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THERMOELASTIC WAVES WITHOUT ENERGY DISSIPATION IN AN ELASTIC PLATE DUE TO A SUDDENLY PUNCHED HOLE

by

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ABSTRACT

The linear theory of thermoelasticity without energy dissipation for isotropic and homogeneous materials is employed to study waves in an elastic plate. The waves are assumed to arise out of a ramp-type stress on the plate's boundary which is maintained at constant temperature. Laplace transforms are used to solve the problem, and the distributions of displacement, temperature, radial and hoop stresses are displayed graphically.

Keywords : Thermoelasticity, Green-Naghdi, punched hole, elastic plate.

INTRODUCTION

Thermoelasticity theories which admit a finite speed for thermal signals have been receiving a lot of attention for the past thirty years. In contrast to the conventional coupled thermoelasticity theory based on a parabolic heat equation (Biot, 1956), which predicts an infinite speed for the propagation of heat, these theories involve a hyperbolic heat equation and are referred to as generalized thermoelasticity theories.

The first generalization, for isotropic bodies, is due to Lord & Shulman (1967) who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier's law. The anisotropic case was later developed by Dhaliwal & Sherief (1980).

The second generalization is known as the theory of thermoelasticity with two relaxation times, or the theory of temperature-rate-dependent thermoelasticity, and was proposed by Green & Lindsay (1972). It is based on a form of the entropy inequality proposed by Green & Laws (1972). It does not violate Fourier's law of heat conduction when the body under consideration has a centre of symmetry, and it is valid for both isotropic and anisotropic bodies.

The theory of thermoelasticity without energy dissipation is another generalized theory and was formulated by Green & Naghdi (1993). It includes the "thermal-displacement gradient" among its independent constitutive variables, and differs from the previous theories in that it does not accommodate dissipation of thermal energy. For a review of the relevant literature, see Chandrasekharaiah (1986, 1998) and Joseph & Preziosi (1989,1990).

The present investigation is devoted to the study of the thermoelastic interactions induced by a suddenly punched hole in an unbounded elastic plate, under the purview of the Green-Naghdi theory (Green & Naghdi, 1993). The plate is considered to be made of a linear, homogeneous and isotropic thermoelastic material, its bounding surface being at constant temperature and subjected to a ramp-type stress. The problem is solved by using the Laplace transform, and exact solutions are obtained in the transform space. Since the response is of more interest in the transient state, the Laplace inversions have been carried out in such a way that the results are particularly applicable in the short-time range; these are justified by the fact that the second-sound effects are short-lived. The derived analytical expressions for the displacement, temperature and stresses are computed numerically for a copper-like material and the results are displayed graphically.

Further, with a view to obtaining more insight into the problem, the Laplace inversions have also been carried out numerically by using the method of Durbin (1972).

We note that the counterpart of our problem in the context of the generalized theory of Green & Lindsay (1972) has been studied by Chand & Sharma (1991). A discussion of the main differences between our results and those of the latter is included.

FORMULATION OF THE PROBLEM

Following Chand & Sharma (1991), we consider a homogeneous and isotropic unbounded elastic plate of thickness *h*, initially unstressed, at rest and at constant temperature θ_0. A flat nose cylindrical projectile of radius r_0, moving with velocity *V*, strikes the plate and begins to punch out a hole of equal radius. The following assumptions are made :

(i) The plastic flow due to punching is localised in the neighbourhood of punching sections.

(ii) The punching begins instantaneously at time $t = 0$ over the whole punched section.

(iii) The punching action starts at velocity $V/2$, the projectile's velocity in the compressional wave that develops in both projectile and plate on impact, i.e., the plate material below the projectile is removed as a plug at velocity $V/2$. Hence, the punching time, $2h/V = T$, is based on a large ratio of diameter of projectile to plate thickness.

We work in cylindrical polar coordinates (r, ϑ, z), choosing the *z*-axis along the axis of the hole, and consider thermoelastic interactions which are symmetrical about the axis. Thus, the displacement vector has only the radial component $u = u(r,t)$, where *r* is the distance measured from the *z*-axis, and the stress tensor has only two components σ_{rr} and $\sigma_{\vartheta\vartheta}$ which are the normal stresses in the radial and transverse directions respectively.

According to the theory of thermoelasticity without energy dissipation, the field equations for a homogeneous and isotropic thermoelastic body, in the absence of heat sources and body forces, are as follows (Green & Naghdi, 1993) :

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla\,\nabla.\mathbf{u} - \gamma\nabla\theta = \rho\ddot{\mathbf{u}}, \qquad \textbf{(1)}$$

$$c\ddot{\theta} + \gamma\theta_0\nabla.\ddot{\mathbf{u}} = k\nabla^2\theta\,. \qquad \textbf{(2)}$$

The constitutive relation, in tensor notation, is given by

$$. \tag{3}$$

Here, **u** is the displacement vector, θ is the temperature change above the uniform reference temperature , ρ is the mass density, c is the specific heat, and are the Lamé constants, being the coefficient of volume expansion, and k is a material constant characteristic of the theory. Further, σ_{ij} and δ_{ij} are the stress tensor and Kronecker delta respectively.

In the present study eqns. (1) and (2) yield the following governing equations for u and θ :

$$, \tag{4}$$

$$k\left[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}\right]=c\frac{\partial^2\theta}{\partial t^2}+\gamma\theta_0\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right) \tag{5}$$

The relation (3) yields the following expressions for σ_{rr} and $\sigma_{\vartheta\vartheta}$:

$$\sigma_{rr}=(\lambda+2\mu)\frac{\partial u}{\partial r}+\lambda\frac{u}{r}-\gamma\theta\,, \tag{6}$$

$$\sigma_{\vartheta\vartheta}=\lambda\frac{\partial u}{\partial r}+(\lambda+2\mu)\frac{u}{r}-\gamma\theta\,. \tag{7}$$

It is convenient to have eqns. (4) and (5) and expressions (6) and (7) cast into dimensionless form. To this end, we follow Chandrasekharaiah (1997), and consider the following non-dimensionalization scheme :

$$r'=\frac{r}{l},\;\; t'=\frac{v}{l}t,\;\; u'=\frac{1}{l}\frac{\lambda+2\mu}{\gamma\theta_0}u,\;\; \theta'=\frac{\theta}{\theta_0},\;\; \sigma'_{rr}=\frac{1}{\gamma\theta_0}\sigma_{rr},\;\; \sigma'_{\vartheta\vartheta}=\frac{1}{\gamma\theta_0}\sigma_{\vartheta\vartheta}, \tag{8}$$

where l is a standard length, and v is a standard speed. Henceforth, non-dimensionalized quantities are used and the primes are omitted for convenience. Substituting (8) into eqns (4)-(7) , we obtain the following dimensionless equations :

$$C_P{}^2\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right]-C_P{}^2\frac{\partial\theta}{\partial r}=\frac{\partial^2 u}{\partial t^2}\ , \tag{9}$$

$$C_T{}^2\left[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}\right]=\frac{\partial^2\theta}{\partial t^2}+\varepsilon\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right), \tag{10}$$

$$\sigma_{rr}=\frac{\partial u}{\partial r}+\eta\frac{u}{r}-\theta\,, \tag{11}$$

$$\sigma_{\vartheta\vartheta}=\eta\frac{\partial u}{\partial r}+\frac{u}{r}-\theta\,. \tag{12}$$

$0<\eta<1$

$u=\frac{\partial u}{\partial t}=\theta=\frac{\partial\theta}{\partial t}=0$

In the above,

$$C_P{}^2=\frac{\lambda+2\mu}{\rho v^2},\ C_S{}^2=\frac{\mu}{\rho v^2},\ C_T{}^2=\frac{k}{cv^2},\ \varepsilon=\frac{\gamma^2\theta_0}{c(\lambda+2\mu)},\ \eta=1-2\frac{C_S{}^2}{C_P{}^2} \tag{13}$$

We also point out that C_P and C_S denote respectively the dimensionless speeds of purely elastic dilatational and shear waves, while C_T denotes that of purely thermal waves. Further, ε is the usual thermoelastic coupling parameter, and .

Let a denote the dimensionless radius of the hole. Assuming the body is at rest and undisturbed initially, then the following initial and regularity conditions hold :

$$, \text{ at } t=0 \quad \text{for } r\geq a\,. \tag{14}$$

The boundary conditions are given by

and $\theta = 0$ at .

We note that this ramp-type stress boundary condition may be written as

,

where σ_0 is a positive constant, and $H(\ )$ is the Heaviside unit step function. On using (11), these conditions become

$$\frac{\partial u}{\partial r} + \eta \frac{u}{r} = \frac{\sigma_0}{T}[(t-T)H(t-T)-t], \text{ and } \theta = 0 \text{ at } . \qquad \textbf{(15)}$$

TRANSFORM SOLUTION

Taking the Laplace transform defined by

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of Eqns (9) and (10) and expressions (11) and (12) under the homogeneous initial conditions (14), the following equations are obtained :

$$[C_P{}^2 DD_1 - s^2]\bar{u} = C_P{}^2\, D\bar{\theta}\,, \qquad \textbf{(16)}$$

$$[C_T{}^2\, D_1 D - s^2]\bar{\theta} = \varepsilon s^2\, D_1 \bar{u}\,, \qquad \textbf{(17)}$$

$$\bar{\sigma}_{rr} = D\bar{u} + \eta \frac{\bar{u}}{r} - \bar{\theta}\,, \qquad \textbf{(18)}$$

$$\bar{\sigma}_{\vartheta\vartheta} = \eta D\bar{u} + \frac{\bar{u}}{r} - \bar{\theta}\,, \qquad \textbf{(19)}$$

where

$$D = \frac{d}{dr},\ D_1 = \frac{d}{dr} + \frac{1}{r}.$$

From eqns. (16) and (17), we obtain

$$(DD_1 - m_1^2)(DD_1 - m_2^2)\,\bar{u} = 0\,, \tag{20}$$

$$(D_1 D - m_1^2)(D_1 D - m_2^2)\,\bar{\theta} = 0\,, \tag{21}$$

where m_1 and m_2 satisfy the bi-quadratic equation

$$C_P{}^2\, C_T{}^2 m^4 - s^2\,[C_T{}^2 + (1+\varepsilon)\,C_P{}^2\,]\,m^2 + s^4 = 0\,. \tag{22}$$

On solving eqns. (20) and (21) and using (14), the following solutions are obtained :

$$\bar{u} = A_1 K_1(m_1 r) + A_2 K_1(m_2 r)\,, \tag{23}$$

$$\bar{\theta} = B_1 K_0(m_1 r) + B_2 K_0(m_2 r)\,. \tag{24}$$

$$[\{C_T{}^2 - (1+\varepsilon)\,C_P{}^2\}^2 + 4\varepsilon\, C_P{}^2 C_T{}^2]^{1/2}$$

Here, K_1 and K_0 are modified Bessel functions of order one and zero respectively, and A_1, A_2, B_1, B_2 are arbitrary constants. Also, m_1 and m_2 are assumed to have positive real parts in order to meet the regularity conditions (14). The solution of eqn. (22) leads to

$$m_\alpha = \frac{s}{V_\alpha}\,, \tag{25}$$

where

$$V_\alpha = \frac{1}{\sqrt{2}}[\{C_T{}^2 + (1+\varepsilon)\,C_P{}^2\} + (-1)^{1+\alpha}\,\Delta]^{1/2}\,, \tag{26}$$

and

$$\Delta =$$

$$= V_1^2 - V_2^2\,.$$

Here and in the expressions that follow, the index α takes values 1, 2.

Substituting for $\overline{u}$ and from (23) and (24) into (17), and equating the corresponding coefficients of the Bessel functions, we have

$$B_\alpha = \frac{\varepsilon s^2 m_\alpha}{s^2 - C_T{}^2 m_\alpha^2} A_\alpha . \tag{27}$$

We now determine A_α by taking the Laplace transform of the boundary conditions (15) and substituting for $\overline{u}$ from (23) into the resulting expression. This yields

, **(28)**

where $\Omega_\alpha = Y_\alpha - Y_{3-\alpha}$, with

$$Y_\alpha = m_{3-\alpha} K_0(m_{3-\alpha} a)[s^2 - C_T{}^2 m_\alpha^2]\left[\frac{1-\eta}{a} K_1(m_\alpha a) + m_\alpha K_0(m_\alpha a)\right]. \tag{29}$$

The substitution of A_α from (28) and of B_α from (27) into (23) and (24) yields explicit expressions for $\overline{u}$ and , which, on taking the inverse Laplace transform, then give *u* and θ. However, finding *u* and θ for arbitrary *t* is a formidable task. We therefore proceed in two ways. First, since the second-sound effects are short-lived, we obtain and analyse the solutions for small *t*, i.e., we take *s* to be large. Secondly, we use a numerical inversion technique to study the long-time behaviour.

SMALL -TIME SOLUTIONS

When *s* is large, so are as given by (25). We therefore make use of the asymptotic expansions of the modified Bessel functions for large arguments; thus

$$K_0(m_\alpha r) = K_1(m_\alpha r) \approx \left(\frac{\pi}{2 m_\alpha r}\right)^{1/2} e^{-m_\alpha r} . \tag{30}$$

Using (25) and (30), we find that expression (28) gets simplified to

$$A_\alpha = \frac{1}{T}\left(\frac{2m_\alpha a}{\pi}\right)^{1/2}\left[\frac{C_\alpha}{s^3}+\frac{E_\alpha}{s^4}\right]e^{as/V_\alpha}\,, \tag{31}$$

where

$$C_\alpha = \sigma_0\left(1-\frac{C_T{}^2}{V_\alpha^2}\right)\frac{V_\alpha^3 V_{3-\alpha}^2}{C_T{}^2(V_\alpha^2 - V_{3-\alpha}^2)}\quad ,$$

$$E_\alpha = \sigma_0\,\frac{1-\eta}{a}\left(1-\frac{C_T{}^2}{V_\alpha^2}\right)\frac{V_\alpha V_{3-\alpha}+C_T{}^2}{C_T{}^4(V_{3-\alpha}-V_\alpha)}\,\frac{V_\alpha^4 V_{3-\alpha}^3}{(V_\alpha + V_{3-\alpha})^2}\quad . \tag{32}$$

Substituting for A_α from (31) and for B_α from (27) into expressions (23) and (24), we obtain $\bar{u}$ and . Substituting the latter into (18) and (19) then yields $\bar{\theta}$ $\bar{\sigma}_{rr}$ and $\bar{\sigma}_{\vartheta\vartheta}$. Taking the inverse Laplace transforms of the expressions for $\bar{u}$, , $\bar{\sigma}_{rr}$ and $\bar{\sigma}_{\vartheta\vartheta}$, we obtain the following solutions for $u,\ \theta, \sigma_{rr}, \sigma_{\vartheta\vartheta}$, valid for small values of t:

$$u(r,t) = \frac{1}{T}\left(\frac{a}{r}\right)^{1/2}\sum_{\alpha=1}^{2}\left[\tfrac{1}{2}C_\alpha + \tfrac{1}{6}E_\alpha\tau_\alpha\right]\tau_\alpha^2\, H(\tau_\alpha)\,, \tag{33}$$

$$\theta(r,t) = \frac{\varepsilon}{T}\left(\frac{a}{r}\right)^{1/2}\sum_{\alpha=1}^{2}\left[C_\alpha + \tfrac{1}{2}E_\alpha\tau_\alpha\right]\frac{V_\alpha}{(V_\alpha^2 - C_T{}^2)}\,\tau_\alpha H(\tau_\alpha) \tag{34}$$

$$\sigma_{rr}(r,t) = -\frac{1}{T}\left(\frac{a}{r}\right)^{1/2}\sum_{\alpha=1}^{2}\left[\frac{1-\eta}{r}\left(\tfrac{1}{2}C_\alpha\tau_\alpha + \tfrac{1}{6}E_\alpha\tau_\alpha^2\right)+\varepsilon(C_\alpha + E_\alpha\tau_\alpha)\frac{V_\alpha}{V_\alpha^2 - C_T{}^2}\right.$$

$$\left. + \frac{1}{V_\alpha}\left(C_\alpha + \tfrac{1}{2}E_\alpha\tau_\alpha\right)\right]\tau_\alpha H(\tau_\alpha)\quad , \tag{35}$$

$$\sigma_{\vartheta\vartheta}(r,t) = \frac{1}{T}\left(\frac{a}{r}\right)^{1/2} \sum_{\alpha=1}^{2} \left[\frac{1-\eta}{r}\left(\tfrac{1}{2}C_\alpha \tau_\alpha + \tfrac{1}{6}E_\alpha \tau_\alpha^2\right) - \varepsilon(C_\alpha + E_\alpha \tau_\alpha)\frac{V_\alpha}{V_\alpha^2 - C_T{}^2} \right.$$

$$\left. - \frac{1}{V_\alpha}\left(C_\alpha + \tfrac{1}{2}E_\alpha \tau_\alpha\right) \right] \tau_\alpha H(\tau_\alpha) \ , \qquad \textbf{(36)}$$

where $\tau_\alpha = t - \dfrac{r-a}{V_\alpha}$.

DISCUSSION

From the short-time solutions obtained above, it can be shown (Chandrasekharaiah, 1997), that they each consist of two distinct coupled waves, one following the other; the faster wave travelling with speed V_1 and the slower wave travelling with speed V_2 . As indicated in Chandrasekharaiah (1997), we note that the faster wave is a predominantly elastic wave (*e*-wave), or a predominantly thermal wave (θ-wave) according as or .

Further, we observe that neither the *e*-wave nor the -wave experiences exponential decay with distance (attenuation). This is in sharp contrast with the results of Chand & Sharma (1991).

Another important point to mention is that all the field variables are continuous at the two wavefronts, unlike those of Chand & Sharma (1991), where only the displacement is continuous and where the radial stress experiences delta-function discontinuities as well.

Lastly, we note that for , all of and $\sigma_{\vartheta\vartheta}$ vanish identically, meaning that at a given instant of time $t_0 > 0$, the points of the region $r > a$ which are beyond the faster wavefront do not experience any disturbance. This is a characteristic feature of the generalized theories and also holds for Chand & Sharma (1991).

NUMERICAL RESULTS

In this section we present some numerical results to illustrate the theoretical ones obtained above. To this end, we choose a hypothetical, copper-like material characterized by the following (dimensionless) parameters :

$$C_P{}^2 = 1, \quad C_T{}^2 = 20, \quad C_S{}^2 = 0.2387, \quad \varepsilon = 0.0168.$$

For this material, we have $C_T > C_P$, so that the -wave is faster than the e-wave. From Eqn. (26) it follows that the (dimensionless) speeds of these waves are and $V_2 = 0.999558$ respectively. By taking $a = 1, \sigma_0 = 1, T = 2$, the behaviour of u, θ, σ_{rr} and $\sigma_{\vartheta\vartheta}$ is analysed at (dimensionless) time $t = 0.1$. It is found that at this instant of time the θ-wavefront is located at $r = r_1 = 1 + tV_1 = 1.447411$, and the e-wavefront at $r = r_2 = 1 + tV_2 = 1.099956$. Further, we have computed the field variables given by Eqns. (33)-(36) for various values of r at time $t = 0.1$. The results are shown in Figs. 1-3. We notice from these figures that the displacement, temperature and stresses all vanish identically beyond the θ-wavefront, thereby indicating that the effects of the waves are confined to the region as predicted by the theoretical results above.

$1 \le r \le 1.447413$ $V_1 = 4.474113$

$$\sigma_{\vartheta\vartheta}(r,t) = \frac{1}{T}\left(\frac{a}{r}\right)^{1/2}\sum_{\alpha=1}^{2}\left[\frac{1-\eta}{r}\left(\tfrac{1}{2}C_\alpha\tau_\alpha + \tfrac{1}{6}E_\alpha\tau_\alpha^2\right) - \varepsilon(C_\alpha + E_\alpha\tau_\alpha)\frac{V_\alpha}{V_\alpha^2 - C_T{}^2}\right.$$

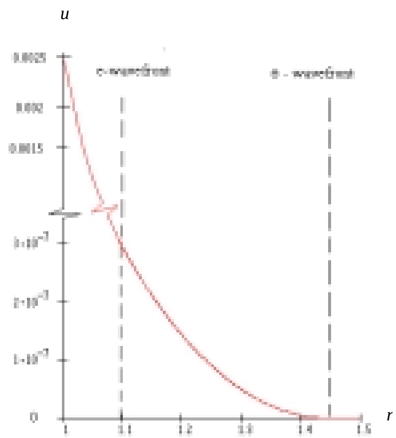


Fig. 1. Variation of u with r at $t = 0.1$.

Fig. 1 shows that *u* is continuous and decreases monotonically throughout the domain . It can be observed that it achieves its maximum value ( ) on the boundary of the hole.

Regarding the temperature field, Fig. 2 shows that is also continuous, rising steadily and attaining its peak value (0.0000321) at the first wavefront and thereafter decreasing uniformly to zero.

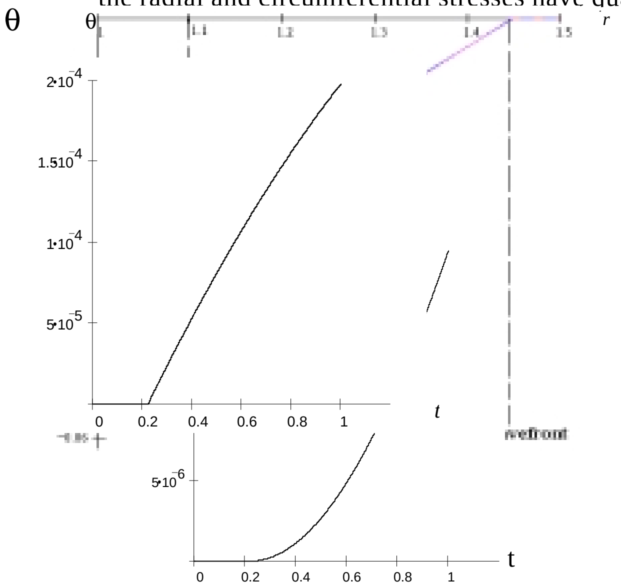
Fig. 2. Variation of θ with r at t = 0.1

As for the stress field, we have from Fig. 3 that both the radial and hoop stresses are compressive throughout the domain of influence , and that their *magnitudes* increase gradually. The maximum value of is found to be 0.04998, while that of $|\sigma_{\vartheta\vartheta}|$ is 0.02433, both maxima occurring on the boundary $r = 1$.

We now turn our attention to the behaviour of the field variables at the position for various values of *t*. At this location the faster wave (-wave) arrives at time 1 and the slower wave (*e*-wave) at $t(= 1/V_2) = 1.00044$. We have worked out the values of the field variables at this location for $t > 0$ using the solutions (33)-(36). These values are displayed in Figs. 4-6 for the interval t ε[0, 1/v2] . From Figs. 4 and 5 we observe that both the displacement and temperature fields are continuous for all $t \geq 0$, including the instants of arrival of wavefronts, as predicted by the theoretical results above. We also note that *u* and θ become positive immediately after the arrival of the faster wave, and increase rapidly.

Fig. 3. Variation of $\overline{\sigma}_{rr}$ and $\sigma_{\vartheta\vartheta}$ with r at $t = 0.1$

As for the stress field, we find from Fig. 5 that upon the arrival of the first wavefront, the stresses become compressive and stay so. Further, as in Fig. 3, we notice that the radial and circumferential stresses have qualitatively the same behaviour.



t

Fig. 4. Variation of u with t at $r = 2$.

Fig. 5. Variation of with t at $r = 2$.

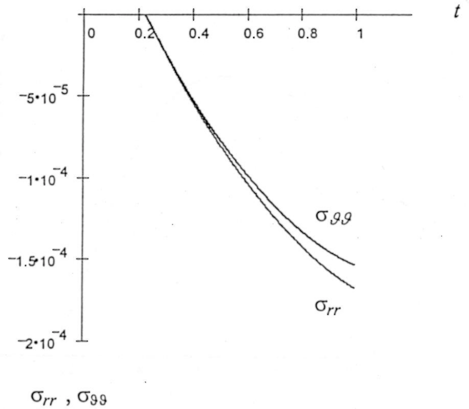


Fig. 6. Variation of σ and with t at r = 2.

LONG-TIME SOLUTIONS

In order to investigate the long-time behaviour of the solutions we perform the numerical inversion of Eqs (23)-(24) and (18)-(19), using the relations given by (27) and (28) as well as the above values for *a*, and *T*. The method employed is due to Durbin (1972).

The Durbin method is based on a Fourier series expansion, where the inverse $f(t)$ of Laplace transform is approximated by

$$f(t) = \frac{e^{\xi t}}{4\mathrm{T}_{\max}}\left[\frac{1}{2}\bar{f}(\xi) + \mathrm{Re}\left\{\sum_{k=1}^{\mathrm{N}} e^{\frac{ik\pi t}{4\mathrm{T}_{\max}}}\,\bar{f}\left(\xi + \frac{ik\pi}{4\mathrm{T}_{\max}}\right)\right\}\right] ,$$

and where

$$\xi = \omega_{\mathrm{opt}} - \frac{\ln(\varepsilon_{\mathrm{opt}})}{8\,\mathrm{T}_{\max}} .$$

The term ω_{opt} is an optimal number greater than the real part of all the singularities of $\bar{f}(s)$ if any, else ω_{opt} is defaulted to zero. $\mathrm{T}_{\max}$ is the maximum time simulated, N is the number of terms in the series and $\varepsilon_{\mathrm{opt}}$ is an optimal tolerance value.
The results are displayed in Figs. 7-9, for the position $r = 2$, and .

Fig. 7 shows that the long-time displacement increases monotonically, reaching a maximum value of 1.3928 at time , and thereafter decreasing to zero.

Fig. 8 shows the long-time thermal field attaining a maximum of 0.00021 at time $t = 1/V_2$, after which it decreases until it reaches a minimum value of -0.000159 at . The temperature then increases until it becomes positive again to finally tend to zero.

Regarding the stress field in Fig. 9 we find that the radial stress remains compressive in the chosen time interval, dropping to a minimum of -0.55407 at $t = 3.023$, and then rising to a maximum of -0.17223 at $t = 8.007$. On the other hand, the hoop stress remains compressive for a while, with a minimum value of -0.10979 occurring at $t = 2.25$, thereafter increasing until it becomes tensile and attaining a peak of 0.40762 when $t = 6.955$.

$\sigma_{rr} = 3.02$

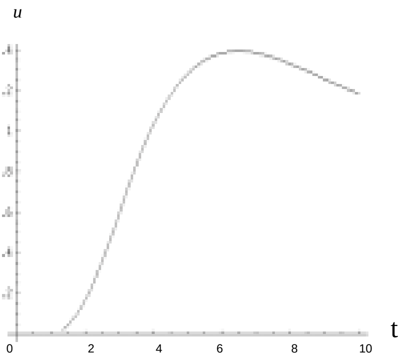


Fig. 7. Variation of *u* with t at *r* = 2.

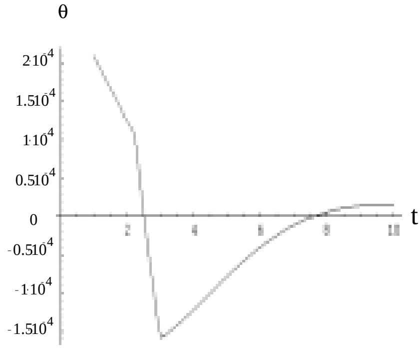


Fig. 8. Variation of θ with *t* at *r* = 2.

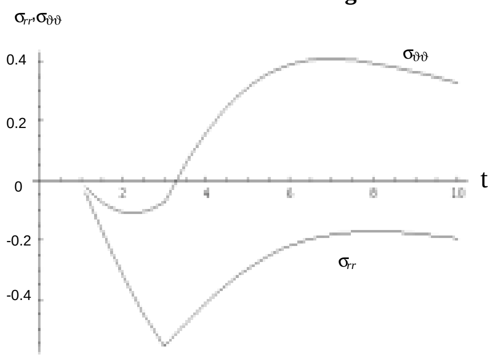


Fig. 9. Variation of and $\sigma_{\vartheta\vartheta}$ with *t* at *r* = 2.

CONCLUDING REMARKS

In this paper we have studied the thermoelastic interactions due to the punching of a cylindrical hole in an elastic plate, using the theory of Green & Naghdi (1993). It has helped bring into focus the similarities and differences between the predictions of the latter theory and those of Green & Lindsay (1972). The main similarities are that the effects of the waves are localized and the displacements are continuous, while the major differences are the absence of discontinuities in the thermal and stress fields, and the absence of attenuation in the Green-Naghdi theory. Finally, it is worth mentioning that the relative merits of one theory over the other cannot be judged upon the basis of these predictions, the latter merely help us in understanding the features inherent in the two theories in the context of the chosen problem.

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