SAMPLE SIZE SIMULATION FOR UNIT ROOT, STRUCTURAL BREAK AND REGIME SHIFTS

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ABSTRACT
Unit root test is an important means to determine the integration order of a variable which has involved different methods of testing for stationarity. Simulation method is adopted in this study to verify whether unit root, structural breaks and regime shifts exist in the sample considered. For sample sizes of 20 and 50 as small, 100 and 250 as medium, and 2500 and 5000 as large, the enhanced Dickey-Fuller test and Zivot-Andrews test were used. The experiment was conducted 5000 times for each sample size, and the results demonstrated that there is presence of unit root at level for all sample sizes taken into consideration, but they were integrated of order 1. This implies that they are stationary at first difference. The results also showed that there are structural breaks at various levels depending on sample size, but it was noted that the breaks remained stable regardless of size when the sample size was large. The MSVAR results demonstrated that regime 1 is more resilient than regime 2, and that regime 1 is projected to last longer than regime 2. As a result, we draw the conclusion that simulation can be utilized to verify a real-world situation.

Keywords: Simulation, Sample size, Unit root, Structural break, Stationarity, MSVAR.

INTRODUCTION
Many macroeconomic time series contain structural breaks, which are widely acknowledged to be of high quality (Stock and Watson 1996; Paye and Timmermann 2006) and to be a significant cause of forecast failure (Hendry 2000; Hendry and Clements 2003). Numerous academics have recognized and proved the fact that the majority of economic variables are never stable at first level but become stationary at first difference (Alehmo and Adenomon, 2022). This study considers a scenario in which a discrete and permanent change in model coefficients may occur during the sample period used for estimation, making unit roots and structural breaks essential.

When determining the integration order or stationarity of variables, the unit root test is crucial. Various techniques, including the Augmented Dickey-Fuller test (ADF), the Phillips-Perron test (PP), the KPSS test, and others, are frequently used to check for stationarity in a given series. The robustness and compatibility of the Augmented Dickey-Fuller (ADF) unit root test with macroeconomic data led us to use it for this study (Adenomon, 2017). The ADF test is based on comparing whether a series is stationary under the statistical premise that errors are white noise with the hypothesis that the series contains unit roots (Zhao, et al, 2020). A series is integrated of order zero if it is stationary; 1 (0).

It is claimed that most macroeconomic data displays unit root processes, it may be essential to select a few special economic events and consider them to have permanently altered the time series' pattern. There are several benefits to testing for unit roots while simultaneously allowing for structural breaks. First of all, it guards against biasing test findings in favour of non-stationarity and unit root. Additionally, these examinations can pinpoint the time that the potential break happened (Allaro, Kassa and Hundie, 2011).

Data from macroeconomic time series shows various trends and patterns over the course of an economic, social, and political cycle. The relationship between the factors in the economy may change when these phases change. This can result in parameter stability during the course of the analysis. Tan (2013) came to the conclusion that these can cause misspecification and false conclusions since the parameters are ineffective and inconsistent. Therefore, using linear models that adopt functional forms that presume the link between variables is constant across the board may not produce reliable results that could have an impact on predictions. Macroeconomic variables typically and persistently fluctuate around high and low levels, hence an unobservable egodric Markov process and the possibility of a regime shift. One appropriate method which captures the unobservable state, the transmission from one regime to another and the duration of stay in a particular regime often ignored by the linear methods is the Markov-Switching Vector Autoregressive model (Tuanh and Essi, 2021).

The MS-VAR model can provide a systematic ability to implementing statistical methods and the model can also estimate efficient and consistent parameters, detect recent changes and correct the VAR model when the regimes change (Wai, et al, 2015). The non-linear data generation method of the Markov-Switching Vector Autoregressive (MS-VAR) model makes it a non-linear model. This is accomplished by restricting the approach to being linear in a specific discrete and unobservable regime. Krolzig introduced it after the Hamilton concept. The simple finite order of the VAR model is additionally generalized by the MS-VAR model. The fundamental idea behind the model is that the time series vectors for the observables depend on an unseen state (Tuanh and Essi, 2021).

The simulation hypothesis is steadily gaining ground as a viewpoint deserving of careful scientific investigation. It asks us to investigate the possibility that we are living in a simulation. While some of its proponents are adamant that it is true and discuss how almost certain it is, others are more circumspect in their statements. The concept has received some criticism, with some in the scientific community criticizing it as unrealistic or pseudoscientific (Ellis,
Although claims of the hypothesis’s near certainty are false, the hypothesis can logically be given a non-zero probability. A concept that is ahead of its time does not exist. The extent to which we can mitigate any threats that may arise from them will depend on how quickly we address them, just like with many other concepts that pose potential existential risks (such as the technological singularity).

There are two types of simulations: Type I simulations, which have no intention of changing the physical rules or contents of the underlying reality, and Type II simulations, which do. All other things being equal, Type II simulations have lesser resolution than Type I simulations because it takes more bits of the simulating reality to produce one bit of the simulated reality compared to Type I simulations. That is, Type I simulations are the only ones we need to take into account if we want to maximize the resolution of a simulation. A Type I simulation may be thought of as a sampling of the quantum world because it does not attempt to change the contents or physical laws of the base reality. A Type I simulation may be considered a sampling of the quantum wave function of the simulating reality (Ozzy, 2023).

This paper aims at simulation study in the presence of unit root, structural break and regime shift for different sample size considered as small, medium and large. The remaining section of this paper is organized as follows: the section 2 is the literature review that related to this study. Section 3 discusses the methodology, model description and specification used. Section 4 presents the results of the study and the paper end with a conclusion.

Aynur (2020), investigated the relationship among economic growth and energy use in G20 countries over the period 1995-2016. Different unit root tests and cointegration tests were adopted. Results of unit-root test indicate that all variables are integrated at I(1).

Adigboye (2017), examined the impact of import and export on economic growth in Nigeria using Vector Autoregressive (VARs) technique through various types of structural analysis of Granger causality tests, impulse response functions, and forecast error variance decompositions to examine the dynamic effects of various shocks on macroeconomic variables. The results of VAR show that the predominant sources of Nigeria economic growth variation are due largely to “own shocks” and import-export trade innovations. Jitendra (2022) examined the relationship between unemployment, public expenditure, and economic growth in India over the period 1990-2021 by estimating the elasticity of economic growth using the Ordinary Least Square econometric approach. The variables taken were the unemployment rate and real gross domestic product as an indicator of economic growth. The results of the descriptive statistics show that the variables were not normally distributed. The stationarity test conducted through the application of the Augmented Dickey-Fuller (ADF) test indicated that the variables were stationary after detrending the series by the Hodrick-Prescott filter at the first difference.

Bildiricia and Turkmen (2020) analyzed the New Monetarist Phillips curve. The study aimed to ascertain the cointegration and causality relationships between inflation, GDP and unemployment in the USA. The Markov Switching –VAR was applied on quarterly data from 1957 second quarter to 2014 third quarter. The study identified 3 regimes and estimated different MS-VAR models and selected the best model based on the AIC and LR test.

Hering, Kazor and Kleiber (2015) introduced MSVAR model and demonstrated its flexibility in simulating wind vectors for 10-min, hourly and daily time series and for individual, locally-averaged and regionally-averaged time series. The parameter estimation and simulation algorithm were also presented along with a validation of the important statistical properties of each simulation scenario. Their result showed that MSVAR is a very flexible in characterizing a wide range of properties in the wind vector, and conclude with a discussion of extensions of this model and modeling choices that may be investigated for further improvements.

**METHODOLOGY, MODEL DESCRIPTION AND SPECIFICATIONS**

This study will use simulation to look at different sample sizes.

**Simulation Procedure**

In this study, simulation is used to explore how structural fractures may affect the test statistics that are used to find a unit root with various sample sizes, with 20 and 50 being considered small, 100 and 250 being considered medium, and 2500 and 5000 being considered large samples, respectively. Using the urca package for R software, the experiment is repeated 5000 times on the basis of Augmented Dickey Fuller test and Zivot-Andrews test, which allow for an endogenous structural break and regime shifts. Variables were created by adjusting the seed to provide results that are similar for a random walk, r, the time trend, dt, and an ar1 process simulation using the arima.sim command with structural breaks. Simulation was also carried out on Markov-Switching vector autoregressive model (MSVAR) to determine the period of recession and expansion in the dataset. The procedure is by giving a command that involves parameters as follows:

```
simulateMSVAR(bigt, m, p, var.beta0, var.betas, e.vcv, Q, seed)
```

Where;

- `bigt` – integer, number of observations to generate
- `m` – number of endogenous variables
- `p` – lag length of the VAR process
- `h` – number of regimes
- `var.beta0` – Array of dimension m x 1 x h of the VAR intercepts for each regime (h)
- `var.betas` – Array of dimension m x mp x h of the autoregressive coefficients. In each element of the array, rows correspond to equations, columns to lags. The first m x m columns are the AR(1) coefficients, etc.
- `e.vcv` – Array of dimension m x m x h of the error covariances. In the Ordinary Least Square approach, the variables are the error covariances for each regime.
- `Q` – h dimensional transition matrix for the MS process. H x h
- `seed` – Integer. A random number seed.

This function simulates a multivariate Markov-switching model (MSVAR) with m equations, p lags and h regimes. The assumption is that the error process is Gaussian.

**Model Description and Specifications**

**The augmented Dickey-Fuller (ADF) Test**

In order to determine the order of integration for each of the variables, it has become customary in the statistical analysis of macroeconomic time series to test the unit root hypothesis first. The augmented Dickey-Fuller (Dickey and Fuller, 1979, 1981) test is the most commonly used test for ascertaining the presence of unit root. It is based on the following regression in the case of trending data:

```
R - A(R - 1)x + b + e
```
\Delta y_t = \alpha + \phi y_{t-1} + \beta + t + \sum_{i=1}^{k} \theta_i \Delta y_{t-i} + \epsilon_t \quad (1)

This involves a regression of \Delta y_t on \Delta y_{t-1}, \Delta y_{t-2}, ..., \Delta y_{t-p} as well as an intercept \alpha and time trend \beta + t, t=1,2,...,T and \epsilon_t, a pure white noise disturbance with variance of \sigma^2. \Delta y_{t-1} is the lagged first differences to correct for serial autocorrelation in the errors. The ADF test is majorly concerned with the estimate of \alpha in the above equation, i.e. we test the hypothesis H_0: \alpha = 0. The rejection of the null hypothesis in favor of the alternative hypothesis implies that \gamma_t is stationary and integrated of order zero, that is, \gamma(0). If the null hypothesis of unit root for the first difference is rejected, the first difference is stationary and the variable is integrated of order one.

Zivot and Andrews Model

A problem common with the conventional unit root tests — such as the ADF, DF-GLS and PP tests, is that they do not allow for the possibility of a structural break. Assuming the time of the break as an exogenous phenomenon, Perron showed that the power to reject a unit root decreases when the stationary alternative is true and a structural break is ignored. Zivot and Andrews propose a variation of Perron's original test in which they assume that the exact time of the break-point is unknown. Instead a data dependent algorithm is used to proxy Perron's subjective procedure to determine the break points. Following Perron's characterization of the form of structural break, Zivot and Andrews proceed with three models to test for a unit root: (2) model A, which permits a one-time change in the level of the series; (3) model B, which allows for a one-time change in the slope of the trend function, and (4) model C, which combines one-time changes in the level and the slope of the trend function of the series. Hence, to test for a unit root against the alternative of a one-time structural break, Zivot and Andrews use the following regression equations corresponding to the three models.

\Delta Y_t = \beta_1 + \beta_2 T + \theta D U_t + \delta Y_{t-1} + \sum_{i=1}^{k} \rho_i \Delta Y_{t-i} + \epsilon_t \quad (2)

Model A

\Delta Y_t = \beta_1 + \beta_2 T + \gamma D T_t + \delta Y_{t-1} + \sum_{i=1}^{k} \rho_i \Delta Y_{t-i} + \epsilon_t \quad (3)

Model B

\Delta Y_t = \beta_1 + \beta_2 T + \theta D U_t + \gamma D T_t + \delta Y_{t-1} + \sum_{i=1}^{k} \rho_i \Delta Y_{t-i} + \epsilon_t \quad (4)

Model C

Where the dummy variable DU captures structural change in the intercept at time TB, DU = 1 if t > TB and zero otherwise; the dummy variable DT represents a change in the slope of the trend function (captures shift in the trend variable at time TB); DT = t-TB if t > TB and zero otherwise; TB denotes the time of break (Glynn et al., 2007).

From the equation above, model (A) allows for a one-time structural break in the intercept, model (B) allows for a one-time structural break in the slope whereas model (C) allows for a one-time structural break in both the intercept and the slope (Narayan and Smyth, 2004).

The null hypothesis under the three models is that the series has a unit root with a drift that excludes any structural breaks whereas the alternative hypothesis is that the series is a trend-stationary process with a one-time break occurring at an unknown point in time (Waheed et al., 2007).

Markov defined a stochastic process as a Markov process if the probabilities of future values in a time series only depend on its most recent value and are independent of earlier periods, that is, the value of the current can capture all information for its prior (Umeh and Anazoba, 2016).

In the Markov switching regime model, time series may change to another state, or stay in the current state at any time. The probability matrix is called the transition matrix. This study looks at the transition matrix for a two state, first order Markov chain is

\[ P = \begin{pmatrix} P_{11} & P_{21} \\ P_{12} & P_{22} \end{pmatrix} \quad (6) \]

where \( P_{i,j} = 1,2 \) denotes the probability that the time series move from regime \( j \) to \( i \). In other words, it is the probability that \( R_t \) is in the regime \( i \) conditional on which \( R_{t-1} \) is in the regime \( j \). \( P_{i,j} = 1,2 \) is,

\[ p_g = \Pr(S_t = i \mid S_{t-1} = j) \]

where \( p_{12} + p_{21} = 1 \) and \( p_{21} + p_{22} = 1 \).

The switching mechanism is controlled by an unobservable state variable \( S_t \), and it follows a Markov process. \( S_t \) is assumed to follow a two-state Markov process and the Markov process is assumed to be ergodic and irreducible.

The transition probabilities allow us with an expected duration that is, the length of time it takes for the system to stay in a particular regime. The expected duration is given as:

\[ E(D) = \frac{1}{1 - p_g} \quad (7) \]

where \( i = 1,2 \) and \( j = 1,2,... \)

D here stands for expected duration.

RESULTS AND DISCUSSION

The analysis was carried out using R software and the result is as below.
The result in table 1 above shows that at various sample size signifies that the null hypothesis cannot be rejected since the test statistics are not less than the critical values at 5% level and this indicates presence of unit root. After being first differenced, it indicates rejection of the null hypothesis of no unit root. Therefore, the variables were stationary of order 1 integration at small, medium and large sample size respectively.

Critical values are -5.08 and -4.8 at 5% for both level and first difference.

The result in table 2 at small, medium and large sample size reveals that there is presence of unit root at level but after first differencing of the dataset, it becomes stationary at order 1. This also allows for endogenous break at various breakpoints with respect to their sample size. It is also evident the plot that the break in the middle of the sample, clearly cuts through all the confidence intervals.

### Table 1: ADF unit root test at different sample size

<table>
<thead>
<tr>
<th>Category</th>
<th>n</th>
<th>Level</th>
<th>CV (5%)</th>
<th>Decision</th>
<th>1st Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>20</td>
<td>-1.699</td>
<td>-3.60</td>
<td>Do not reject</td>
<td>-3.120</td>
</tr>
<tr>
<td>Small</td>
<td>50</td>
<td>-2.590</td>
<td>-3.50</td>
<td>Do not reject</td>
<td>-4.574</td>
</tr>
<tr>
<td>Middle</td>
<td>100</td>
<td>-2.794</td>
<td>-3.45</td>
<td>Do not reject</td>
<td>-7.071</td>
</tr>
<tr>
<td>Middle</td>
<td>250</td>
<td>-1.952</td>
<td>-3.43</td>
<td>Do not reject</td>
<td>-11.485</td>
</tr>
<tr>
<td>Large</td>
<td>25000</td>
<td>-3.036</td>
<td>-3.41</td>
<td>Do not reject</td>
<td>-35.189</td>
</tr>
<tr>
<td>Large</td>
<td>50000</td>
<td>-2.578</td>
<td>-3.41</td>
<td>Do not reject</td>
<td>-49.833</td>
</tr>
</tbody>
</table>

### Table 2: Zivot-Andrews Unit Root Test

<table>
<thead>
<tr>
<th>Category</th>
<th>n</th>
<th>Level</th>
<th>TB</th>
<th>Decision</th>
<th>1st Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>20</td>
<td>-4.072</td>
<td>11</td>
<td>Do not reject</td>
<td>-6.524</td>
</tr>
<tr>
<td>Small</td>
<td>50</td>
<td>-4.708</td>
<td>31</td>
<td>Do not reject</td>
<td>-7.106</td>
</tr>
<tr>
<td>Middle</td>
<td>100</td>
<td>-4.009</td>
<td>59</td>
<td>Do not reject</td>
<td>-10.246</td>
</tr>
<tr>
<td>Middle</td>
<td>250</td>
<td>-4.573</td>
<td>73</td>
<td>Do not reject</td>
<td>-16.058</td>
</tr>
<tr>
<td>Large</td>
<td>25000</td>
<td>-5.051</td>
<td>889</td>
<td>Do not reject</td>
<td>-49.453</td>
</tr>
<tr>
<td>Large</td>
<td>50000</td>
<td>-4.629</td>
<td>2953</td>
<td>Do not reject</td>
<td>-70.825</td>
</tr>
</tbody>
</table>
Figures 1: Zivot-Andrews unit root plots at various sample size.

The figure 1 above represents the different break time at various sample size respectively demonstrating a shock in the process. Figure 2 below shows the plot for the values for a pure random walk, \( r \), the trend stationary process, \( dt \), includes just the deterministic trend and noise and an \( \text{ar1} \) process with a coefficient of 0.8. This implies that as the sample size increases, the process becomes more significant, reliable and stationary. Also, the plot has similar behaviour based on the sample size from small to medium and large sample size respectively.
Figure 2: Simulated plot of the samples.

Table 3: MSVAR model estimation

<table>
<thead>
<tr>
<th>Sample</th>
<th>Transition Probability</th>
<th>Expected Duration</th>
<th>Standard error</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>S=1 S=2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.92 0.82</td>
<td>11.97 5.60</td>
<td>0.099 0.226</td>
<td>0.672 0.903</td>
</tr>
<tr>
<td>50</td>
<td>0.82 0.85</td>
<td>6.66 6.81</td>
<td>0.164 0.048</td>
<td>0.575 0.696</td>
</tr>
<tr>
<td>100</td>
<td>0.94 0.92</td>
<td>15.96 12.09</td>
<td>0.041 0.078</td>
<td>0.681 0.729</td>
</tr>
<tr>
<td>250</td>
<td>0.86 0.84</td>
<td>6.94 6.16</td>
<td>0.017 0.022</td>
<td>0.629 0.776</td>
</tr>
<tr>
<td>2500</td>
<td>0.89 0.89</td>
<td>8.84 8.95</td>
<td>0.005 0.005</td>
<td>0.747 0.665</td>
</tr>
<tr>
<td>5000</td>
<td>0.85 0.87</td>
<td>8.69 7.83</td>
<td>0.003 0.004</td>
<td>0.654 0.757</td>
</tr>
</tbody>
</table>

Table 3 presents the estimation outcomes of the MSVAR model. The outcomes show that the simulated data was divided into two separate regimes. With the exception of sample sizes 50 and 2500, the standard error in regime 1 is lower than in regime 2. As a result, regime 1 can be regarded as having low volatility, and regime 2 as having high volatility. The table also shows that the average duration in the high-volatility regime is between 5 and 12 years, whereas the duration for the observations in the low-volatility regime ranges from 5 to 16 years. St=1 and St=2 imply that the low-volatility regime was more enduring than the high-volatility regime, respectively, validating the transition regime. Our results concur with those of Chkili and Nguyen (2014) and Kanas (2005). The data also demonstrates that the predicted coefficients of both regimes 1 and 2 are substantial and have a beneficial impact across all observations. At a 5% level of significance, the P-Values for all observations, regardless of sample size, reveal that they are highly significant, showing a strong rejection of the null hypothesis that there was no switching. This suggests that there is proof of regime changes.

The different moments in time a regime occurs are specified by the filtered probability charts. Due to their capacity to display the nature and timing of important changes in the data series, these MSVAR probability plots provide additional insights beyond those offered by the linear VAR framework (Okereke and Uwaeme, 2018). Additionally, it demonstrates that the filtered likelihood of its existence in regime shift behavior increases with sample size, making larger samples more trustworthy and meaningful.
Conclusion
This paper investigated the simulation of sample sizes having a unit root and structural breaks behaviour that was replicated five thousand times. The results revealed that both at small, medium and large sample size, there is presence of unit root at level but stationary at first difference. Zivot-Andrews result indicates that there is structural break according to the sample size which expresses shocks at one time and the other. The MSVAR results demonstrated that regime 1 is more resilient than regime 2, and that regime 1 is projected to last longer than regime 2. As a result, we draw the conclusion that simulation can be utilized to verify a real-world situation.

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