THE GEOMETRIC MEAN MODEL IN FINANCE

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The financial manager is increasingly involved in global product and financial markets. The globalization movement means that investment and financing decisions must be made in an international arena. International capital budgeting for example, embraces

... (1)

... (2)

... (3)

... (4)

... (5)

... (6)

If \( w_i = 1/n \) for each \( i \) then \( A_x, G_x \) and \( H_x \) are the usual unweighted means. Subsequently let the \( y \)- variate of the arithmetic, geometric and harmonic be given respectively by \( A_y, G_y \) and \( H_y \). It is easily shown that only the geometric mean possesses the reciprocity property, i.e.

... (7)

The relationships for the arithmetic geometric and harmonic are given in the form of lower and upper bounds, such that

... (8)

for any non-negative variate (Francis, 1995). Hence,

... (9)

... (10)

equality is attained only when the \( x' s \leq y' s \).

Daily average exchange rates
Relevant information about the daily fluctuations of the official exchange rate of a home currency with respect to the standard currencies, such as US dollar, Euro and the Pound Sterling, is important to the global banking community, researchers, potential investors, businessmen and traders.
Such information can be provided by the central bank of the country under consideration at the end of a given period (daily, weekly, etc.) rate or as average rate for a period (week, month, quarter, etc.).

Suppose there are \( n \) working days in the period and \( x_{i,t} \) is the exchange rate of the standard currency in terms of the home currency, i.e. one unit of the standard currency equals \( x_{i,t} \) units of the home currency in day \( t \). The geometric mean ensures that the average a unit of the foreign currency equals \( G_x \) units of the domestic currency and also equals \( G_x^{-1} \), units of the foreign currency.

**Nominal effective exchange rate index**

The price of a unit of foreign currency in terms of the domestic currency is regarded as the exchange rate of the domestic currency. In the exchange rate policy, a country might wish to determine whether its currency is overvalued or undervalued. If it is overvalued, the price of a unit of the foreign currency is relatively low and this could lead to excessive imports and have an adverse effect on the economy of the country. The extend of appreciation or depreciation of a domestic currency with respect to the foreign currencies, various exchange rates indices can be computed depending on the monetary systems of the country (Pandey, 1999).

**Geometric**

\[
GR_x = \prod_{i=1}^{n} x_{i,t}^{w_i} \quad \text{Arithmetic} \quad \quad AR_x = \sum_{i=1}^{n} w_i x_{i,t}
\]

**Harmonic**

\[
GR_y = \prod_{i=1}^{n} y_{i,t}^{w_i} \quad \text{Arithmetic} \quad \quad AR_y = \sum_{i=1}^{n} w_i y_{i,t}
\]

\[
HR_x = \left( \frac{\sum w_i}{\sum \frac{1}{x_{i,t}}} \right)^{-1} \quad \text{Harmonic} \quad \quad HR_y = \left( \frac{\sum w_i}{\sum \frac{1}{y_{i,t}}} \right)^{-1}
\]

It can easily be verified that:

\[
GR_x / GR_y = 1 \quad \text{... (15)}
\]

It is observed that only the geometric nominal effective exchange rate index possesses the reciprocity property given by equation (3). From equations (8) and (9) it is observed that

\[
AR_y \geq 1/AR_x \quad \text{... (16)}
\]

equality being attained when \( x_{i,t} \) and \( y_{i,t} \) are equal. The indices in real situation are computed by using the representative period's average exchange rates for \( x_{i,t} \) (or \( y_{i,t} \)) and comparing these with the corresponding base period averages.

According to Jean (1976), a commonly used nominal exchange rate index is one that measures the movements of the currencies of all the trading partners of a country relative to a base period, taking into account the weight of each currency as reflected by the volume of trade conducted in each standard currency.

Suppose that a given country trades with \( n \) foreign currencies. Let \( x_{i,t} \) represent the price of one unit of currency of the \( i \) th trading partner at time \( t \) in terms of the home currency. Let \( y_{i,t} \) be the price of the home currency at time \( t \) in term of the currency \( i \). The two exchange rates are joined by the equation

\[
x_{i,t} = 1/y_{i,t} \quad \text{... (11)}
\]

We shall express exchange rates relative to an initial period (t=0) in terms of \( x_{i,t} \) and \( y_{i,t} \) as

\[
x_{i,t} = x_{i,0}^{t} \quad \text{... (12)}
\]

\[
y_{i,t} = y_{i,0}^{t} \quad \text{... (13)}
\]

The effective exchange rate indices of a given set of weights \( w_i \) are defined either by the \( x \)-variate or the \( y \)-variate. The geometric effective exchange rate and the arithmetic effective rate indices can be defined as shown below:

**Numerical example**

Consider the computation of a nominal effective exchange rate index for the domestic currency \( N \) of a given country in 2007 relative to 2006.

We shall assume for simplicity that the country trades in only three foreign currencies B1, B2 and B3 and that the weights of these two currencies in 2006 are given by \( w_1 = 1/6 \), \( w_2 = 1/3 \) and \( w_3 = 1/2 \) respectively (assuming there is no variation in 2007), based on the share attributable to each currency in 2007 trade transactions. We shall also assume that exchange rates averaged for 2006 and 2007 are as given next page:
The computation of the arithmetic, geometric and the harmonic exchange rates indices gives the following values with the corresponding reciprocal of each.

**TABLE 1. EXCHANGE RATE INDICES AND THEIR RECIPROCAL.**

<table>
<thead>
<tr>
<th>Computed index</th>
<th>Reciprocal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$GR_x = 0.7595$</td>
<td>$GR_y = 1.3166$</td>
</tr>
<tr>
<td>$AR_x = 0.8367$</td>
<td>$AR_y = 1.4028$</td>
</tr>
<tr>
<td>$HR_x = 0.6698$</td>
<td>$HR_y = 1.8947$</td>
</tr>
</tbody>
</table>

The $GR_x$ value of 0.7595 is interpreted to indicate a depreciation of 0.2405 in 2007 of foreign currencies with respect to the domestic currency. $GR_y$ shows an appreciation of 0.3166 in 2007 of the domestic currency with respect to the foreign currencies. The geometric indices (unlike the arithmetic and harmonic indices) are consistent because of their reciprocity property. This implies that $GR_x$, is on average, the domestic currency value of 1 unit of the foreign currency in 2007 equals 0.7595 times the domestic currency value of 1 unit of the foreign currency in 2006.

Consistently the value of $GR_y$ implies that, on the average, the foreign currency value of 1 unit of the domestic currency in 2007 equals 1.7595 times the foreign currency value of 1 unit of the domestic currency in 2006.

**Exchange rate index**

The nominal effective exchange rate is a measure of the changes in a country’s exchange rate with respect to its trading partners relative to a based period.

These changes do not have any connection with the purchasing power of the currency, nor do they indicate the level of changes in the competition on tradable goods of the country over the periods. The real effective exchange rate index can be used to measure the changes in the purchasing power of a currency over time by determining relative index movements (Howard & Gareth, 1991). The real effective exchange rate index for a domestic currency known as geometric real effective exchange rate

$$GRR_y = \prod_{i=1}^{n} \left( \frac{y_{i,t}}{R_{i,t}} \right)^{w_i} \ldots (17)$$

where $y_{i,t}$ is given in (12), and $R_{i,t}$ is price index ratio of the first trading partner to the price index of the domestic currency in period $t$.

The applications of geometric mean discussed in this paper, could prove useful in the financial industry for international trade and business. It can also be used in monitoring the stability of the economy of a country with respect to its trading partners. It is a tool for making very useful discoveries.

**REFERENCES**

