected at 0.05 level of significance for both models. Therefore, based upon the correlation test, the constructed models are deemed to be adequate for forecasting since the models have no presence of serial correlation.

For SETAR(2;5,2); the P-values of the corresponding F-statistic (0.934608) and Chi-square (62.63754) are 0.6084 and 0.4179, respectively; since both P-values are greater than 0.05 level of significance we do not reject the null hypothesis of no serial correlation and conclude that SETAR(2;5,2) model is adequate for prediction. While, the P-values of the corresponding F-statistic (0.6387) and Chi-square (50.6341) for SETAR(3;5,3) are 0.9699 and 0.8254, respectively. Since, the corresponding P-values of both test statistics (i.e. F-statistic and Chi-square) are greater than 0.05 level of significance we do not reject the null hypothesis and conclude that SETAR(3;5,3) model is also free from serial correlation or autocorrelation.

Table 13: ARCH Heteroscedasticity Test on Monthly Exchange Rate of NGN/EUR

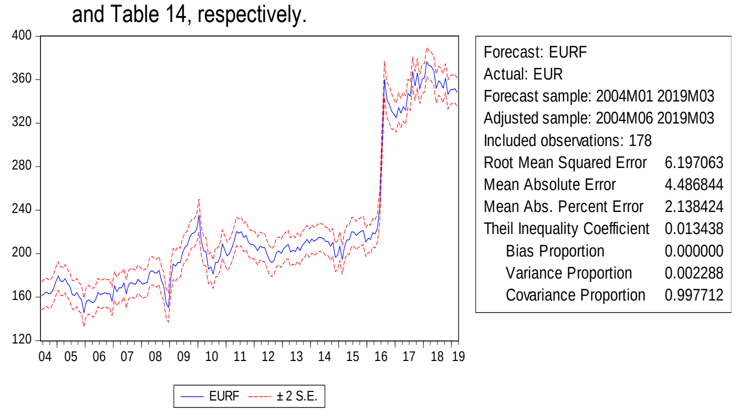
Models	F-Statistic	Prob.	Chi-Square Statistic	Prob.
SETAR(2;5,2) without Dummy Variable	0.719817	0.8944	51.94007	0.7892
SETAR(3;5,3) with Dummy Variable	0.141667	1.0000	15.88703	1.0000

* Significant at the 0.05 level.

The results of the ARCH test for heteroscedasticity are summarized in Table 15. The P-values obtained for both models that is, 0.7892 and 1.000 for SETAR(2;5,2) and SETAR(3;5,3) models, respectively which are both greater than 0.05 level of significance indicating that the null hypothesis of no heteroscedasticity is not rejected. Therefore, SETAR(2;5,2) and SETAR(3;5,3) models are free from a heteroscedasticity problem and are adequate for forecasting NGN/EUR exchange rate.

3.7 Forecast Performance Measure Indices

After determining the adequacy of SETAR(2;5,2) and SETAR(3;5,3) models, the static forecasting method is employed to perform a series of one-step ahead forecasts of the dependent variable taking into account the entire observations. In order to compare the forecasting performance of both models, the criteria summarized in section 4.11 are employed as shown in Figure 6 and Table 14, respectively.



One-Step Ahead Forecast of SETAR(2;5,2) Model without Dummy Variable

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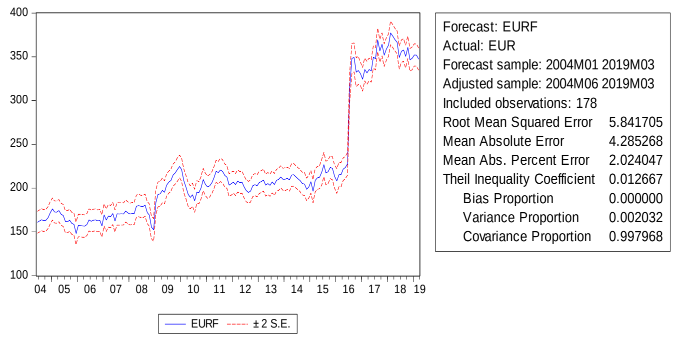

One-Step Ahead Forecast of SETAR(3;5,3) Model with Dummy Variable

Figure 6: One-Step Ahead Forecast of SETAR(2;5,2) and SETAR(3;5,3) Models on Exchange Rate of NGN/EUR

Figure 6 shows the one-step ahead forecast of the dependent variable after determining the adequacy of SETAR(2;5,2) and SETAR(3;5,3) models.

Table 14: Comparison of the Predictive Power of SETAR(2;5,2) and SETAR(3;5,3) Models

Predictive/Selection Criteria	SETAR(2;5,2) without Dummy Variable	SETAR(3;5,3) with Dummy Variable
Root Mean Squared Error (RMSE)	6.197063	5.841705
Mean Absolute Error (MAE)	4.486844	4.285268
Mean Absolute Percentage Error (MAPE)	2.138424	2.024047
Theil's u inequality coefficient (TIC)	0.013438	0.012667

To compare the fit of the SETAR(2;5,2) and SETAR(3;5,3) models, we have regrouped the selection criteria obtained from the models. Table 14 present the completeness and precision in the predictive estimates of the models.
More precisely, the RMSE, MAE, MAPE and TIC are used to evaluate and determine the predictive power/or forecast evaluation (i.e. Evaluation of the quality of a forecast requires comparing the forecast values to actual values of the target value over a forecast period) of each model generated from the series. Based on the result in Table 14, it can also be infer that SETAR(3;5,3) model has the smallest values of RMSE, MAE, MAPE and TIC when compared to the SETAR(2;5,2) model. This results shows that, SETAR(3;5,3) model have a better predictive power compared to SETAR(2;5,2) model in this study. Furthermore, based on this study SETAR(3;5,3) model with dummy variable is a better model for analyzing the fluctuations of Naira to Euro exchange rates in Nigeria. That is, invariably that the SETAR(3;5,3) model with dummy variable to curtile and solve the problem of structural breaks/changes or shifts gives a better fit for the Nigeria exchange rate of the Naira to Euro than the SETAR(2;5,2) model without dummy variable.

4.0 DISCUSSION

Our findings reveals that the monthly exchange rate of the Naira per unit of Euro ranges from 144.97 to 377.84 with an average of 219.764 and a standard deviation of 64.8537 for the period under reviews. The three (3) nonlinearity test (i.e. Brock, Dechert and Scheinkman (BDS) nonlinearity test, Keneen's one-degree test for nonlinearity, Tsay's test for quadratic nonlinearity and Likelihood ratio test for threshold nonlinearity), all indicates that the series under study follows a nonlinear process. Since, fairly good estimates of parameters of time series are obtained if the series is stationary, a stationarity test using Augmented Dickey Fuller (ADF) and Ng–Perron unit root were conducted on monthly exchange rate of the Naira – Euro so as to ascertain these claims. Thus, the results of this test showed that NGN/EUR is non-stationary at all levels. Vogelsang and Perron (1998) points out that structural change and unit roots are closely related, and researchers should bear in mind that conventional unit root tests are biased towards a false unit root null when the data are trend stationary with a structural break. Based on the results obtained, the structural break point test reveals that only one significant break date in the exchange rate of NGN/EUR exist (i.e. The break date identified is 2016M05). The structural break dates results obtained seem to point to asymmetric transmission of economic and political related shocks. In addition, another domestic shock are the new policies (i.e. new administration) era which might be the explanation for the break date in May 2016. Note that, the structural break date identified in this study is not independent of the sample period or of the data frequency. Also, the breakpoint unit root test rejected the null hypothesis of unit root and thereby conclude that the series is stationary. Based on this result, we can ascertain that the ADF unit root tests is biased towards the non-rejection of non-stationarity. Hence, it is imperative to account for structural break in the time series for monthly exchange rate of Naira to European Euro. And most importantly, examine the presence of unit root more carefully if the series under consideration exhibit structural break.
The nonlinear least squares method was used in estimating the parameters of both models (i.e. SETAR model with and without dummy variables). Thus, the SETAR model established without dummy variable is given as SETAR(2;5,2) which is the baseline model. While, the SETAR model established with the inclusion of the dummy variable is given as SETAR(3;5,3). Both models are statistically significant and also, the R-squared shows that about 99.09% variability in exchange rate is explained by changes in its lags of SETAR(2;5,2) model. While, The R-squared shows that about 99.19% variability in exchange rate is explained by changes in its lags of SETAR(3;5,3).
We can state categorically, that there is no much difference between the R-squared values of both models. The diagnostic tests confirm that both models are suitable for forecast (i.e. they are both free from serial correlation and heteroscedasticity). Therefore, SETAR(2;5,2) and SETAR(3;5,3) models are both adequate for forecasting NGN/EUR exchange rate in Nigeria. Finally, the measures of accuracy that is, RMSE, MAE, MAPE and TIC showed that, SETAR(3;5,3) have a better predictive power compared to SETAR(2;5,2) model in this study. Furthermore, based on this study SETAR(3;5,3) model with dummy variable is a better model for analyzing the fluctuations of Naira to Euro exchange rates in Nigeria.

5.0 Conclusion and Recommendations

5.1 Conclusion

This study focuses on building a model for Nigeria exchange rates of Naira to the European Euro using Self exciting threshold autoregressive (SETAR) model. We used monthly data of Naira to

Euro exchange rates from January, 2004 to March, 2019. The monthly data was employed to endogenously determine structural breaks in the series since structural breaks can be observed more closely in a low frequency data set. We used the Vogelsang and Perron (1998) and Bai and Perron (1998) frame work, and provide evidence of structural break which are associated with unstable political, economic and trade liberalization. The economy is susceptible to internal shocks and some policy implications, which may be linked to the 2015 general elections in Nigeria. Also, the date break is associated with external shocks and this identified break date might be linked to international commodity price and global financial crises. The findings of structural break in the series are useful for future empirical studies using exchange rates in Nigeria. Self-exciting threshold autoregressive models have been used in order to be able to model the volatile changes in currency market, since nonlinear time series models gives better fit in-wake of structural breaks in the time series data. Test for nonlinearity of Nigerian monthly exchange rate of the Naira to Euro was investigated before the modelling approach was applied, the approaches employed to investigate nonlinearity are BDS nonlinearity test, Tsay nonlinearity, Keenan nonlinearity test and Likelihood Ratio nonlinearity test. Based on the results obtained, the various test indicated that, the exchange rate of the Naira to Euro showed that both non-linearity and threshold non-linearity occurred in the series.

Furthermore, the unit root test in the presence of structural breaks for Nigeria exchange rates of the Naira to Euro showed that the ADF unit root test without accounting for the breaks could lead to false non-rejection of unit root. That is, the unit root test in the presence of structural break showed that the time series data set is stationary after the ADF unit root tests indicates that the series has unit root (i.e. the series is non-stationary).

In performing the modelling procedures two SETAR models were considered and these are SETAR model without dummy variable and SETAR model with dummy variable. After performing the modelling procedures, SETAR(2;5,2) and SETAR(3;5,3) were generated and applied in estimating Nigerian monthly exchange rate of the Naira to Euro. The model generated without dummy variable is SETAR(2;5,2). While, SETAR(3;5,3) takes into account dummy variable. Based on the following diagnostic test; Breusch-Godfrey correlation LM test and ARCH test, it can be seen that both SETAR models developed for modelling Nigeria monthly exchange rates of NGN/EUR are free from both serial correlation and heteroscedasticity.

Finally, based on the forecasting performance indices generated by both models, it can be seen that SETAR(3;5,3) with dummy variable outperform the SETAR(2;5,2) without dummy variable. This results indicates that SETAR(3;5,3) model fits the series better than SETAR(2;5,2) model, as such, the SETAR(3;5,3) model is superior to the SETAR(2;5,2) model in addressing structural breaks in the face of non-linearity in a data set. This result is own to the fact that breakpoint/or structural break is resolved by adding a dummy variable. The inclusion of dummy variable as shown in this study is capable of resolving structural breaks, regimes or shocks of any certain dimension on exchange rate of the Naira to Euro. The effectiveness of SETAR(3;5,3) model over the SETAR(2;5,2) model in terms of the time series data set characterized by structural instability have been supported by the application of real data of univariate time series here investigated.

5.2 Recommendations

In this research work, it is established that SETAR model with dummy variables shows significant precision than SETAR model without dummy variables in forecasting Naria-Euro exchange rates. Therefore, this study presents the following recommendations:

i. This study has shown the need to always examine the occurrence, or otherwise, of non-linearity and threshold non-linearity in the time series data before adopting any model estimation.
ii. It is imperative to investigate unit root more carefully especially when the data set exhibit structural break; otherwise, ignoring the breakpoint unit root test can lead to model misspecification and spurious results of model parameters.
iii. Dummy variables should be considered in the case of structural breakpoint as it will help mitigate and solve the problem of breaks/changes/or shifts that may exist in a series and there-by increase precision of estimate in a model.
iv. Since, nonlinear time series is characterized by jumps, discontinuity, asymmetry, irregularities, structural instability etc., therefore policies geared towards financial stabilization should be well structured and carefully implemented.
v. This study helps to provide break dates that are attached to economic, political, natural shocks etc. More supportive evidence is found in line with the structural break test; the breakpoint occurred in May, 2016 which was barely a year of the new transition of government, it shows that financial shocks tends to be heightened around this particular period, were the Naira tend to fare poorly to foreign currencies especially the currencies in the nation's foreign reserves. By implication, the government/policy makers may use the historical information to forecast future movements in financial/climatic/macroeconomic time series.

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