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Regular Research Manuscript

Optimization of Vehicle Mix for Three-and Four-Wheel Passenger Transportation System in Nigeria

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ABSTRACT

Efficient transportation systems are essential for reducing passengers waiting times in densely populated areas. However, there is a dearth of information on the optimization of vehicle mix for three-and four-wheel transportation systems in Nigeria. The present study was designed to optimize the vehicle mix for three-and four-wheel passenger transportation to improve transportation efficiency. Twenty purposively selected vehicles were observed, while a queuing model (M/M/s) was used to determine passenger arrival rates during different periods (morning, afternoon, and evening). An integer linear programming model was developed to minimize passenger waiting times. Data on vehicle types, capacities, travel cycles, and average passenger waiting times were collected over five working days for three months at a Nigerian tertiary institution. These data were input into the model and solved in the Python PuLP environment. The obtained optimal vehicle mix for morning were 4, 2, 1, and 13 at either travel cycles (C_3)=1, 2, or 3, respectively; afternoon, and evening were 6, 2, 1, 11; 4, 2,1, 13; 4, 2,1, 13, and 7, 2, 3, 8; 5, 2, 1, 12; 4, 2,1, 13 for C_3=1, 2, or 3, respectively, for cars, shuttle bus, omnibus, and tricycle. The reduced total passenger waiting times were up to 41.4, 12.8, and 13.6% for morning, afternoon, and evening, respectively, by the optimized vehicle mix compared to existing transportation system. The per-passenger time savings were 0.11-14.55 minutes depending on period and travel cycle. The model provides a framework for real-time vehicle allocation, reducing congestion, enhancing passenger satisfaction, and supporting sustainable transport operations. The approach has broad applicability for transportation planning and management.

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Highlights:

- The research explores the vehicle mix for three-and four-wheel passenger transportation system
- An integer linear programming model was developed to minimize passenger waiting times.
- The model provides a framework for real-time vehicle allocation, reducing congestion, enhancing passenger satisfaction, and supporting sustainable transport operationsns

INTRODUCTION

In an ever-evolving world, the rapid growth of cities has led to increased congestion, air pollution, and energy consumption. In response, there is a growing consensus that embracing diverse forms of transportation, ranging from traditional four-wheeled cars to compact three-wheeled vehicles, can help mitigate these challenges. The choice between different types of vehicles, with respect to their capacity, speed, and flexibility, has had significant implications for the global transportation system. As the population continues to surge and environmental concerns intensify, purchasing new vehicles poses a significant expense for many transporters, so there is a need for careful consideration when making decisions concerning the types and number of vehicles to be purchased (Lvovich *et al.*, 2022). Public transportation systems, particularly in developing countries, often lack efficiency due to improper vehicle allocation. Finding an optimal, efficient, and sustainable mix of three-and four-wheel vehicle types has become a challenge. However, the three-and four-wheel vehicles such as cars, shuttlebuses, omnibuses, and tricycles are important for public transportation to ensure movement of passengers from one location to another (Abate and Kveibory, 2013). The public transport timetable concept has been applied to ensure reliability, comfort, and an attractive transportation system, which was found to be inadequate and inaccurate (Tolic *et al.*, 2020; Liu

and Ceder, 2016). The ideal mix of vehicles in a transport park is usually diverse, and the main challenge lies in determining how many of each vehicle type should be allocated to ensure efficient utilization while minimizing passenger waiting times, avoiding excess vehicles or insufficient capacity. Therefore, this study is designed to optimize the vehicle mix for three-and four-wheel transportation systems to improve efficiency and utilization by considering passenger arrival rate during three periods (morning, afternoon, and evening), waiting time, vehicle capacity, and travel cycle at a maximized revenue for the drivers.

METHODS AND MATERIALS

Abate and Kveibory (2013) looked into the capacity utilization of freight vehicles by reviewing a literature model to maximize revenue for the freight operators. A constant challenge observed was the inability of the available vehicle capacity to meet demand; therefore, monitoring of the arrival rate of passengers was necessary. However, the present study considers passenger arrival rates to optimize passenger vehicle mix. Onyeka *et al*. (2021) developed an optimization model to obtain an optimal mix for vehicle types such as saloon cars, wagon vehicles, minibuses, coaster buses, and big buses of capacities of 3, 5, 9, 20, and 32, respectively, as well as to minimize air pollution. The model was constrained to minimize air pollution generated by the vehicle types by 40% for carbon monoxide, 50% for nitrogen dioxide, and hydrocarbons, and methane, respectively. The pollutants were measured using standard Aeroquip/Crowcon gasman monitors, and values were compared with recommended standards. The existing vehicle mix for passengers was 56:64:20:2:1; the optimal vehicle mix obtained from the developed model to reduce the pollutants was 10:33:53:14:1 for saloon cars, wagon vehicles, minibuses, coaster buses, and big buses, respectively, as well as reduced passengers' time for four-wheel vehicles. Sadrani *et al*. (2021) formulated a Mixed-Integer Nonlinear Programming (MINLP) model to optimize vehicle distribution schemes by following dispatching order and time to minimize passenger waiting time by developing a stochastic travel time between stops, as well as considering vehicle capacity and travel time uncertainty using a Monte Carlo simulation framework to solve the model. However, in the present study, the queuing theory of single queue, multiple servers (M/M/s), passenger arrival rate and waiting time, and travel time were considered for each vehicle type as observed to maximize revenue generation and minimize passenger waiting time to obtain optimal vehicle mix for the periods in the developed Integer Linear programming model. The common queue disciplines include First-In, First-Out (FIFO), Last-In, First-Out (LIFO), Service In Random Order (SIRO), and Priority-Based Selection (PBS). Single-server queues are among the most common queuing scenarios encountered in various sectors, including business, industry, transport, telecommunications, and computing. Yu *et al*. (2024) considered passenger waiting and on-board time to construct a synchronous optimization model for line network and scheduling four wheel vehicle (buses), passengers were considered for different choice of route through shortest traversal algorithm to analyze the number and weight of the shortest paths between the same origin destination, the synchronous optimization model was solved using generic algorithm. The model achieved passenger time, bus operation, and total bus system cost reduction of 21.5, 13.7, and 18.0%, respectively. While the present study developed a model that optimizes three-and four-wheel vehicle types to reduce passenger waiting times and maximize revenue for the operators by obtaining optimal vehicle mix, it did not include passenger route but vehicle travel time, which was solved in PuLP Python programming environment. Abuaisha and Abu-Eishen (2023) presented an optimization model to optimize modal fleet size for urban public transportation, considering constraints such as expected passenger demand, travel cycle time, route, available seats, and determined the optimal combination of different vehicle classes and the number of trips based on the stated constraints. A combinational scheduling model was developed for an enterprise supplies zonal service based on the regular service of buses. This is to minimize total passenger travel and operation cost, for which the simulated annealing algorithm was designed to solve the optimization model. It was revealed that the model developed satisfied passengers' demand and diminished total cost by 0.07% compared with the normal operation system. Noor *et al*. (2021); Gong *et al*. (2019) used artificial neural network and support vector machine to predict fuel cost, gas emission and travel time based on a four-wheel vehicle (shuttlebus) travel distance of 2.85, 5.60, 6.44, 7.19, and 7.43 km, which resulted in a reduced time error rate in transportation system in a university, root mean square error revealed that artificial neural network performed better than support vector machine during the analysis. PuLP is a high-level modelling library that utilizes the power of the Python programming language to avoid special syntax and keywords wherever possible. By describing linear programming of a product-mix optimization problem using PuLP Python by writing a short program in Python to call PuLP modelling functions called solver to obtain a best mix to maximize profit (Parganilia, 2018), maximization of generated revenue by the three-and four-wheel vehicle operators is part of the consideration in the present study. The category of mathematical

programming concerned with scarce resource allocation, workforce, investment capital, or time to many competing activities, with the need for optimality, was integer and linear programming. These techniques have been applied in various areas of application, such as industry, economics, transportation, etc. The framework for rapid conversion and global optimum is integer programming, for which PuLP software was introduced to solve a transportation problem in a Python programming environment to minimize cost (Jha *et al*., 2023). PuLP allowed the use of Python to quickly develop and solve linear and integer programming models by allowing access to all the available tools in the Python standard library (Mitchell, 2009). The PuLP package in Python has been used to solve the facility location planning problem. The facility location problems are applicable in telecommunication, industrial transportation, and distribution. The problem requires locating *p*-facilities in a given space that fulfill *n*-demand points in ways that the total distance between each demand point and the closest facility is minimized (Anand *et al*., 2017). By contrast, the present study minimizes total passenger waiting time to determine optimal vehicle-mix for three-and four-wheel passenger transport vehicles during three periods of the day (morning, afternoon, and evening). Lvovich *et al*. (2022) researched optimizing transport systems by focusing on areas that involved vehicle type choice and the storage terminals layout, but specifically did not address the vehicle mix for three-and four-wheel passenger transportation systems, rather emphasizing the significance of choosing optimal vehicles and locations for loading and unloading.

The adopted queuing theory with a single queue multiple server (M/M/s) model, which was chosen as it has characteristics related to the present study, where multiple vehicle types serve a single queue of passengers on a first-come, first-served basis. Purposive sampling technique was used to select twenty vehicles representing 48% of the available registered vehicles plying the study route based on the vehicle types activity levels, availability for full-day study as well to ensure proportional representation across vehicle types. This was used to obtain and record total number of passengers arriving during three segmented periods of the five working days (morning, afternoon and evening) excluding weekends and holidays for three months to reflect the dynamic nature of transportation operation throughout the day in a tertiary institution in Nigeria. Each period was analyzed separately to optimize vehicle mix based on observed demand patterns. The capacities of the tricycle, car, shuttlebus, and omnibus were three, four, nine, and eighteen passengers, respectively. The observed vehicle capacity data were cross-referenced with information provided by the drivers. Observations were made during the periods to record occupancy levels of the vehicles. Data on passenger arrival within three hours (7:00 – 10:00 am, 12:00 – 15:00 pm, and 16:00 – 19:00 pm) were collected and recorded. The time intervals aligned with class schedule, staff shifts and preliminary observations showed that it captured 85% of daily passenger movements as well to avoid research fatigue. An integer linear programming model was implemented in Python 3.12.4 using the PuLP package. Decision variables representing the number of vehicles of each type per period were declared as integer type using the LpVariable class. The objective function was to minimize total passenger waiting time across vehicle types, which was formulated to sum the products of waiting time and number of vehicles. Constraints were stated to ensure vehicle capacities meet passenger arrival rates, and revenue generated per vehicle type did not exceed the benchmark revenue of the omnibus. The observed data, such as passenger arrival rates, vehicle type capacities, and travel cycle times, were organized into dictionaries and used as model parameters. The model was solved using PuLP's default solver (CBC), solution status and variable values were extracted to identify the optimal vehicle mix period. However, advanced solver turning was not required because the problem is a simple computation. Model verification was done by comparing solver outputs with manually computed small-scale test cases, ensuring all constraints were satisfied. A pseudocode representation of the model setup is provided in Appendix A to support the model reproducibility.

Table 1 presents types, wheel numbers, and maximum passenger capacity for various vehicles.

Table 1: Vehicle characteristics

Vehicle types	Vehicle Wheel number	Vehicle maximum capacity
Car	4	4
Shuttlebus	4	9
Omnibus	4	18
Tricycle	3	3

Table 2 shows the number of passengers arriving on different days and periods in a three-hour observation.

Table 2: Observed passengers' arrival per period per day

Days	Morning	Afternoon	Evening
Monday	52	229	509
Tuesday	48	225	519
Wednesday	57	252	581
Thursday	64	290	604
Friday	62	440	355
Total	283	1436	2568

Passengers' arrival average	57	287	514

Model Assumptions

The following assumptions were made:

i. Service was rendered to passengers on a first-come, first-served basis.
ii. Each vehicle type must not generate revenue exceeding that of the benchmark vehicle (omnibus) per period.
iii. Service times are between 8:00 am and 9:00 pm.
iv. Vehicles depart the park when it reach maximum passenger capacity, this allow for consistency and accommodate park policy, however, hybrid iteration that consider scheduled departure can be considered in future review of the model.
v. All 20 selected vehicles registered on the route are operational and work only on the route assigned.
vi. No data were collected during the weekend and holidays
vii. The observed travel times for cars, shuttlebuses, omnibuses, and tricycles are 30, 20, 40, and 27 minutes, respectively
viii. For fairness and consistency, the revenue per trip of each vehicle type in the model is constrained not to exceed the revenue per trip of the omnibus that serves as reference benchmark due to its highest seating capacity.

Model Notations

λ_t: Passenger arrival rate during period t
W_{it}: Average waiting time for passengers by using vehicle type i during period t
T_i: observed travel time of vehicle type i (30, 20, 40, and 27 for car, shuttlebus, omnibus, and tricycle, respectively)
X_{it}: Number of vehicles of type i allocated during period t
Z_t: Total waiting time for all vehicle types during period t
C_{it}: Number of travel cycles of vehicle type i during period t
FFC: Fixed fare cost per passenger is ₦200
K_i= Capacity of each vehicle type
R_i = Revenue per vehicle type
P_t=Average number of passengers arriving during period t
t= Morning, Afternoon, and Evening
i=1, 2, 3, and 4 (car, shuttlebus, omnibus, and tricycle, respectively)

Model development

The three-hour Average passenger arrival per 3 hours (P_t) = $\frac{\text{Total number of passengers arriving during period t}}{5}$ (1)

Passenger arrival per minute $(\lambda_t) = \frac{P_t}{180}$ (2)

Passenger waiting time per vehicle type

$$W_{it} = \frac{\text{capacity of vehicle}}{\lambda_t} \quad (3)$$

Revenue per vehicle type $(R_i) = C_{it} \times K_i \times FFC$ (4)

Average passenger arrival revenue per period = $\frac{Total\ passenger\ in\ period\ t}{5} \times FFC$ (5)

The objective function of the integer linear programming was to minimize the total passenger waiting time across all vehicle types using the omnibus travel cycle C_{3t} as the reference benchmark for revenue constraint. The general integer linear programming is given as:

Minimize total passenger waiting time

$$Min\ Z_t = \sum_{i=1}^{4}[(W_{it} \times X_{it}) + (T_i \times C_{it})] \quad (6)$$

Subject to the following constraints:

Vehicle types capacity constraint:

Total served passengers ≥ Average passenger arrival

$$\sum_{i=1}^{4} K_i \times X_{it} \times C_{it} \geq P_t \quad (7)$$

Variables must be integers

$$X_{it} \epsilon \mathbb{Z} \geq 0 \quad (8)$$

The omnibus was selected as the revenue benchmark due to its highest seating capacity (18 passengers), also, generates highest revenue per trip among all the vehicle types. Therefore, this ensures that smaller vehicle types are evaluated against the maximum achievable per trip revenue, which allows fairness and avoided unrealistic overperformance expectations for lower-capacity vehicles.

Vehicle types capacity revenue constraints:

Total revenue of the vehicle types ≥ Benchmark vehicle revenue (omnibus)

$$\sum_{i=1}^{4} R_{it} \geq R_{Omnibus} \quad (9)$$

where

$$R_{it} = FFC \times K_i \times C_{it} \quad (10)$$

Substituting equation (10) into (9) become:

$$\sum_{i=1}^{4} FFC \times K_i \times C_{it} \geq R_{Omnibus} \quad (11)$$

Revenue of the selected vehicle type ≤ Revenue of the benchmark vehicle (omnibus)

$$R_{it} \leq R_{Omnibus} \quad (12)$$

Substituting equation (10) into (12) become:

$$FFC \times K_i \times C_{it} \leq R_{Omnibus} \quad (13)$$

Hardware and Software Requirements

The integer linear programming model was performed in Python 3.12.4 using the PuLP library (version 2.8). The default CBC MILP solver (version 2.10.3) was employed, running on a Windows 11 platform with an Intel Core i7-11800H CPU at 2.30GHz, 8GB RAM, and 228GB storage. The solver used its default optimality tolerance (relative MIP gap = 1×10^{-5}) and found optimal solutions with zero enumerated nodes in an average wall-clock time of approximately 0.03 seconds per scenario.

RESULTS AND DISCUSSION

Table 3 presents passenger arrival rate per 3 hours (p/3hrs), passenger arrival rate per minute (p/min), and passenger arrival rate per second (p/sec) calculated using Equations (1) and (2) for the three periods of morning, afternoon, and evening.

Table 3: Arrival rate of passengers (λ_t)

Period	Average arrival rate per 3 hours (p/3hrs)	Average arrival rate per minute (p/min)	Average arrival rate per second (p/sec)
Morning	56.60	0.32	0.005
Afternoon	287.00	1.60	0.027
Evening	514.00	2.86	0.048

The passenger waiting times for different vehicle types were calculated using Equation (3) during the morning, afternoon, and evening periods as presented in Table 4.

Table 4: Waiting time for different vehicle types (W_{it})

Vehicle types	Morning (min)	Afternoon (min)	Evening (min)
Car	12.50	2.50	1.40
Shuttlebus	28.10	5.60	3.15
Omnibus	56.30	11.25	6.29
Tricycle	9.38	1.88	1.05

Equation (4) was used to generate revenue per vehicle type:

Revenue for the omnibus (C_3=1) for morning, afternoon, and evening period = 1×18×200=3,600 naira(₦)

Revenue for the omnibus (C_3=2) for morning, afternoon, and evening period = 2×18×200=₦7,200

Revenue for the omnibus (C_3=3) for morning, afternoon, and evening period = 3×18×200=₦10,800

Equation (5) using Table 2 generate average passenger arrival revenue per period

Average passenger arrival revenue for morning period = $\frac{283}{5} \times 200 = ₦11{,}400$

Average passenger arrival revenue for the afternoon period = $\frac{1436}{5} \times 200 = ₦57{,}400$

Average passenger arrival revenue for evening period = $\frac{2568}{5} \times 200 = ₦102{,}800$

Demonstration of the developed Model

The Integer Linear Programming (ILP) model developed to minimize passenger waiting time is illustrated for morning periods for benchmarked vehicle (omnibus) C_3:

Substituting $W_{1,morning}$=12.5, $W_{2,morning}$=28.1, $W_{3,morning}$=56.3, and $W_{4,morning}$=9.38, T_1= 30 minute for car, T_2= 20 minute for shuttlebus, T_3= 40 minute for omnibus, and T_4= 27 minute for tricycle into equation (6) become:

$$Min\ Z_t = [[(12.5 \times X_{1t}) + (30 \times C_{1t})] + [(28.1 \times X_{2t}) + (20 \times C_{2t})] + [(56.3 \times X_{3t}) + (40 \times C_{3t})] + [(9.38 \times X_{4t}) + (27 \times C_{4t})]] \quad (14)$$

Subject to:

Vehicle types capacity constraint

Substituting K_1=4, K_2=9, K_3=18, and K_4=3 into equation (7) become:

$$[(4 \times X_{1t} \times C_{1t}) + (9 \times X_{2t} \times C_{2t}) + (18 \times X_{3t} \times C_{3t}) + (3 \times X_{4t} \times C_{4t})] \geq 0 \quad (15)$$

$$X_{1t} \geq 4 \quad (16)$$

$$X_{2t} \geq 2 \quad (17)$$

$$X_{3t} \geq 1 \quad (18)$$

$$X_{4t} \geq 6 \quad (19)$$

$$X_{1t}, X_{2t}, X_{3t}, X_{4t} \in \mathbb{Z}^+ \quad (20)$$

$$X_{1t} + X_{2t} + X_{3t} + X_{4t} = 20 \quad (21)$$

Vehicle types capacity revenue constraints:

Substituting FFC=₦200, K_1=4, K_2=9, K_3=18, K_4=3, and $R_{Omnibus}$=₦11,400 into equation (11) and (13) become:

$$[(200 \times 4 \times C_{1t}) + (200 \times 9 \times C_{2t}) + (200 \times 18 \times C_{3t}) + (200 \times 3 \times C_{4t})] \geq 11400 \quad (22)$$

Revenue constraint of each vehicle type:

$$200 \times 4 \times X_{1t} \leq 11400 \quad (23)$$

$$200 \times 9 \times X_{2t} \leq 11400 \quad (24)$$

$$200 \times 18 \times X_{3t} \leq 11400 \quad (25)$$

$$200 \times 3 \times X_{4t} \leq 11400 \quad (26)$$

where

i= 1, 2, 3 and 4 for car, shuttlebus, omnibus and tricycle, respectively, while t represents morning period.

$X_{1,morning}$= require optimal vehicle mix for car during period t

$X_{2,morning}$= require optimal vehicle mix for shuttlebus during period t

$X_{3,morning}$= require optimal vehicle mix for omnibus during period t

$X_{4,morning}$= require optimal vehicle mix for tricycle during period t

$C_{1,morning}$= require travel cycle for car during period t

$C_{2,morning}$= require travel cycle for shuttlebus during period t

$C_{3,morning}$= require travel cycle for omnibus during period t

$C_{4,morning}$= require travel cycle for tricycle during period t

The optimal vehicle mixes types for different periods (morning, afternoon, and evening) obtained from the Python PuLP using the Omnibus travel cycle (C_{3t}) at 1, 2, and 3, where t = morning, afternoon, and evening, are presented in Table 5.

Table 5 shows the vehicle types, capacity, travel cycle, and optimal vehicle mix for the three periods (morning, afternoon, and evening).

Table 5: Optimal distribution of vehicles

Periods	Vehicle types	Vehicle capacity (K_i)	Travel cycle (C_i)	Optimal vehicle mix (X_i)
Morning				
$C_3 = 1$	Car	4	5	4
	Shuttlebus	9	2	2
	Omnibus	18	1	1
	tricycle	3	6	13
	Objective function in minutes = 586.44 minutes			
$C_3 = 2$	Car	4	9	4
	Shuttlebus	9	4	2
	Omnibus	18	2	1
	tricycle	3	12	13
	Objective function in minutes = 886.44 minutes			
$C_3 = 3$	Car	4	14	4
	Shuttlebus	9	6	2
	Omnibus	18	3	1
	tricycle	3	18	13
	Objective function in minutes = 1170.44 minutes			
Afternoon	**Vehicle types**	**Vehicle capacity (K_i)**	**Travel cycle (C_i)**	**Optimal vehicle mix (X_i)**
$C_3 = 1$	Car	4	5	6
	Shuttlebus	9	2	2
	Omnibus	18	1	1
	tricycle	3	6	11
	Objective function in minutes = 360.13 minutes			
$C_3 = 2$	Car	4	9	4
	Shuttlebus	9	4	2
	Omnibus	18	2	1
	tricycle	3	12	13
	Objective function in minutes = 640.89 minutes			
$C_3 = 3$	Car	4	14	4
	Shuttlebus	9	6	2
	Omnibus	18	3	1
	tricycle	3	18	13
	Objective function in minutes = 942.89 minutes			
Evening	**Vehicle types**	**Vehicle capacity (K_i)**	**Travel cycle (C_i)**	**Optimal vehicle mix (X_i)**
$C_3 = 1$	Car	4	5	7
	Shuttlebus	9	2	2
	Omnibus	18	1	3
	tricycle	3	6	8
	Objective function in minutes = 345.37 minutes			
$C_3 = 2$	Car	4	9	5
	Shuttlebus	9	4	2
	Omnibus	18	2	1
	tricycle	3	12	12
	Objective function in minutes = 616.19 minutes			
$C_3 = 3$	Car	4	14	4
	Shuttlebus	9	6	2
	Omnibus	18	3	1
	tricycle	3	18	13
	Objective function in minutes = 917.84 minutes			

Table 6 shows comparison between existing and optimized operational vehicle mix for three-and four-wheel vehicle types. The existing total waiting time (minute) for morning, afternoon, evening was 739.55, 1292.00, and 2000.00; 413.15, 724.30, and 1062.00; 399.50, 694.00, and 1031.00, optimized total waiting time (minute) were 586.44, 886.44, and 1170.44; 360.13, 640.89, 942.89; 345.37, 616.19, and 917.84, reduction time (minute) were 153.11, 405.56, and 829.56; 53.02, 83.41, and 119.11; 54.13, 77.81, and 113.16, reduction (%) were 20.71, 31.35, and 41.40; 12.84, 11.56, 11.27; 13.55, 11.24, and 10.97, while per-passenger saving (minute) were 2.69, 7.11, and 14.55; 0.18, 0.29, and 0.42; 0.11, 0.15, and 0.22 when C_3=1, 2, and 3, respectively.

Table 6. Comparison between current operational mix and optimized mix

Period	Existing total waiting time (minute)	Optimized total waiting time (minute)	Reduction waiting time (minute)	Reduction waiting time (%)	Per-passenger saving (minute)
Morning (C_3=1)	739.55	586.44	153.11	20.71	2.69
Morning (C_3=2)	1292.00	886.44	405.56	31.35	7.11
Morning (C_3=3)	2000.00	1170.44	829.56	41.40	14.55
Afternoon (C_3=1)	413.15	360.13	53.02	12.84	0.18
Afternoon (C_3=2)	724.30	640.89	83.41	11.56	0.29
Afternoon (C_3=3)	1062.00	942.89	119.11	11.27	0.42
Evening (C_3=1)	399.50	345.37	54.13	13.55	0.11
Evening (C_3=2)	694.00	616.19	77.81	11.24	0.15
Evening (C_3=3)	1031.00	917.84	113.16	10.97	0.22

Figures 1 - 3 show the frequency and time plot of the observed passengers' traffic flow rate for the five

days at intervals of 10 minutes for the different periods (morning, afternoon, and evening).

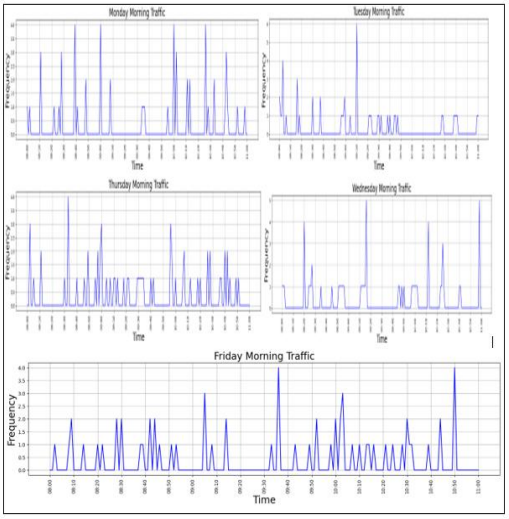

Figure 1: Passengers' arrival rate for the five-day morning period.

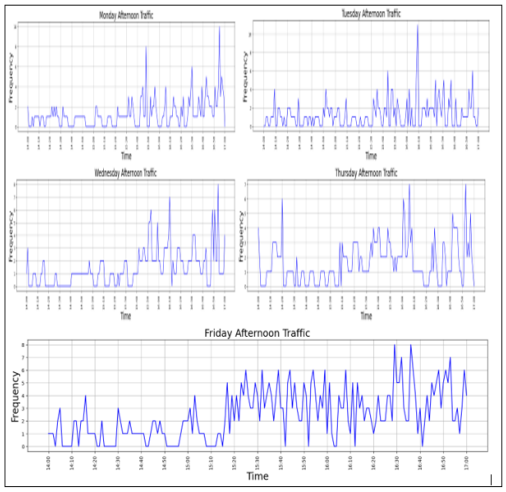

Figure 2: Passengers' arrival rate for a five-day afternoon period.

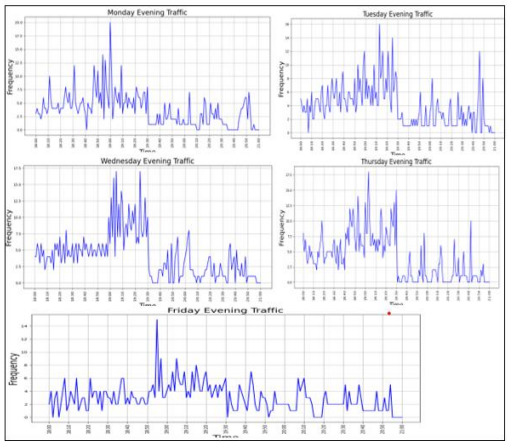

Figure 3: Passengers' arrival rate for a five-day evening period.

Discussion

Table 5 shows that the three-and four-wheel vehicle types of cars, shuttlebus, omnibus, and tricycle capacities were 4, 9, 18, and 3, respectively. The assigned travel cycles for the vehicle types for morning, afternoon, and evening periods were 5, 2,1, 6; 9, 4, 2, 12; and 14, 6, 3, 18, respectively, when $C_3 = 1$, 2, and 3 for the three periods. The omnibus travel cycle (C_3) was used as a reference point, as it has the highest loading capacity and generates the highest revenue with the lowest travel cycle. The omnibus driver could generate revenue of 3,600, 7,200, and 10,800 naira (₦) when $C_3 = 1$, 2, and 3, respectively. However, for a car, shuttlebus driver, and tricycle rider to generate the same revenue as omnibus driver the travel cycles were 5, 2, and 6 for $C_3 = 1$; 9, 4, and 12 for $C_3 = 2$; 14, 6, and 18 for $C_3 = 3$, respectively, at full capacity of various vehicle types. Hence, the C_3 was assigned 1, 2, and 3 for morning, afternoon, and evening periods, respectively. The optimal vehicle mix obtained for the three periods: morning at the assigned travel cycle of C_3 were 4, 2, 1, and 13, respectively to minimize total travel time of 586.44, 886.44, and 1170.44 minutes, these translate to 20.71, 31.35, and 41.40% reduction compared to existing vehicle types travel time, respectively; afternoon were 6, 2, 1, 11; 4, 2, 1, 13; and 4, 2, 1, 13, respectively to minimize total travel time of 360.13, 640.89, and 942.89 minutes, therefore translate to 12.84, 11.56, and 11.27% reduction in vehicle types travel time of existing transportation system, respectively; evening were 7, 2, 3, 8; 5, 2, 1, 12; and 4, 2, 1, and 13, respectively to minimize total travel time of 345.37, 616.19, and 917.84 minutes, which means that the existing vehicle types travel time had been reduced by 13.55, 11.24, and 10.97%, respectively. The per-passenger time saving (minute) were 2.69-14.55, 0.18-0.42, and 0.11-0.22 depending on period and travel cycle.

The passenger traffic flow rate in the morning, afternoon, and evening periods of Monday, Tuesday, Wednesday, Thursday, and Friday, as presented in Figures 1 – 3, highlights the dynamic nature of passenger behavior. The plot shows the frequency versus the arrival time of passengers at 10-minute intervals for a three-hour observation of service time from 8 am to 9 pm for the three periods of the five days. The highest frequency of passenger arrival rate occurred on Tuesday morning (6) and evening (11), and Monday evening (20) at 9.10 am, 16.10 pm, and 19.10 pm, respectively, emphasizing the need for real-time vehicle allocation to match demand. Therefore, the passenger arrival rate influences the optimal allocation of the vehicle mix during the periods. Onyeka *et al*. (2021) emphasized the importance of vehicle mix optimization in reducing waiting times and environmental impacts, which revealed that achieving vehicle mix led to reduction in air pollution due to smoke produced by vehicles, the authors observed reduction ranged of

40-50%, while optimal vehicle mix in the present study minimized passengers' waiting time and maximize revenue generation for operators. Yu *et al*. (2024) optimization model was based on the passenger route choice and on-board time of buses only, which achieved cost reductions, while present study looked into four vehicle types of three-and four-wheels (cars, shuttlebus, omnibus, and tricycle) capacity, travel time, and travel cycle time to minimize passenger total waiting time and maximize revenue generated by the drivers. Feng *et al*. (2014) studied varying passenger utilization rates and target speed on changed traction cost intensity on passenger efficiency in train transport. The vehicle types' cost of energy and speed were not studied in the present study; instead, vehicle travel cycle time and capacity utilization were used for optimal vehicle mix to minimize total passenger waiting time and maximize revenue generation. Abuaisha and Abu-Eishen (2023) reported an optimal solution for passengers and service providers to comprise different modes of fleet for urban public transport instead of a single mode fleet. This supported the present study, which revealed an optimal solution for the three-and four-wheel vehicle mix. The present study aligns with the findings of Gong *et al*. (2019), emphasizing the potential for optimization models to improve service efficiency. Noor *et al*. (2021) carried out a study during the semester and excluded examination and weekend breaks due to reduced passenger arrival rate, which supported the present study, where weekends and any other holidays were not included to obtain data.

Limitation

The observed data were limited to five working days (Monday, Tuesday, Wednesday, Thursday, and Friday) during the three-month observation for data collection because the case study used was an institution where passenger activities were high during the selected days; however, the absence of the weekend days did not influence the result. The model did not consider the speed and energy cost of the various vehicle types studied to obtain an optimal vehicle mix; rather, the travel cycle times of the different vehicle types were used in the developed model. One of the present model assumptions is based on vehicle types full capacity departure for consistency and align with the transport park policy, however, in practice, departure can also occur after a maximum waiting threshold. This hybrid approach can be considered in the future model iterations.

CONCLUSION AND RECOMMENDATIONS

Vehicle mix optimization can be achieved using integer linear programming, which helps rationalize revenue generation of the drivers by reducing vehicle congestion in transport parks. The issue of drivers being violent in the park when calling for passengers will be minimized. Also, passengers will be sure of getting to their destination with minimal delays by reducing passenger waiting time, as the available vehicle is based on the passenger flow rate. The drivers would be aware of the number of travels cycle time they can complete in a given period each day, which will help drivers plan their generated revenue. The model can serve as a park management and decision-making tool in any designated route where more than two vehicle types travel to achieve an optimal vehicle mix. Hence, it minimizes vehicle congestion, reduces driver conflict and operational inefficiency, and decreases passenger waiting times, ultimately enhancing the productivity and efficiency of the passengers and drivers. Future research could explore the impact of vehicle speed and energy consumption to further refine the optimization models. In addition, the optimized vehicle mix model should be tested in a real-time transport system operation to confirm the real-world applicability of the developed model. The required assumption modifications to this present model should be considered before implementation.

Competing Interest

The authors have no competing interests to declare that are relevant to the content of this article.

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Appendix A
Pseudocode code

```
  # Solve the model
   status = model.solve()
   if status == pulp.LpStatusOptimal:
      print("Solution Found:")
      print("X1 =", X1.varValue, "X2 =", X2.varValue,
"X3 =", X3.varValue, "X4 =", X4.varValue)
      print("Objective Value:",
pulp.value(model.objective))
   else:
      print("No solution found.")

# Example Usage
user_c3 = float(input("Enter the value for C3: "))
c1, c2, c4 = find_c_values(user_c3)

if c1 is not None:
   print(f"Using C1 = {c1}, C2 = {c2}, C3 =
{user_c3}, C4 = {c4}")
   solve_lp(c1, c2, user_c3, c4)
else:
   print("No feasible C values found for the given
C3.")

Algorithm: Optimize Vehicle Mix using ILP in PuLP

Input:
  - Travel cycles C1, C2, C3, C4
    - Vehicle capacities K1, K2, K3, K4
    -Observe travel time T1, T2, T3, T4
    - Passenger waiting times W1, W2, W3, W4
    - Revenue benchmark R_bench (Omnibus)
    - Passenger arrival data

  Define decision variables:
    X1 = number of cars
    X2 = number of shuttlebuses
    X3 = number of omnibuses
    X4 = number of tricycles

  Objective:
    Minimize Total_Waiting_Time =
       W1*X1 + W2*X2 + W3*X3 +
W4*X4+T1*C1+T2*C2+T3*C3+T4*C4
  Constraints:
    Capacity constraint:
 K1*X1*C1 + K2*X2*C2 + K3*X3*C3 + K4*X4*C4
>= Passenger_Arrivals
  Total revenue constraint for all vehicle types
FFC*K1*C1+FFC*K2*C2+FFC*K3*C3+FFC*K4*C
4
  Revenue constraint for each vehicle type:
       Revenue_i <= R_bench
   Total vehicles available constraint:
       X1 + X2 + X3 + X4 = Total_Available
   Integer constraint:
       X1, X2, X3, X4 are integers
  Solve using PuLP with CBC solver.

  Output:
    - Optimal vehicle mix (X1, X2, X3, X4)
    - Minimum total waiting time
    - Reduction compared to current allocation
```