Science World Journal Vol 10 (No 3) 2015
www.scienceworldjournal.org
ISSN 1597-6343

Full Length Research Article

A CASH FLOW EOQ INVENTORY MODEL FOR NON-DETERIORATING ITEMS WITH CONSTANT DEMAND

Dari S. and Ambrose D.C.

Department of Mathematical Sciences,Kaduna State University,Kaduna**.**

Corresponding author E-mail: **sanisdari@yahoo.com**

ABSTRACT
This study presents an inventory model to determine an optimal ordering policy for non-deteriorating items and time independent demand rate with delay in payments permitted by the supplier under inflation and time discounting, and the rate is assumed to be constant.This study determines the best cycle period and optimal payment period for items so that the annual total relevant cost is minimized. The main purpose of this study is to investigate the optimal (minimum) total present value of the costs over the time horizon H for both cases where the demand is fixed (constant) at any time. This study work is limited to only non-deteriorating goods with constant demand and with a permissible delay in payment.Numerical example and sensitivity analysis are given to evince the applicability of the model.

Keywords: Demand, Inventory, Non-Deterioration, Inflation, Delay in payments, Replenishments.

INTRODUCTION
In a continuous, or fixed-order-quantity, system when inventory reaches a specific level, referred to as the *reorder point,* a fixed amount is ordered. The most widely used and traditional means for determining how much to order in a continuous system is the economic order quantity (EOQ) model, also referred to as the economic lot-size model. The earliest published derivation of the basic EOQ model formula in 1915 is credited to Ford Harris, an employee at Westinghouse. The economic order quantity (EOQ) is the order quantity that minimizes total holding and ordering costs for the year. Even if all the predictions don't hold accurately, the EOQ gives us a good indication of whether or not present order quantities are reliable (Bozarth, 2011). The economic order quantity, as mentioned, is the order size that minimizes the sum of carrying costs and ordering costs. These two costs react inversely to each other. As the order size increases, fewer orders are required, causing the ordering cost to decline, whereas the average amount of inventory on hand will increase, resulting in an increase in carrying costs. Thus, in effect, the optimal order quantity represents a compromise between these two inversely related costs.
Of recent, Ouyang *et al.,* (2006) developed a model for non-instantaneous deteriorating items under permissible delay in payment. An EOQ model under conditionally permissible delay in payments was developed by Huang (2007). Jaggi *et al.,* (2008) developed a model retailer's optimal replenishment decisions with credit- linked demand under permissible delay in payments. Optimal retailer's ordering policies in the EOQ model for deteriorating items under trade credit financing in supply chain was developed by Mahata and Mahata (2009). Musa and Sani (2009) developed an EOQ model for items that exhibit delay in deterioration. Hou and Lin (2009) developed "a Cash Flow Oriented EOQ model with delay in payment for deteriorating goods". Chiu *et al.,* (2010) jointly determined economic batch size and optimal number of deliveries for EPQ model with quality assurance issue. Feng *et al.,* 2010 presents an algebraic approach adopted to re-examine Chiuet *et al.,*'s model (2010).Tripathi *et al.,* (2010a), obtained and ordering policy for non-deteriorating items and time dependent demand rate under inflation and time discounting. Tripathi *et al.,* (2010b) developed EOQ model credit financing in economic ordering policies of non-deteriorating items with time- dependent demand rate in the presence of trade credit using a discounted cash-flow (DCF) approach. Tripathi *et al.,* (2011) developed an inventory model for deteriorating items over a finite planning horizon considering the effects of inflation and permissible delay in payment.Musa and Sani (2012) also developed an EOQ model for items that exhibit delay in deterioration under permissible delay in payment. Also, Amutha *et al.,* (2013) presented an inventory model for constant demand with shortages which are completely backloggedThere are also inventory model with permissible delay in payment for linear demand rate of non-deteriorating goods. Dari and Sani (2013) developed an EPQ model for items that exhibit delay in deterioration with reliability considerationfor which the demand before and after deterioration are assumed to be constant.
In this paper, we develop an EOQ inventory model that determine an optimal ordering policy for non-deteriorating items and time independent demand rate with delay in payments permitted by the supplier under inflation and time discounting, and the rate is assumed to be constant. Thus, this paper is an extension of Tripathi *et al.,*(2010a).

NOTATIONS
Y (*t*): Inventory at any time *t*
Q: Order quantity, units/cycle
H: Length of planning horizon
L: Replenishment cycle time
n: Number of replenishment during the planning horizon, *n* = *H/n*
5 and rate per unit time
ler cost at time 't' is zero, ₦/order
c: Per unit cost of the item, ₦ /unit
h: Inventory holding cost per unit per unit time excluding interest charges,
r: Discount rate represent the time value of money
f: Inflation rate
k: The net discount rate of inflation ($k = r - f$)
Y_e: The interest earned per dollar in stocks per unit time by the

Science World Journal Vol 10 (No 3) 2015
www.scienceworldjournal.org
ISSN 1597-6343

supplier
Y_c: The interest charged per dollar in stocks per unit time by the supplier,
m: The permissible delay in settling account
ip: Interest payable during the first replenishment cycle
$z_1(n)$: The total present value of the cost over the time horizon *H*, for $m \leq L = H/n$
$z_2(n)$: The total present value of the cost for $m > L = H/n$
E: The interest earned during the first replenishment cycle
E_1: The present value of the total interest earned over the time horizon
Y_p: The total interest payable over the time horizon *H*

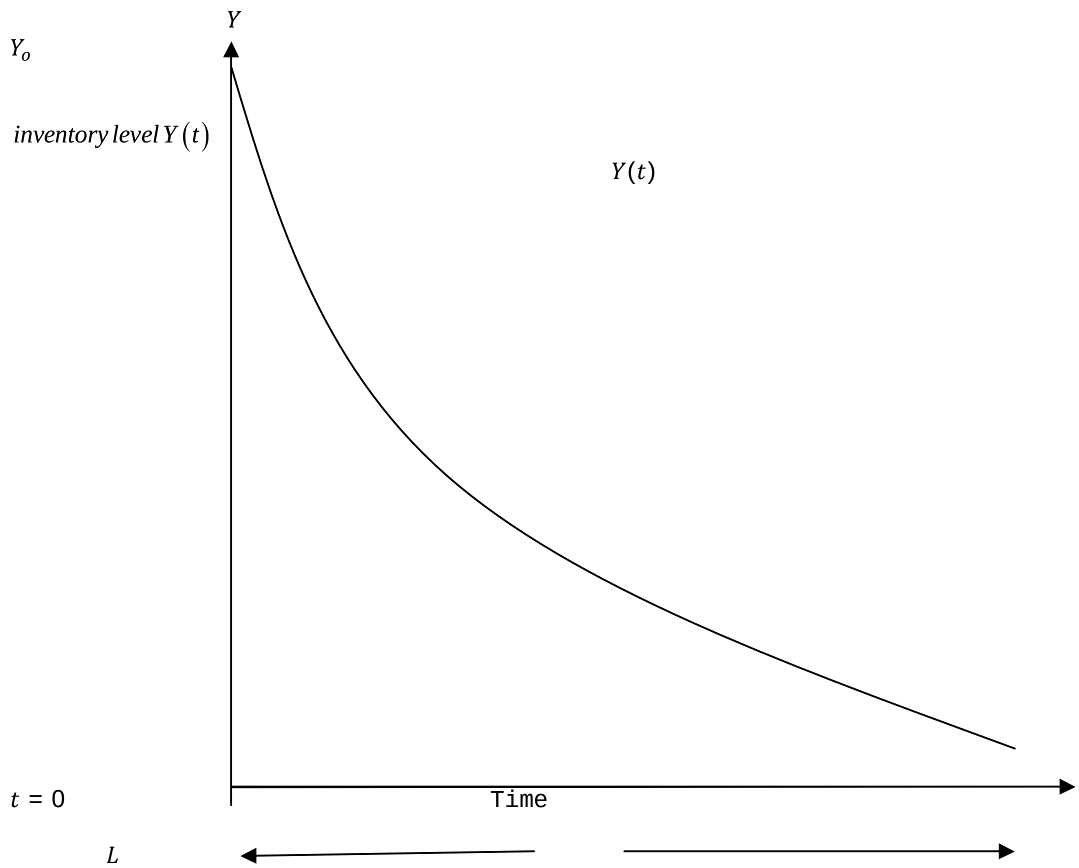

Fig 1: Inventory movement in a non-instantaneous deterioration situation

MATHEMATICAL FORMULATION
To develop the mathematical model, the following assumptions are being made:

- The demand *D*is constant.
- Shortages are not allowed.
- Lead time is zero.
- The net discount rate of inflation rate is constant.
- **caseI**: *m*(permissible delay in settling account) ≤ *L* (Replenishment cycle time).
- **caseII**: *m*(permissible delay in settling account) >*L* (Replenishment cycle time).

The inventory *Y* (*t*) at any time *t* is depleted by the effect of demand only. Thus the variation of *Y*(*t*) with respect to 't' is governed by the following differential equation

$$\frac{dY(t)}{dt} = -D \qquad 0 \leq t \leq L = H/n \qquad (1)$$

Solving(1), we have

$$Y(t) = -Dt + k \qquad \text{(where } k \text{ is a constant)} \tag{2}$$

From *fig1,* we have the initial condition $Y\left(t\right) = Y_0$ at $t = 0$. Substituting this in (2), we get

$$Y(t) = -Dt + Y_0 \tag{3}$$

From *fig1,* Y(*L*) = 0. Substituting in (3), we have

$$Y_0 = DL \tag{4}$$

Substituting (4) into (3), we get

$$Y(t) = -Dt + DL \tag{5}$$

The initial inventory Y_0 (order quantity) after replenishment is given by

$$Q = Y\left(0\right) = DL \tag{6}$$

The total present value of the replenishment cost is given by:

$$C_1 = \sum_{j=0}^{n-1} A_0 e^{-jkL} = A_0 \left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] \qquad (L = H/n) \tag{7}$$

The total present value of the purchasing costs is given by:

$$C_2 = c\sum_{j=0}^{n-1} Y(0) e^{-jkL} = cQ\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] \qquad (L = H/n) \tag{8}$$

The present value of the total holding costs over the time horizon *H* is given by

$$A = h\sum_{j=0}^{n-1} e^{-jkL} \int_0^L Y(t) e^{-kt} dt$$

$$= h\sum_{j=0}^{n-1} e^{-jkL} \left[D\left(\frac{L}{k} + \frac{e^{-kL}-1}{k^2}\right)\right]$$

$$A = h\left[D\left(\frac{L}{k} + \frac{e^{-kL}-1}{k^2}\right)\right]\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] \tag{9}$$

Case I. *m* ≤ t ≤*L= H/n*

The total value of the interest payable during the first replenishment cycle is given by

$$i_p = cl_c \int_m^L Y(t) e^{-kt} dt$$

$$= cl_c \left[-DL\frac{e^{-kt}}{k} + Dt\frac{e^{-kt}}{k} + \frac{D}{k^2}e^{-kt}\right]_m^L$$

$$i_P = cl_c\left[Q\left(\frac{e^{-km}}{k}\right) - D\left(\frac{me^{-km}}{k} + \frac{e^{-kL}-e^{-km}}{k^2}\right)\right] \quad \text{Where } Q = DL \tag{10}$$

The total present value of the interest payable over the time horizon *H* is given by

$$Y_p = \sum_{j=0}^{n-1} i_p e^{-jkL} = i_p\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] \tag{11}$$

Also, the total value of the interest earned during the first replenishment cycle is

$$E = cl_e \int_0^L Dte^{-kt} dt$$

$$= cl_e D\left[-t\frac{e^{-kt}}{k} - \frac{e^{-kt}}{k^2}\right]_0^L$$

$$E = cl_e\left[D\left(-L\frac{e^{-kL}}{k} + \frac{1-e^{-kL}}{k^2}\right)\right] \quad (12)$$

Thus the present value of the total interest earned over the time horizon *H* is

$$E_1 = \sum_{j=0}^{n-1} Ee^{-jkL} = E\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] \quad (13)$$

Therefore, the total present value of the costs over the time horizon *H* is

$$Z_1(L) = C_1 + C_2 + A + Y_P - E_1 \quad (14)$$

Substituting *(7),(8),(9),(11)*and*(13)* into *(14),* we have

$$Z_1(L) = A_0\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] + cQ\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] + h\left[D\left(\frac{L}{k} + \frac{e^{-kL}-1}{k^2}\right)\right]\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] + i_p\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right]$$
$$-E\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] \quad (15)$$

Case II. *m >L= H/n*

The interest earned in the first cycle is the interest during the time period (0, *L*) plus the interest earned from the cash invested during the time period (*L*, *m*) after the inventory is exhausted at time *L*and it is given by:

$$E_2 = cl_e\left[\int_0^L R(t)te^{-kt}dt + (m-L)e^{-kT}\int_0^L R(t)dt\right]$$

$$= cl_e\left[D\left(-L\frac{e^{-kT}}{k} + \frac{1-e^{-kL}}{k^2}\right) + (m-L)e^{-kT}\left[Dt\right]_0^L\right]$$

$$E_2 = cl_c\left[D\left(-L\frac{e^{-kL}}{k} + \frac{1-e^{-kL}}{k^2}\right) + (m-L)e^{-kL}DL\right] \quad (16)$$

Hence, the present value of the total interest earned over the time horizon *H* is

$$E_3 = \sum_{j=0}^{n-1} E_2 e^{-jkL} = E_2\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] \quad (17)$$

Therefore, the total present value of the costs $Z_2(n)$ over the time horizon *H* is

$$Z_2(L) = C_1 + C_2 + A - E_3 \quad (18)$$

Substituting *(7),(8),(9)* and *(17)* into *(18),* we have

$$Z_2(L) = A_0\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] + cQ\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] + h\left[D\left(\frac{L}{k} + \frac{e^{-kL}-1}{k^2}\right)\right]\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right]$$
$$-cl_c\left[D\left(-T\frac{e^{-kL}}{k} + \frac{1-e^{-kL}}{k^2}\right) + (m-L)e^{-kL}DL\right]\left[\frac{1-e^{-kH}}{1-e^{-kL}}\right] \quad (19)$$

OPTIMAL SOLUTION

To find the best value of L which optimizes (minimizes/maximizes) $Z_1(L)$ and $Z_2(L)$, we take the first derivative of $Z_1(L)$ and $Z_2(L)$ with respect to *L* and equate the result to zero.Also, to find if $Z_1(L)$ or $Z_2(L)$ is optimal(minimization/maximization) solution,we take the second derivative of $Z_1(L)$ and $Z_2(L)$ with respect to *L*. If the result is less than zero is a maximization case and if the result is greater than zero is a minimization case.

Taking the first derivative of (18) with respect to *L*, we have

$$\frac{dZ_1}{dL}=\frac{dC_1}{dL}+\frac{dC_2}{dL}+\frac{dA}{dL}+\frac{dY_P}{dL}-\frac{dE_1}{dL} \tag{20}$$

Now from (7), using Mclauren's expansion of exponential powers, we have

$$C_1 = A_0\left(1-e^{-KH}\right)\left(1-e^{-KL}\right)^{-1}$$

$$=A_0\left[1-\left(1-KH+\frac{(KH)^2}{2!}-\frac{(KH)^3}{3!}\right)\right]\left(1-\left[1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right]\right)^{-1}$$

$$C_1 = A_0\left[KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1} \tag{21}$$

Taking the first and second derivative of (21)with respect *L* respectively, we have

$$\frac{dC_1}{dL}=-A_0\left[KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right]\left[K-K^2L+\frac{K^3L^2}{2!}\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2} \tag{22}$$

and

$$\frac{d^2C_1}{dT^2}=-A_0\left[KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right]\left[\begin{array}{l}-2\left(K-K^2L+\frac{K^3L^2}{2!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-3}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\\ +\left(-K^2+K^3L\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\end{array}\right] \tag{23}$$

From (8), we have

$$C_2 = CDL\left(1-e^{-KH}\right)\left(1-e^{-KL}\right)^{-1}$$

$$=CDL\left(KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right)\left(KL-\frac{(KL)^2}{2!}\frac{(KL)^3}{3!}\right)^{-1} \tag{24}$$

Taking the first and second derivative of (24)with respect *L* respectively, we have

$$\frac{dC_2}{dL}=CD\left(KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right)\left[\begin{array}{l}\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\\ -L\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(K-K^2L+\frac{K^3L^2}{2!)}\right)\end{array}\right] \tag{25}$$

and

$$\frac{d^2C_2}{dT^2}=CD\left(KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right)\begin{bmatrix}-2\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\\+2L\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-3}\left(K-K^2L+\frac{K^3L^2}{2!}\right)^2\\-L\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(-K^2+K^3L\right)\end{bmatrix} \quad (26)$$

From (9), we get

$$A=\frac{D}{K^2}\left(KL+e^{-KL}-1\right)\left(1-e^{-KH}\right)\left(1-e^{-KL}\right)^{-1}$$

$$A=\frac{D}{K^2}\left[KL+1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}-1\right]\left(1-\left[1-KH+\frac{(KH)^2}{2!}-\frac{(KH)^3}{3!}\right]\right)\left[1-\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\right]^{-1}$$

(27)

Taking the first and second derivative of (27)with respect *L* respectively, we have

$$\frac{dA}{dL}=\frac{D}{K^2}\left(KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right)\begin{bmatrix}\left(K^2L-\frac{K^3L^2}{2!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\\-\left(\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\end{bmatrix}$$

(28)

and

$$\frac{d^2A}{dL^2}=\frac{D}{K^2}\left(KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right)\begin{bmatrix}\left(K^2-K^3L\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\\-2\left(K^2L-\frac{K^3L^2}{2!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\\+2\left(\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-3}\left(K-K^2L+\frac{K^3L^2}{2!}\right)^2\\-\left(\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(-K^2+K^3L\right)\end{bmatrix}$$

(29)

From (13), we get

$$Y_P = Cl_c \frac{D}{K^2}\left(KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!}\right)\left[\left(\left(\begin{array}{l} -KL - \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \\ +Km - \frac{(Km)^2}{2!} + \frac{(Km)^3}{3!} \end{array}\right) + K\left(L-m\right)\left(1 - Km + \frac{(Km)^2}{2!} - \frac{(Km)^3}{3!}\right)\right)\right]\left(KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right) \tag{30}$$

Taking the first and second derivative of (30) with respect *L* respectively, we have

$$\frac{dY_P}{dL} = Cl_c \frac{D}{K^2}\left(KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!}\right)\left(\begin{array}{l} \left[\left(\left(-K + K^2L - \frac{K^3L^2}{2!}\right) + K\left(1 - Km + \frac{(Km)^2}{2!} - \frac{(Km)^3}{3!}\right)\right)\right]\left(KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right)^{-1} \\ + \left[\left(\left(\begin{array}{l} -KL - \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \\ +Km - \frac{(Km)^2}{2!} + \frac{(Km)^3}{3!} \end{array}\right) + K\left(L-m\right)\left(1 - Km + \frac{(Km)^2}{2!} - \frac{(Km)^3}{3!}\right)\right)\right]\left(KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right)^{-1}\left(K - K^2L + \frac{K^3L^2}{2!}\right) \end{array}\right) \tag{31}$$

and

$$\frac{d^2Y_P}{dL^2}=Cl_c\frac{D}{K^2}\left(KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right)\left(\begin{array}{l}\left(K^2-K^3L\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\\ -\left[\left(\begin{array}{l}\left(-K+K^2L-\frac{K^3L^2}{2!}\right)\\ +K\left(1-Km+\frac{(Km)^2}{2!}-\frac{(Km)^3}{3!}\right)\end{array}\right)\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\\ +\left[\left(\begin{array}{l}\left(-K+K^2L-\frac{K^3L^2}{2!}\right)\\ +K\left(1-Km+\frac{(Km)^2}{2!}-\frac{(Km)^3}{3!}\right)\end{array}\right)\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\\ -\left[\left(\begin{array}{l}\left(\begin{array}{l}-KL-\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\\ +Km-\frac{(Km)^2}{2!}+\frac{(Km)^3}{3!}\end{array}\right)\\ +K\left(L-m\right)\left(1-Km+\frac{(Km)^2}{2!}-\frac{(Km)^3}{3!}\right)\end{array}\right)\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(K-K^2L+\frac{K^3L^2}{2!}\right)^2\\ +\left[\left(\begin{array}{l}\left(\begin{array}{l}-KL-\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\\ +Km-\frac{(Km)^2}{2!}+\frac{(Km)^3}{3!}\end{array}\right)\\ +K\left(L-m\right)\left(1-Km+\frac{(Km)^2}{2!}-\frac{(Km)^3}{3!}\right)\end{array}\right)\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\left(-K^2+K^3L\right)\end{array}\right) \qquad (32)$$

From (16), we have

$$E_1=Cl_e\left(KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right)\left[\begin{array}{l}-\frac{D}{K}\left(L\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\right)\\ +\frac{D}{K^2}\left(KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\end{array}\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1} \qquad (33)$$

Taking the first and second derivative of (33) with respect *L* respectively, we have

$$\begin{aligned}\frac{dE_1}{dL} = {} & Cl_e\left(KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right)\begin{bmatrix}-\frac{D}{K}\left(\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\right)\\ -\frac{D}{K}\left(L\left(-K+K^2L-\frac{K^3L^2}{2!}\right)\right)\\ +\frac{D}{K^2}\left(K+K^2L-\frac{K^3L^2}{2!}\right)\end{bmatrix}\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\\ & -Cl_e\left(KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right)\begin{bmatrix}-\frac{D}{K}\left(L\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\right)\\ +\frac{D}{K^2}\left(KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\end{bmatrix}\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\end{aligned} \tag{34}$$

and

$$\frac{d^2E_1}{dL^2} = Cl_e\left(KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right)\left[\begin{aligned} & \begin{bmatrix}-\frac{2D}{K}\left(\left(-K+K^2L-\frac{K^3L^2}{2!}\right)\right)\\ -\frac{D}{K}\left(L\left(K^2-K^3L\right)\right)\\ +\frac{D}{K^2}\left(K^2-K^3L\right)\end{bmatrix}\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\\ & -2\begin{bmatrix}-\frac{D}{K}\left(\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\right)\\ -\frac{D}{K}\left(L\left(-K+K^2L-\frac{K^3L^2}{2!}\right)\right)\\ +\frac{D}{K^2}\left(K+K^2L-\frac{K^3L^2}{2!}\right)\end{bmatrix}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\\ & +2\begin{bmatrix}-\frac{D}{K}\left(L\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\right)\\ +\frac{D}{K^2}\left(KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\end{bmatrix}\left(K-K^2L+\frac{K^3L^2}{2!}\right)^2\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-3}\\ & -\begin{bmatrix}-\frac{D}{K}\left(L\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\right)\\ +\frac{D}{K^2}\left(KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\end{bmatrix}\left(-K^2+K^3L\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\end{aligned}\right] \tag{35}$$

From (21), we get

$$E_3 = Cl_e D\left(KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!}\right)\left(\begin{bmatrix} -\frac{L}{K}\left(1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!}\right) \\ +\frac{1}{K^2}\left(KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right) \\ +(m-L)L\left(1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!}\right) \end{bmatrix}\left(KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right)^{-1}\right) \quad (36)$$

Taking the first and second derivative of (36) with respect L respectively, we have

$$\frac{dE_3}{dL} = Cl_e D\left(KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!}\right)\left(\begin{array}{l} \begin{bmatrix} -\frac{1}{K}\left(1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!}\right) \\ -\frac{L}{K}\left(-K + K^2L - \frac{K^3L^2}{2!}\right) \\ +\frac{1}{K^2}\left(K - K^2L + \frac{K^3L^2}{2!}\right) \\ -L\left(1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!}\right) \\ +(m-L)\left(1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!}\right) \\ +(m-L)L\left(-K + K^2L - \frac{K^3L^2}{2!}\right) \end{bmatrix}\left(KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right)^{-1} \\ -\begin{bmatrix} -\frac{L}{K}\left(1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!}\right) \\ +\frac{1}{K^2}\left(KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right) \\ +(m-L)L\left(1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!}\right) \end{bmatrix}\left(K - K^2L + \frac{K^3L^2}{2!}\right)\left(KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right)^{-2} \end{array}\right) \quad (37)$$

and

$$
\begin{aligned}
\frac{d^2E_3}{dL^2} = Cl_eD\left(KH-\frac{(KH)^2}{2!}+\frac{(KH)^3}{3!}\right)\Bigg(
&\left[\begin{aligned}
&-\frac{2}{K}\left(-K+K^2L-\frac{K^3L^2}{2!}\right)-\frac{L}{K}\left(K^2-K^3L\right)\\
&+\frac{1}{K^2}\left(-K^2+K^3L\right)-2\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\\
&+2\left(m-2L\right)\left(-K+K^2L-\frac{K^3L^2}{2!}\right)+\left(m-L\right)L\left(K^2-K^3L\right)
\end{aligned}\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\\
&-\left[\begin{aligned}
&-\frac{1}{K}\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)-\frac{L}{K}\left(-K+K^2L-\frac{K^3L^2}{2!}\right)\\
&+\frac{1}{K^2}\left(K-K^2L+\frac{K^3L^2}{2!}\right)-L\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\\
&+\left(m-L\right)\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\\
&+\left(m-L\right)L\left(-K+K^2L-\frac{K^3L^2}{2!}\right)
\end{aligned}\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\\
&-\left[\begin{aligned}
&-\frac{1}{K}\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)-\frac{L}{K}\left(-K+K^2L-\frac{K^3L^2}{2!}\right)\\
&+\frac{1}{K^2}\left(K-K^2L+\frac{K^3L^2}{2!}\right)+\left(m-2L\right)\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\\
&+\left(m-L\right)L\left(-K+K^2L-\frac{K^3L^2}{2!}\right)
\end{aligned}\right]\left(K-K^2L+\frac{K^3L^2}{2!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\\
&-\left[\begin{aligned}
&-\frac{L}{K}\left(1-KL+\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)+\frac{1}{K^2}\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)\\
&+\left(m-L\right)L\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)
\end{aligned}\right]\left(-K^2+K^3L\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\\
&+2\left[\begin{aligned}
&-\frac{L}{K}\left(1-KL+\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)\\
&+\frac{1}{K^2}\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)\\
&+\left(m-L\right)L\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)
\end{aligned}\right]\left(K-K^2L+\frac{K^3L^2}{2!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-3}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\Bigg)
\end{aligned}
\tag{38}
$$

Substituting (22), (25), (28), (31) and (34) into (20), and setting the result to zero, we have

$$-A_0\left[K-K^2L+\frac{K^3L^2}{2!}\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}$$
$$+CD\left[1-L\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(K-K^2L+\frac{K^3L^2}{2!)}\right)\right]$$
$$+\frac{D}{K^2}\left[\left(K^2L-\frac{K^3L^2}{2!}\right)-\left(\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\right]$$
$$+Cl_c\frac{D}{K^2}\left(\begin{array}{l}\left[\left(-K+K^2L-\frac{K^3L^2}{2!}\right)+K\left(1-Km+\frac{(Km)^2}{2!}-\frac{(Km)^3}{3!}\right)\right]\\ +\left[\begin{array}{l}\left(\begin{array}{l}-KL-\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\\ +Km-\frac{(Km)^2}{2!}+\frac{(Km)^3}{3!}\end{array}\right)\\ +K\left(L-m\right)\left(1-Km+\frac{(Km)^2}{2!}-\frac{(Km)^3}{3!}\right)\end{array}\right]\left(K-K^2L+\frac{K^3L^2}{2!}\right)\end{array}\right)$$
$$-Cl_e\left[\begin{array}{l}-\frac{D}{K}\left(\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\right)-\frac{D}{K}\left(L\left(-K+K^2L-\frac{K^3L^2}{2!}\right)\right)\\ +\frac{D}{K^2}\left(K+K^2L-\frac{K^3L^2}{2!}\right)\end{array}\right]$$
$$-Cl_e\left[\begin{array}{l}-\frac{D}{K}\left(L\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\right)\\ +\frac{D}{K^2}\left(KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\end{array}\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\left(K-K^2L+\frac{K^3L^2}{2!}\right)=0$$

(39)

If other variables of (39) are known, then (39) can be use to determine the best (optimal) value of *L* which minimize the total variable cost per unit time, provided that $\frac{d^2Z_1}{dL^2}>0$

Now the second derivative of (15)with respect to L is

$$\frac{d^2Z_1}{dL^2}=\frac{d^2C_1}{dL^2}+\frac{d^2C_2}{dL^2}+\frac{d^2A}{dL^2}+\frac{d^2Y_P}{dL^2}-\frac{d^2E_1}{dL^2} \quad (40)$$

Substituting (23), (26), (29), (32) and (35) into (40), we have

$$\frac{d^2Z_1}{dL^2}>0 \quad (41)$$

Thus, $Z_1$is a minimization function.

Similarly for **case II**, taking the first derivative of (19)with respect to *L*, we have

$$\frac{dZ_2}{dL}=\frac{dC_1}{dL}+\frac{dC_2}{dL}+\frac{dA}{dL}+\frac{dY_P}{dL}-\frac{dE_3}{dL} \tag{42}$$

Substituting (22), (25), (28), (31) and (37) into (42), and setting the result to zero, we have

$$\begin{aligned}
&-A_0\left[K-K^2L+\frac{K^3L^2}{2!}\right]\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1} \\
&+CD\left[1-L\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-2}\left(K-K^2L+\frac{K^3L^2}{2!)}\right)\right] \\
&+\frac{D}{K^2}\left[\left(K^2L-\frac{K^3L^2}{2!}\right)-\left(\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\left(K-K^2L+\frac{K^3L^2}{2!}\right)\right] \\
&+Cl_c\frac{D}{K^2}\left(\begin{array}{l}\left[2\left(-K+K^2L-\frac{K^3L^2}{2!}\right)+K\left(1-Km+\frac{(Km)^2}{2!}-\frac{(Km)^3}{3!}\right)\right] \\ +\left[\begin{array}{l}\left(\begin{array}{l}-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!} \\ +Km-\frac{(Km)^2}{2!}+\frac{(Km)^3}{3!}\end{array}\right) \\ +K\left(L-m\right)\left(1-Km+\frac{(Km)^2}{2!}-\frac{(Km)^3}{3!}\right)\end{array}\right]\left(K-K^2L+\frac{K^3L^2}{2!}\right)\end{array}\right) \\
&-Cl_e\left[-\frac{D}{K}\left(\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\right)-\frac{D}{K}\left(L\left(-K+K^2L-\frac{K^3L^2}{2!}\right)\right)\right] \\
&-Cl_eD\left(\begin{array}{l}\left[\begin{array}{l}-\frac{1}{K}\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)-\frac{L}{K}\left(-K+K^2L-\frac{K^3L^2}{2!}\right) \\ +\frac{1}{K^2}\left(K-K^2L+\frac{K^3L^2}{2!}\right)-L\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right) \\ +(m-L)\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)+(m-L)L\left(-K+K^2L-\frac{K^3L^2}{2!}\right)\end{array}\right] \\ -\left[\begin{array}{l}-\frac{L}{K}\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right) \\ +\frac{1}{K^2}\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right) \\ +(m-L)L\left(1-KL+\frac{(KL)^2}{2!}-\frac{(KL)^3}{3!}\right)\end{array}\right]\left(K-K^2L+\frac{K^3L^2}{2!}\right)\left(KL-\frac{(KL)^2}{2!}+\frac{(KL)^3}{3!}\right)^{-1}\end{array}\right)=0
\end{aligned}$$

(43)

If other variables of (43) are known, (43) can be use to determine the best (optimal) value of L which minimize the total variable cost per unit time, provided that $\frac{d^2Z_2}{dL^2} > 0$

$$\frac{d^2Z_2}{dL^2} = \frac{d^2C_1}{dL^2} + \frac{d^2C_2}{dL^2} + \frac{d^2A}{dL^2} + \frac{d^2Y_P}{dL^2} - \frac{d^2E_3}{dL^2} \quad (44)$$

Substituting(23), (26), (29), (32) and (38) into (44), we have

$$\frac{d^2Z_2}{dL^2} > 0 \quad (45)$$

Thus, $Z_2$is a minimization function.

Now, the second derivative of (19) with respect to T is

NUMERICAL EXAMPLES

The following data gives an example to illustrate the result of the model developed in this study: demand (D) = 700unit, A_0 = ₦80 unit^{-1}, holding cost (h) = 2.6/unit /order, unit cost an item (c) = ₦15, net discount rate of inflation (k) = ₦0.2 /year, interest charged per naira in stocks per year by the supplier(Y_c)= ₦ 0.16, interest earned per naira per year, (Y_e)=₦ 0.14, and the planning horizon, H is 5 year. The permissible delay in settling account, m =70/365 years. The computational outcome is shown in table1. In the numerical example we see that the case I is optimal in credit policy. The minimum total present value of cost is obtained when the number of replenishment, n, is 85. With 85 replenishment the optimal (minimum) cycle time is 0.233 year, the optimal (minimum) order quantity, Q= 163.01 units and the optimal (minimum) total present value of cost, Z= ₦ 35233.518. This means that the marketing department, production department and finance department are in an enterprise jointly to find the policy. Therefore, the policy involves inventory financing and marketing issue. So, we investigate that this model is very important and valuable to the enterprise.

Table1: Table of numerical examples

Cases	Order no (n)	Cycle Time L	Order Quantity(Q) units	Total Cost Z(n)
I	81	0.221917808	155.3424658	35235.03808
	82	0.224657534	157.260274	35233.9747
	83	0.22739726	159.1780822	35233.37771
	84	0.230136986	161.0958904	35233.23067
	85*	**0.232876712***	**163.0136986***	**35233.51787***
	86	0.235616438	164.9315068	35234.22438
	87	0.238356164	166.8493151	35235.33592
	88	0.24109589	168.7671233	35236.83887
	89	0.243835616	170.6849315	35238.72022
	90	0.246575342	172.6027397	35240.96755
II	91	0.249315068	174.5205479	35901.22265
	92	0.252054795	176.4383562	35907.759
	93	0.254794521	178.3561644	35914.46419
	94	0.257534247	180.2739726	35921.32909
	95	0.260273973	182.1917808	35928.34487

***OPTIMAL SOLUTION:** Table1 above shows the results of the two cases (case I and caseII) of the numerical examples at different point to test the applicability of the model developed. The optimal option in credit policy and the optimal total present value of the costs is obtain when the number of replenishment, n is 85, order quantity, Q=163.01 with a total present value of the cost, z=35233.51787.

SENSITIVITY ANALYSIS

Taking the parameters at the optimal point, changes in decrease of 5%, 10% and an increase in 5%, 10% of the demand, D, the holding cost, A_O, the per unit cost, c, net discount rate of inflation, k, can seen in the table2 below.

Table 2: Table of sensitivity analysis

Parameters	Change in the Parameters	% Change in Z
D	-10	-0.01384
	-5	-0.00692
	+5	0.006917
	+10	0.013835
A	-10	-0.01384
	-5	-0.00197
	+5	0.001971
	+10	0.003942
H	-10	-0.07368
	-5	-0.03684
	+5	0.036841
	+10	0.073682
C	-10	-0.63287
	-5	-0.31643
	+5	0.316434
	+10	0.632868
K	-10	-9.17528
	-5	10.64787
	+5	-9.17528
	+10	-17.1108
L_e	-10	1.088215
	-5	0.544108
	+5	-0.54411
	+10	-1.08822
L_c	-10	-0.03355
	-5	-0.01677
	+5	0.016774
	+10	0.033548

From table2 above, it can be seen that the holding and ordering costs react inversely to each other. As the order size increases, fewer orders are required, causing the ordering cost to decline, whereas the average amount of inventory on hand will increase, resulting in an increase in carrying costs.

ACKNOWLEDGEMENTS
The authorswishtothanktheanonymousrefereesandthe editor(s) in chief for their constructive comments on thepaper.

REFERENCES

Agrawal, R., Rajput, D. and Varshney, N.K. (2009). Integrated Inventory System with the Effect of Inflation and Credit Period.*International Journal of Applied Engineering Research*4: 2334-2348.

Amutha, R. and Chandrasekaran. E. (2013). An Inventory Model for Constant Demand with Shortages under Permissible Delay in Payment. *International Journal of Applied Engineering Research*6:28-33.

Cecil, B. (2011). Supply Chain Resource Cooperative Poole College of Management NC State University.*A Hillsborough Street Raleigh, NC*27: 695-729.

Chiu, Y., Lin C., Chang, H.H. and Chiu, V. (2010). Mathematical modelling for Determining Economic Batch Size and Optimal number of Deliveries for EPQ Model with Quality Assurance. *Math. Comp. Model. Dyn. Sys.* 16: 373-388.

Dari, S. and Sani, B. (2013). An EPQ Model for Items that Exhibit Delay in Deterioration with Reliability Consideration.*Journal of the Nigerian Association of Mathematical Physics*24:163-172.

Feng-Tsung C. and Chia-Kuan T.(2010). Determining Economic Lot Size and Number of Deliveries for EPQ Model with Quality Assurance using Algebraic Approach. *International Journal of the Physical Sciences*5:2346-2350.

Ghare, P.M. and Shrader, G.F. (1963). A Model for Exponentially Decaying Inventory. *J. Ind. Eng.* 14:238-243.

Hou, L. and Lin .L. (2009). A Cash Flow Oriented EOQ Model with Deteriorating Items under Permissible Delay in Payment. *Journal of Applied Sciences*9:1791-1794.

Huang, Y.F. (2007). Economic Order Quantity under Conditionally Permissible Delay in Payments. *European Journal of Operational Research*176:911-924.

Jaggi, C.K., Aggarwal, K.K. and Goel, S.K. (2007). Retailer's Optimal Ordering Policy under Two Stage Trade Credit Financing. *Advanced Modelling and Optimization*9: 67-80.

Jaggi, C.K., Goyal, S.K. and Goel, S. K. (2008). Retailer's Optimal Replenishment Decisions with Credit-linked Demand under Permissible Delay in Payments. *European Journal of Operational Research*190:130-135.

Liao, H.C., Tsai C.H. andSu C.T.(2000). An Inventory Model with Deteriorating Items under Inflation when a Delay in Payment is Permissible. *Int.J.Prod.Econ.* 63:207-214.

Mahata, G.C. and Mahata, P. (2009). Optimal Retailer's Ordering Policies in the EOQ Model for Deteriorating Items under Trade Credit Financing in Supply Chain. *International Journal of Mathematical, Physical and Engineering Sciences*110:100-135.

Musa, A. and Sani, B. (2011), An EOQ Model for some Tropical Items that Exhibit Delay in Deterioration with Shortages, *the Journal of Mathematical Association of Nigeria (abacus)* 38:54-62.

Ouyang, L. Y., Wu, K. S. and Yang, C. T. (2006). A study on an Inventory Model for Non-instantaneous Deteriorating Items with Permissible Delay in Payments, *Computers and Industrial Engineering* 56: 719-726.

Roychowdhury, M. and Chaudhuri, K.S. (1983). An Order Level Inventory Mode for Deteriorating Items with Finite Rate of Replenishment, *Opsearch*20:99-106.

Tripathi, R.P., Misra,S. S. and Shukla. H. S.(2010). A Cash Flow Oriented EOQ Model with Deteriorating Items under Permissible Delay in Payment. *International Journal of Engineering, Science and Technology*2:123-131.

Tripathi, R.P. and Misra, S.S. (2010). Credit Financing in Economic Ordering Policies of Non- deteriorating Items with Time –dependent Demand Rate.*International Review of Business and Finance*2:47-55.

Tripathy, C.K. and Mishra, U. (2011). An EOQ Model for Linear Deteriorating Rates with Shortage and Permissible Delay in Payment. *The IUP Journal of Operations Management*4:7-20.