

The Impact of the Interaction between Verbal and Mathematical Languages in Education

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Thought and Practice: A Journal of the Philosophical Association of Kenya (PAK)

New Series, Vol.2 No.2, December 2010, pp.79-99

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<http://ajol.info/index.php/tp/index>

Abstract

Since the methods employed during teacher-learner interchange are constrained by the internal structure of a discipline, a study of the interaction amongst verbal language, technical language and structure of disciplines is at the heart of the classic problem of transfer in teaching-learning situations. This paper utilizes the analytic method of philosophy to explore aspects of the role of language in mathematics education, and attempts to harmonize mathematical meanings exposed by verbal language and the precise meanings expressed by the mathematics register (MR) formulated in verbal language. While focusing on the integration of language use and meaning construction in mathematics education, the paper explores the relationship between the conceptual understanding revealed by the mathematics register and the procedural knowledge that refers to the mathematical content through

ordinary discourse.

Keywords: mathematics register (MR); Mathematics Problem Solving Strategy (MPSS); Mathematical language; Verbal language; conceptual schemata; Text-driven processing; Conceptually-driven processing.

Introduction

The fact that mathematical language formulated as mathematics register (MR) is expressed in grammatically well-formed sentences and phrases in verbal language shows that mathematics and verbal language interact. Every language seems to have ways in which it expresses mathematical operations. For instance, the mathematics register in English is the distinct way in which mathematical meaning is expressed in that language despite the Hindu-Arabic numerical system that it uses. Dale & Cuevas (1987) describe MR in terms of the unique vocabulary and syntax (sentence structure), and discourse (whole text features) in which it is expressed. Some scholars have even gone further to claim that mathematical language could be subsumed under verbal language (Sidhu 1984; Eshiwani 1987; Hjelmslev 1974; Pimm 1987; Mutio 1989; Ernest 1991; Huang & Normandia 2007). According to Schindler & Davison (1975), Mathematics Register (MR) is the sense of the meanings by which a natural language accommodates and integrates the mathematical system in natural language as a sub-system of the same linguistic system, thus defining the mathematical use of the natural language. It is the meanings, including the styles of meaning and modes of argument, that constitute the register, rather than the words and natural language structures as such.

While this study concurs with the above scholars that mathematics is akin to verbal language, the isomorphism that is claimed between mathematics and verbal language is yet to be established or disproved. Indeed, that mathematics is a tool of communication which uses a special language, or at least that it uses language in a special way, is intelligible considering the formation of MR. However, the more radical view that there is a direct equation between mathematical language and verbal language, and that the teaching of mathematics involves, to some extent, the teaching of certain linguistic patterns, needs closer attention.

Works that treat mathematics as a language leave the following questions unanswered:

(1) What linguistic meanings do mathematics embody, and of what consequence could such

conceptions be towards the development of a solid foundation for mathematics, and for improvement of mathematical pedagogy?

(2) Does mathematical language expressed in mathematics register (MR) suggest appropriate Mathematics Problem Solving Strategy (MPSS) as a dialogical tool for tackling mathematical problems?

It may be realized that if we accept the claim that “Mathematics (A) is a language (B)”, then there is reason to suppose that A relates specifically to a whole sub-class of B, or that “B is A” is also true. The claim that “Mathematics is Language” may need to be justified using the foregoing logic if the claim is to be intelligible.

A is B

$A \subset B$

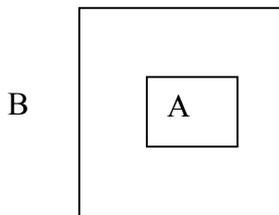


Figure 1: A is a subset of B

A is B and B is A

$(A \subset B) \wedge (B \subset A)$

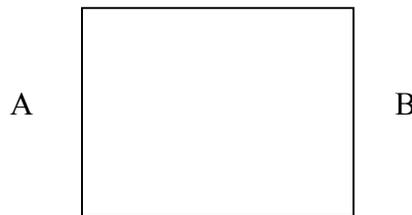


Figure 2: A is a subset of B and B is a subset of A

This position raises the question concerning the existence of significant or partial similarity between what is ordinarily called language on the one hand and mathematics on the other, or whether they are so alike as to fall under the same definition.

The Significance of the Discipline of Mathematics

Mathematics is one of the most important subjects in educational curricula. It is estimated

that in most school systems of the world, between 12 and 15 percent of students' time is devoted to it (Travers *et.al.*, 1989). The importance of Mathematics for potential future careers of students cannot be overemphasized. Mathematics is fundamental to national prosperity in providing tools for understanding science, technology and economics (Brown & Porter 1996). A student who chooses to ignore mathematics, or to treat it casually, forfeits many future career opportunities.

Mathematics is crucial not only for success in school, but also in producing informed citizens, productive in their careers and in their personal endeavours. In today's technology-driven society, demands are placed on individuals to be able to interpret and use mathematics to make sense of information in diverse situations. The study of mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and rewarding participation in society. Learning mathematics results in more than a mastery of basic skills: it equips students with a concise and powerful means of communication. Mathematical structures, operations, processes, and language provide students with a framework and tools for reasoning and expressing ideas clearly. Through mathematical activities that are practical and relevant to their lives, students develop insight, problem-solving skills, and related technological skills that they can apply in their daily lives and, eventually, in the workplace (Ministry of Education 2005, 3).

Mathematics has been conceived as a system of problem solving. By reflecting the laws of the universe, mathematics serves as a powerful instrument for human knowledge and mastery of nature. It reveals and predicts order in the universe, and as far as education is concerned, its importance arises from its inherent power to describe, explain and predict natural trends. Other than possessing practical utility as a means to technological advancement for the improvement of the human condition, mathematics also possesses analytic utility. It is a tool for exploring the possible world of existence and a precise means of communication that employs the logic of relational thought which gives us intellectual independence to engage in abstract thinking.

Mathematics in the Kenyan Context

In Kenya, the central place of mathematics in education has been demonstrated through periodic reviews of curricula in order to make the mathematical content and experiences

consistent with current developmental and technological demands. “Traditional Mathematics” was in vogue in the 1960s. “New Mathematics” was introduced in the 1970s as a response to strategic and computational needs for global technological advancement. However, this change of emphasis did not produce the expected results. Features of the “New Mathematics” curriculum were, apparently, least understood and not applied by teachers as expected. The world over, dissatisfaction began to be voiced with the low arithmetical ability of the new crop of students, more so in developing countries like Kenya (Eshiwani 1981). The failure of students to meet societal demands with respect to their numeric capacities created disillusionment among educators and employers alike.

In the early 1980s, the Kenya government introduced “Appropriate Mathematics”, and changed the education system from 7-4-2-3¹ to 8-4-4². It was assumed that the new structure of education and the new curriculum would not only improve pupils’ performance in mathematics, but also solve problems related to unemployment. The programme, which is still running currently, is yet to be evaluated effectively, although a few changes have been introduced into it lately, especially after the implementation of free primary education in 2003 (UNESCO 2004).

The challenges faced by the 8-4-4 system of education have been pointed out and its conceptual validity questioned by several scholars (e.g. D’Souza 1987; Sifuna 1990; Kibera 1993; Nyaigotti-Chacha 2004). Criticisms levelled against its mathematics curriculum are similar to the ones earlier averred against “New Mathematics” and “Traditional Mathematics” worldwide. A close look at the developments in terms of pupils’ competence and level of numeracy measured in the form of comparative performance vis-a-vis performance in other subjects shows a stable failure trend. This is not surprising, as it is noteworthy that the trend of poor performance in mathematics has been a global problem (Bockarie 1993; Aguele & Usman 2007).

¹ 7-4-2-3 means Seven years of Primary or basic education, Four years of lower Secondary education, Two years of Upper (Higher) Secondary education and a minimum of Three years of University education.

² 8-4-4 means Eight years of Primary or basic education, Four years of Secondary education and a minimum of four years of University education.

From the foregoing observations, it is apparent that perpetual poor performance in mathematics may not be adequately addressed by intermittent changes in the structure of education. Neither do changes in the “types” of mathematics curricula such as “New Mathematics” and “Appropriate Mathematics” seem to solve the problem. It should be noted that the mathematics register (MR) in the English language, for instance, is an international ‘medium’ for expressing mathematical considerations among English language speakers. The use of MR in the English language and its relation to curricula formulation is a matter of global concern, and does not have to be unduly varied by Kenyan curricula developers in the name of making mathematics more ‘appropriate’.

One of the reasons for poor performance in mathematics arises from language considerations (Eshiwani 1983) rather than just from inherent conceptual difficulties of mathematics itself. It should be noted that verbal language spoken by children outside mathematics classrooms is not directly formulated as Mathematics Register (MR) (Halliday 1975, 61-72), which always uses words of the verbal language more precisely. The interaction between Mathematical language (expressed in the form of mathematics register) of a verbal language, and the well-formed phrases of the verbal language apparently creates difficulty for pupils. The difficulty experienced in understanding a mathematics problem is further heightened when a pupil has to interpret the MR into appropriate mathematics problem solving strategy (MPSS)³.

Analysis of the Concept of Mathematical Language

The concept of mathematical language brings to the fore the following questions:

- What is mathematics?
- What is language?

The necessity of definitions cannot be over emphasized, because the beliefs of teachers and pupils about what mathematics is frequently influence their approach to mathematics. If children believe that mathematics is a collection of rules, for example, then their learning might be influenced by their search for rules to memorize and attempt to apply. If teachers think of mathematics as a rigid formal system, they might remain unaware of alternative

³ *Mathematics problem solving strategy* (MPSS) is an intelligible principle which is capable of yielding an algorithm whose transformation through computation gives the solution to a mathematics problem.

concepts or ways of perceiving mathematical ideas.

A definition need not be absolutely explicit, since the *definiens*⁴ has to match the conceptual capacity of pupils. For example, at the primary school level of education, it is not misleading to define a circle as a round figure and to accompany the verbal definition with a model of a ring, or an illustration on the black board. Consequently, in ordinary language, it is normal to talk of the area of a circle. As the pupils acquire competence in higher level mathematics, the definition is modified. A circle is then defined as the locus of a moving point whose position is equidistant from a fixed point called the centre of the circle. With such a definition, the concept of “area of a circle” is mathematically meaningless and instead we talk of “the area enclosed by the circle”.

One of the major problems facing attempts to give definitions is the choice of vocabulary to be used in the *definiens*, which should be clearer than the term it defines, that is, the *definiendum*⁵. When it comes to defining ‘mathematics’ and ‘language’, the terms that constitute the *definiens* fall short of ideal clarity. For example, a definition of mathematics as “the logical study of shapes, arrangement, quality and many related concepts” (Mathematics Dictionary, 1976) is so vague a phrase that it does not explicate what mathematics is. Others define it as the science of abstract form (Sidhu 1984, 1). It is largely due to lack of precision that mathematicians have tended to approach the characterization problem indirectly through axiomatics (the art of using self-evident truths).

Similarly, Strang (1962, 2) observes that there are countless definitions of language, simply because the semantic spread of the word ‘language’ in ordinary usage is so great that any manageable definition will leave out or distort something. So, while this paper attempts to provide guiding definitions of language and mathematics, the main focus will be to provide a working account or description of each. Accordingly, it is expected that such an approach would bring out those characteristics most important to the understanding of how they function, and thereby explicate the concept of mathematical language.

⁴ *Definiens* is the symbol or group of symbols used to explain the meaning of a term (Copi 1986, 41).

⁵ *Definiendum* is the term being defined by the *definiens* (Copi 1986).

Since the method of mathematics is basically argumentation and computation, language considerations play a significant role in mathematical exposition. The first step in resolving any mathematical issue is to translate it into everyday language. Teaching and learning mathematics, therefore, involves a rather complex interaction between a highly stable old knowledge structure and permanent verbal linguistic mechanisms on the one hand, and new knowledge structure and symbol systems on the other (Kaput 1982). In this context, Miller (2008) believes that mathematics is indeed a universal language:

... mathematics is indeed a powerful language ... mathematical symbols - including numbers - are no more or less than the symbols (letters) we string together to make words and sentences, to communicate our thoughts and feelings, to articulate and illustrate our imaginations. Mathematics as a language also has a powerful quality that it shares with music and art: that of crossing cultural and language barriers.... in general, a mathematics equation or expression means the same thing to someone whose native language is Mandarin Chinese or American English (Miller 2008, par. 4).

Mathematics seems to be an outgrowth of verbal language (Barton & Neville-Barton 2004). It is a language which works with ideograms (symbols for ideas) rather than phonograms (symbols for sounds) (Pimm 1987). Ideograms make algorithmic manipulation accurate and efficient, thereby serving as mental labouring devices for expressing the formal relations that are implicit in the verbal medium.

Any particular verbal language expresses thought which has already been formulated indifferently and non-linguistically before the verbal expression of it. Mathematics is seen to be concerned with universal formulation of thought guided by the principles of logic. While verbal language is guided by a grammar which conforms to norms of conventional and social correctness, mathematical language is guided by a grammar consistent with intelligible forms of rational thought. While mathematics operates in the realm and laws of pure thought, verbal language operates according to acceptable social conventions.

The view that mathematics is a language is held by those who believe that mathematics, like any other language, has its own symbols, and its expressions conform to a unique grammar. Although mathematical language is not a language in the conventional philological sense, it is functionally isomorphic to verbal language as a transactional device rather than an interactional one. Thompson & Chappell (2007) observe that both mathematics and English

share words that have distinct meanings in the different contexts, such as *product*, *volume*, and *difference*. Some words are shared with other disciplines, but have different technical meanings in the various disciplines (for example, “*radical*” in mathematics has a different meaning from what it has in the social and natural sciences). Even within mathematics, some words have different meanings depending on the context (for example, “*median*” in geometry versus statistics). Furthermore, technology has rendered particular meanings to certain words and symbols (for example, “*log*” on a calculator always means “logarithm in base 10” and “*ln*” always means logarithm in base e).

The general theory of mathematical language provides a scheme and notation for grammatical description - a precise formulation of grammatical rules. The rules of grammar in mathematics are functions variously expressed in the form of algorithms generated by various operations and allowable transformations. Mathematical discourse involves quantification of the givens within a problem situation, expression of such quantities into condensed relationships called formulae, synthesis of formulae into explanatory systems, and the testing of the ensuing conclusions against intelligible data.

Mathematical language has its syntax (sentence structure), semantics (meaning structure), logic and pragmatics, albeit relational. The functional isomorphism between the two systems is revealed through their dealings with relational properties within categories of abstract experience. Symbolic language, for instance, is used in mathematics to express mathematical parts of ‘speech’ analogous to the way verbal language is constructed. When a ‘number sentence’ is read aloud, it appears in spoken medium as mathematics register (MR) which obeys all the grammatical rules of the particular verbal language in a way that is philologically sound. In mathematics, it is not the verbal sentences in MR that are important, but the sentence-forms expressing only the essential relations. Symbolism in mathematics is just a short-hand for otherwise cumbersome word-names: it is simply a means of manipulating concepts according to precise rules, since it condenses a hierarchy of concepts into manageable form.

While verbal language describes actual or imagined existence, mathematical language describes all logically possible existence. While verbal language describes the sorts of things

in the actual world, mathematical language describes relational properties of pattern, order, sizes and shapes of intelligible entities in possible worlds. Just as verbal language develops through the need to talk about categories of things that are important in everyday life like food, bed et cetera, mathematical language is adequate to describe and analyse the experiences of shape, space and order found in active play and observation. While verbal language is guided by a grammar which conforms to norms of conventional and social correctness, mathematical language is guided by a grammar consistent with the logic of intelligible forms of relational thought. Indeed, while verbal language operates according to acceptable social conventions, mathematical language operates in the realm and laws of pure thought.

Mathematical discourse is largely argumentation and computation, since the first step in solving a mathematical problem in MR is to express it in verbal language. It may also be realized that language, thought and calculation are interwoven. All thought expressed by whatever means acquire intelligibility in a linguistic medium (Urban 1971, 300-340). The first step in solving any mathematical problem is to look at it intuitively by verbalizing it. Looking at problems from different angles and asking questions are strategies for solving any problem whatsoever; it is such approaches that always trigger and order thought processes and conceptual schemata. By putting down our arguments extracted from MR, we come to verbally present the rationale that underlies various transformations within the solution process. For instance, before we can apply a given relation and associated transformations to a particular problem situation, we must first apprehend a structural isomorphism between the problem situation and the accompanying relations and transformations. It is in this sense that mathematical conceptions acquire intelligibility in discourse, and therefore the general interchange between linguistic and mathematical systems suggests isomorphic functions.

The solution to a mathematics problem requires a transformation procedure. The transformation starts from formulation in colloquial language to MR, to MPSS through computation, to the solution of the problem. This procedure requires some level of competence in identification of the logical form of linguistic patterns that guide the process.

Communication of Mathematical Experience in the Classroom

The first stage in general human perception is the attempt to fit experience into verbal

language. It is precisely in this sense that concepts are consequently developed and experiences categorized. The special sub-division within categories of experience managed by precise use of verbal language becomes the starting point of partitioning collections of things, identification of positions and relations of things which lead to development of mathematical concepts (Liebeck 1984). For instance, in an attempt to describe the experience of shapes and positions, concepts such as ‘nearness’ related to distance and “move” related to translation come to be developed.

In education, all teaching and learning of mathematics involves an understanding of relational properties of elements within the modes of categories of human experience. In pedagogy, there is interaction between such modes of experience and verbal linguistic mechanisms on the one hand, and knowledge of the levels of the structure of mathematics and the role of symbolic systems on the other. To discern the subtleties of this sort of interaction, engagement in mathematics education has to involve the use of instructional procedures that discern phases of intelligibility within discourse expressed in MR as follows:

(1) Problem exposition to identify the givens and relations that subsist in a problem situation.

The relations amongst the givens may be necessary, causal or contingent. Functionally, mathematics and verbal language are isomorphic, that is, although they differ in content, they are morphologically identical. Therefore, just as we have sentences in English language, so we also have number sentences in mathematics. The only subtle difference is that with mathematics, there are distinctions amongst hierarchies of language layers with respect to abstraction, formalization, precision, symbolization and generalization. A choice often has to be made as to which language layer is appropriate for specific tasks within a given problem situation. Necessary transactions have to be carried out by rephrasing the problem situation through verbal language, which already contains mathematical relations expressible in the form of the mathematics register (MR).

(2) Problem representation to discover structural properties of operations that give significance to the semantic relations in terms of allowable transformations. There is the development of number sentences ‘hidden’ in the verbal text, and identification of the necessary syntactic actions that ought to be performed to produce other acceptable transformations. Generally, under problem representation, there is the formulation of

number sentences through syntactic and semantic actions, thus initiating algorithmic transformations, that is, specification of “what to do” and “how to do” rules.

Problem representation is the discovery of the number sentence ‘hidden’ in the verbal text. It involves the processing of verbal input as well as the activity of the pupil’s cognitive schemata. The major aim of problem representation is to yield number sentences which function in two ways: as a formal mathematical representation of the semantic relations between quantities involved in the problem, and as an algorithmic expression which reveals the syntactic actions that ought to be performed to produce acceptable transformations.

The successful solution of a mathematical problem depends on the appropriate formulation of problem representation, which in turn presents semantic relations between word problems and number sentences. It is important that the underlying semantic relations between the givens and the unknown quantities be made explicit and expressed in appropriate sentences. It is also crucial to realize that the level of difficulty of a problem is determined by its semantic structure (Oldham 1989). More specifically, the relationship between children’s solution strategies and the semantic structure of word problems holds regardless of the kind of strategy adopted, that is, whether use is made of concrete objects, mental-solution or recalled number facts. These considerations reveal that failure to solve word problems is due to lack of appropriate schema rather than poor arithmetical or logical skills. Schemata clarify the problem by identifying the sequences of steps of a problem representation.

Solving a Mathematics Problem

To illustrate the subtleties of problem representation, we utilize the results of an empirical investigation which was undertaken by the authors. Note that “ Q_1 ” is a standard verbal problem, and “ Q'_1 ” is the reworded problem derived from the former. Although this paper utilises a philosophical method, it is our contention that drawing its implications from practical educational experience is consistent with its objectives.

Q_1 : “When I multiply a certain number by ten and subtract the product from ninety two, the answer I get is four less than twice the number. Find the number.”

Q'_1 : “When I multiply a certain number by ten and subtract the product from ninety two, I get

another number. If I take the initial number and double it, I get yet a different number. If I add four to the number I got in the first case, I get the same number as the one I got in the second case. Find the initial number."

The questions listed above were presented to pupils in their standard verbal forms, and interviews conducted to determine how they processed problem solving strategies. The question was then reworded by the pupils and the interviews repeated in order to enable the researcher to perform a comparative analysis of problem representation by the pupils.

The interview revealed that the conceptual reality of mathematics is analysable in terms of some logical form - a study of relational invariants that define the structure of mathematics. The understanding of problems expressed in the mathematics register (MR) is not easily accessible to pupils. The surface structure of MR does not readily reveal the semantic relation within a mathematics problem. However, when MR is translated into its equivalent reworded verbal text, pupils solve problems relatively better as table 1 reveals.

	A= Percentage of pupils who solved the problems correctly	B= Percentage of pupils who translated number sentences correctly into corresponding verbal problems
Standard verbal problems	68	48
Reworded verbal problems	92	60

Table 1: Relative Ability to Translate and Solve Problems

When the problem Q_1 is in its standard verbal form, its surface structure does not make obvious the semantic relations within the problem. While 68% of the pupils got the problems right in its standard verbal form, 92% got it right after it was reworded. It is therefore clear that reworded problems are solved significantly better than standard verbal problems. An explanation for this state of affairs is that semantic schemata of categories of relations are not

easily developed in children. Teachers who were interviewed believe that pupils tend to depend more on text driven processing to construct an appropriate problem representation.

Concerning syntactic translation, pupils assume that the sequence of words maps directly onto a corresponding sequence of literal symbols implied by a number sentence. For example, in an attempt to solve Q_1 , 60% of the pupils wrote the following number sentence which is rather misconceived, despite the fact that solving it gives the same numeral as the right answer:

X	*	10	-	92	=	4	-	2X
Unknown number X	Multiply	Ten	Subtract	Ninety Two	Result is	Four	Less than	Twice the number X

Table 2: Sequence of Literal Symbols of a Number Sentence

Although when the number sentence $10x - 92 = 4 - 2x$ is solved it yields a figure which is the same as the right answer, the conceptual inclinations that generated it are misplaced and cannot be replicated in a different problem with the same level of consistency. This is an example of a wrong mathematics problem-solving strategy (MPSS). It should be noted that the right MPSS gives the number sentence as $92 - 10x = 2x - 4$.

It is apparent that pupils tend to depend on text-driven processing of problem representation while in the ideal sense, they need to process MR in a conceptually-driven way using their semantic schemata. It seems that pupils tend to assume that the sequence of words in MR maps directly onto corresponding sequences of literal symbols implied by a number sentence, thus misrepresenting the syntactic translation. By asking students to explain their thinking, write their own problem, or compare and contrast concepts, teachers can pinpoint difficulties students are having with content. They can then adjust instruction to address those misconceptions early, rather than waiting until an assessment to determine what students do not know (Thompson & Chappell 2007). Although attempts at bridging the apparent dichotomy between text-driven and conceptually-driven processing is beyond the scope of this paper, we have suggested the movement from problem exposition, through problem

representation, to problem solution as an alternative solution procedure. This implies that pupils should be given a chance to reconstruct the word problem, which should in turn be assessed for correctness. It is the reconstructed word problem which would yield the corresponding number driven problem that would be transformed into a solution.

The elements of mathematical discourse are as follows:

- * Ordinary terms of verbal language made technical by giving them precise conceptual meanings.
- * Stipulation of distinctive modes of representing concepts. For instance, an ordinary fraction has been presented as

a Numerator
b Denominator

Figure 3: Fraction

- * Usage of stylized icons called pictograms.
- * Usage of symbols as signs having no significance in themselves, except as code elements representing concepts unambiguously.
- * Formulation of mathematical sentences expressed as semi-formal language consisting of verbal language supplemented by special symbols.
- * Development of mathematics register which involves expression of relations, operations, positions, sequences and patterns through special vocabulary within discourse of common speech.

The consequent procedure for problem-solving is here stipulated as a possible aid to the development of effective instructional designs as follows:

- Problem exposition.
- Problem representation.
- Identification of the givens within the problem situation.
- Identification of the goal state.
- Increasing specificity of the goal state by deriving its properties from the givens.
- Development of necessary working concepts for reaching the goal.

- Identification of sub-problems and sub-goals which test some specific level of competence.
- Generation of materials which act as inputs and Showing that the goal is (or is not) a possible derivative of the givens acts as the solution to the problem situation.

In performing a semantic translation, there is a tendency for pupils to link the equation being generated to the perceivable meaning of the problem. This is evident in their response to specific words and phrases in the problem. Instead of generating the equation as an expression of equivalence, they do it as a description of words and phrases in the order they appear in the problem, and therefore misrepresent it.

The foregoing observations indicate that whether the cause of difficulty is syntactic or semantic, verbal language interferes with translation of Mathematics Register (MR) into number sentence and vice versa, thereby leading to occasional misrepresentation of the Mathematics Problem Solving Strategy (MPSS). When mathematical problems are presented to pupils in their standard verbal forms, their surface structures do not make obvious the semantic relations within each problem. However, reworded problems are significantly solved better, as pupils tend to depend on text-driven processing to construct an appropriate problem representation. Only the competent pupils process the verbal text in a conceptually-driven way using their well developed semantic schemata. It is noteworthy that while text driven processing causes syntactic confusion due to the literal and lineal description of words in a problem situation, conceptually-driven processing emanates from a developed semantic schemata which generates number sentences as expressions of equivalence.

Problem Representation

Adequate problem representation is a prerequisite for successful problem-solving, the latter being a succession or sequence of problem states which terminate with a goal state. Each successive state is obtained from a preceding state by means of an allowable action. A solution procedure is an ordered succession of events which involve building sub-goals with a range of possible given materials and operations which have to deal with constraints specified in the problem situation. Problem solving procedure may be ordered as follows:

- Identification of the givens and the goal.

- Identification and attainment of sub-goals.
- Performance of operations and transformations toward a solution.

Let us briefly examine these elements.

The Givens and Goal

The first step toward solving a problem is the identification of the ultimate goal. A goal is the expression to be arrived at as the solution to a problem. For instance, in the number sentences generated by Q_1 , " $92 - 10x = 2x - 4$, find x ", the goal would be of the form $x = ?$ Where "?", becomes the solution to the problem and may be evaluated as right or wrong when it is found.

It is helpful to have a detailed representation and understanding of the goal. This may be done by increasing the specificity of the goal by deriving its additional properties using either the statement of the properties of the goal as given in the original problem, or by using given information to derive properties of the goal. The purpose of increasing the specification of the goal is to introduce the necessary working concepts for reaching it, which in turn reveals the necessary sub-goals.

Sub-problems, Sub-goals and Solutions

Between problem representation and identification of the goal, there are sub-problems which are solved by reaching the corresponding sub-goals. These sub-goals are always determined to have intermediate values between the givens and goal state, according to some explicitly defined evaluation functions.

Suppose we represent $SG_1, SG_2 \dots SG_n$ as the first to the n^{th} sub-goal respectively, then we may develop the following picture.



Figure 4: Successive Sub-goals from Problem Representation to Goal

Each sub-goal tests some specific competence, and normally generates materials which act as inputs for the subsequent sub-goals.

Although in utilizing our intuitive capacities we tend to formulate a problem-solving procedure from the givens state to the goal state, the initial approach to obtaining a solution may involve identification of sub-goals in the reverse order. Working backwards is a problem solving strategy in which the problem solver starts from the goal and determines the preceding statements which do not necessarily belong to the givens, but which when taken together will produce the goal.

While, in a theoretical sense, the solution to a problem requires working from the first sub-goal to the last sub-goal, actual problem-solving often requires identification of the order and forms of sub-goals from the last to the first. In this sense then, pupils may have to know the approximate form of the n^{th} sub-goal first and then to determine other sub-goals in the reverse order until the first one is reached. The first sub-goal is usually reached as a solution to the first sub-problem which is the immediate product of the relation between the givens at face value. Polya (1962), for instance, advises that it is useful to imagine that a problem is already solved, and then to ask oneself "what have I used to get this if I have these 'givens' and these 'operations?'"

Consider, for instance, the backward method of solving Q₂: A trader bought 60kg of maize flour at Sh. 4 per kg and another 40kg of millet flour at Sh. 6 per kg. She mixed the two types of flour and sold the mixture at Sh. 6 per kg. What percentage profit did she make?

$$\text{Percentage profit} = (\text{Profit} / \text{Buying Cost}) * 100\%$$

$$\text{Profit} = \text{Cost of mixture} - \text{Buying cost (of maize and millet)}$$

$$\text{Cost of sale} = \text{Mass of mixture} * \text{price (kg}^{-1}\text{) of the mixture}$$

$$= (60 + 40) \text{ kg} * \text{Sh. } 6 \text{ kg}^{-1}$$

$$= \text{Sh. } 600$$

$$\text{Buying cost} = \text{Cost of maize} + \text{cost of millet}$$

$$= \text{Mass of maize} * \text{price of maize} + \text{mass of millet} * \text{price of millet}$$

$$= 60\text{kg} * \text{sh. } 4 \text{ kg}^{-1} + 40\text{kg} * \text{sh. } 6 \text{ kg}^{-1}$$

$$= \text{Sh. } 240 + \text{Sh. } 240$$

$$= \text{Sh. } 480$$

$$\text{Profit} = \text{Sh. } 600 - \text{Sh. } 480$$

$$= \text{Sh. } 120$$

$$\text{Percentage profit} = \frac{\text{Sh. } 120}{\text{Sh. } 480} \times 100\% = 25\%$$

$$\text{Sh. } 480$$

The advantage of the backward method of problem-solving is that it is possible to detect contradictions. A contradiction would suggest that the goal is not a possible derivative of the givens. The givens in a problem may have a conjunctive or disjunctive relationship. When there are large numbers of given statements which have conjunctive relationships to one another, then working up a problem ‘backwards’ from the goal to the sub-goals is effective on condition that there is a single specified goal in the problem. However, the ‘forward’ method, which is working from the givens through sub-goals to the ultimate goal, is the conventional approach to problem solving, albeit not always efficient.

Conclusion

The foregoing discussion has observed that mathematics education in general and mathematical communication in classroom in particular is a discourse which oscillates between understanding verbal language, mathematics register (MR) and Mathematics Problem Solving Strategy (MPSS). The interaction develops into a dialogue which culminates in ‘mathematical dialectics’. The discourse is carried out in MR which contains sentences and phrases specifying components of a problem situation, namely the givens, operations, functions, relations, sub-goals and goals. These components are discerned through the semantic and syntactic analysis of the problem. Mathematical pedagogy is therefore the development of an understanding of the interaction amongst verbal language, MR and MPSS through the following procedure:

- Problem exposition.
- Problem representation.
- Problem solution.

Moreover, since the method of mathematics is basically argumentation and computation,

language considerations play a significant role in mathematical exposition (Kaput 1982; Pimm 1987). As such, the first step in resolving any mathematical problem is to translate it into everyday language. Consequently, curriculum development should not only concentrate on individual mathematical entities, but also on explicating the structural features of the mathematical system through an analysis of mathematical language. In this case then, it is the concern with translation of the structure of mathematics through mathematical language into instructional procedures that should pre-occupy teachers.

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