A Comparison of Existing Optimisation Techniques with the Univariate Marginal Distribution Algorithm for the Channel Assignment Problem

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Abstract
The channel assignment problem has an important role to play in mobile telephone communication. Since the frequency band width is limited, the optimal assignment problem of channels has become increasingly important. In this paper we give a review of methods that have been used to solve the channel assignment problem and propose a new algorithm, namely the univariate marginal distribution algorithm. The existing techniques that are described are the simulated annealing, graph colouring, neural networks, genetic algorithms and tabu search. We discuss the drawbacks of each of these optimization techniques and we show how the proposed algorithm performs in terms of the number of generations required for convergence. We apply the univariate marginal distribution algorithm to a benchmark problem known as the Philadelphia problem or the 21-cell problem so that we can compare our simulation results. We consider a series of different problems by altering the channel assignment constraints, which are the co-channel constraint, the adjacent channel constraint and the co-site constraint. We show how the proposed technique also achieves the theoretical lower bounds in each of the cases.

Keywords: Channel Allocation, Estimation of Distribution Algorithms

1. INTRODUCTION

The rapid development of various forms of mobile communications and their increasing popularity assured its high growth rate. This growth rate is due to the ability to provide users instant connectivity anytime and anywhere in the world. Starting from an analog standard known as the 1st generation, one can see the full migration from an analog into a digital standard known as the 2nd generation. The 2nd generation mobile standard (GSM, D-AMPS and CDMA), with high quality voice and the capability to provide high speed data services to mobile user as an additional service has been the major factors of high growth of sales in communication technology. Subsequently, the 3rd generation mobile wireless technology with the aim of having global standard for all applications and countries has been introduced. In mobile cellular network the geographical area covered by the network is divided into a number of hexagonal cells. Communications to and from mobile users in each cell are serviced by a base station (BS) located at the center of each cell. Each channel can support a call. When a channel is allocated to a cell, a mobile user in that cell can use that channel. A channel can be simultaneously used by multiple base stations, provided that their distances are more than the minimum re-use distance, in other words there is enough distance so that there will be no interference. The minimum physical distance after which the same channel could be used is an important parameter for the mobile network design and is called the re-use distance which is normally expressed in units of number of cells. The interference pattern between any two pair of cells of the mobile system is fixed and is specified by an $n \times n$ matrix, known as the compatibility matrix, where n is the number of base stations.

The channel assignment problem (CAP) is to assign carriers to cells so that, as far as possible, the traffic demand for the cell is met. Thus it is to minimize the overall rate of blocked calls. Blocked calls are those calls where the BS has not been able to allocate a channel to a mobile user. There are three types of carrier allocation strategies:

(1) Fixed Channel allocation, where depending on a priori available static demand of different cells the carriers are assigned.
(2) Dynamic Channel allocation, where due to change in traffic pattern with time, the carrier demands at different cells change dynamically, a free channel is allocated to a BS requiring it.
(3) Hybrid Channel allocation, where each cell is allocated a fixed set of permanent carriers and a number of channels are set aside to be dynamically allocated depending on changing requests from base stations.

The fact that the available frequency bandwidth is limited, an efficient use is necessary. Frequency reuse is highly recommended but at the same time interference among the users must be avoided and the traffic demand should be met as well. However, the concept of cellular system enables the discrete channels assigned to a specific cell to be reused in different cells separated by a distance sufficient enough to bring the value of co-channel interference to a tolerable level thereby reusing each channel many times. Therefore this is a constraint optimization problem. Our aim is to minimize the frequency bandwidth and the corresponding channel assignment, satisfying given channel demands for different cells without violating interference constraints.

The paper is organized as follows: section 2 gives a description of the mathematical formulation of the channel assignment problem. In section 3 a review of the different algorithms that have been used to solve the channel assignment problem is given. Section 4 gives a brief description of the Estimation of Distribution Algorithms and in section 5 the methodology is described. In section 6 we present the simulation results and we finally wrap up with some concluding remarks.

2 CHANNEL ASSIGNMENT PROBLEM IN CELLULAR RADIO NETWORKS.

According to the model described in (Kim 1994; Sivaranjan 1986) we define a cellular network by means of the following five components:

(i) a set of n distinct cells;

(ii) a demand vector $\boldsymbol{m} = (m_i)$, $1 \le i \le n$;

(iii) a frequency separation matrix or interference matric $C = (c_{ij})_{n\times n}$;

(iv) a frequency assignment f_{ik}, $1 \le i \le n$, $1 \le k \le m_i$, where each frequency f_{ik} is represented by a positive integer;

(v) a set of frequency separation constraints

$$\left|f_{ik} - f_{jl}\right| \ge c_{ij} \quad \forall\, i, j, k, l$$

Each entry $c_{ij} \in C$ represents the required frequency separation between each pair of system channels. If, for example, $c_{ij}=0$, then no frequency separation is needed between f_{ik} and f_{jl}: cells i and j are co-channel cells and f_{ik} may be reused in cell j.

We now describe the frequency separation constraints:

(i) Co-channel constraint – when $c_{ij}= 1$, meaning that the same frequency cannot be assigned to certain pairs of cells simultaneously.

(ii) Adjacent channel constraint- when $c_{ij}= 2$, meaning that adjacent frequencies for example f_i and f_{i+1} cannot be assigned to certain pairs of cells simultaneously.

(iii) Co-site constraint c_{ii} – is the minimum distance of separation of frequencies of two carriers assigned to the same cell x_i . Its value depends on the communication system used.

Given the above conditions, the channel assignment problem consists of finding a channel assignment, that is, the f_{ik}s for the cellular network such that the system bandwidth, that is,

$$\max_{ik} f_{ik}$$

is minimized.

3. SEARCH AND OPTIMISATION ALGORITHMS

In this section we give a description of the different optimization techniques used to solve the CAP by different authors. The CAP has interested many researchers since the 70s (Box, 1978) and still now it presents an active topic (San Jose, 2007). Several methods have been proposed based on the following algorithms:

- Graph Theory- (Hale 1980; Gamst 1982; Sivaranjan 1986; Sengoku 1991)
- Neural Networks - (Chan 1994; Funabiki 1992; Smith 1998)

- Simulated Annealing - (Duque 1994; Kirkpatrick 1983; Li 2001; Mathar 1993)
- Genetic Algorithm - (Beckmann 1999; Fu 2003; Ghosh 2003; Kim 1995; San Jose, 2007)
- Tabu Search- (Capone 1999; Hao 1998).

Hale(1980) showed that the channel assignment problem is equivalent to graph colouring problem, when only co-channel constraints are considered: that is, *C* matrix are either zero or one. Several ways of modeling the CAP as a graph coloring has been proposed in (Sen, 1997; Sengoku, 1991; Wang, 1997). The graph colouring problem, which is a simpler version of this channel assignment problem is a well-known NP-complete problem.

The graph theoretic approach has been extensively studied as mentioned above. Based on the heuristic of assigning channels to the cell with the highest assignment difficulty first, Box (1978) proposed an iterative algorithm with an initial set of randomly generated numbers to represent the assignment difficulties of individuals cells. This algorithm was shown to have a slow convergence rate and a high running time complexity especially when the system size is large. In (Gamst, 1982), a heuristic measure of the assignment difficulty was proposed and cells are ordered into a list by either node-color ordering or node-degree ordering. Based on the list, channels are assigned by either frequency exhaustive or requirement exhaustive strategies. Later Sivaranjan (1986) proposed an improved heuristic measure for channel assignment difficulty and a new cell ordering method called column-wise cell ordering was also introduced. The algorithms proposed in (Sivaranjan, 1986) gave the best performance overall existing algorithms on the 21-cell landmark examples adopted. The simulation results in Sivaranjan (1989) were compared with the theoretical lower bounds derived by Gamst (1986) .

The Neural Network approach has been proposed by many authors (Kunz 1991; Fernandes 2001; Funabiki 2000) but the main drawback of this method is that it has been shown to be inappropriate for channel assignment as it generates poor solutions even in simple cases and converges at a slow rate. The use of simulated annealing avoid the problem of getting trapped by the local minimum solutions but at the expense of very long running time. Furthermore the quality of the solution is difficult to control. Tabu Search methods have not been that popular but the algorithms have shown to give good solutions but at a rather slow speed. A comparison between Simulated Annealing and Tabu Search method shows that Tabu Search algorithm is not only capable of matching, but it even outperforms Simulated Annealing, in locating the minimal number of frequencies for channel allocation, and it also constitutes a faster procedure (Hao, 1998).

Genetic algorithms (GAs) are iterative optimization procedures and work with a number of solutions known as a population in each iteration. Genetic algorithms depend to a large extend on associated parameters like operators and probabilities of crossing and mutation, size of population, rate of generational reproduction, the number of generations. Beckmann, (1999) and Ghosh, (2003) have proposed GA algorithms that performed well in CAP.

4. ESTIMATION OF DISTRIBUTION ALGORITHMS

Researchers using GAs require lots of experience to be able to choose suitable values for GA parameters. Thus there been a need to find better algorithms. The "Estimation of Distribution Algorithms" (EDAs) (Larranaga, 2002) helps to make prediction of the movements of the population in the search space easier besides avoiding the need of many parameters like those required in GAs. EDAs are population based search algorithms based on probabilistic modeling. The new population of individuals are generated without using neither crossover nor mutation operators. Instead the new individuals are sampled starting from a probability distribution estimated from the database containing only selected individuals from the previous generation. The interrelations between the different variables representing the individuals are expressed explicitly through the joint probability associated with the individuals selected at each iteration. The pseudo code of EDA is as follows:

1. $D_0 \leftarrow$ Generate M individuals (the initial population randomly).

Repeat for l = 1, 2,… until a stopping criteria is met.

2. $D_{l-1}^N \leftarrow$ Select $N \le M$ individuals from D_{l-1} according to a selection method.
3. $\rho_l(x) = \rho(x \mid D_{l-1}^N) \leftarrow$ Estimate the probability distribution of an individual being among the selected individuals.
4. $D_l \leftarrow$ Sample M individuals (the new population from $\rho_l(x)$).

This pseudo-code of EDA involves four main steps:

(i) The first population D_0 of M individuals is generated, usually by assuming a uniform distribution (either discrete or continuous) on each variable, and evaluating each of the individuals.
(ii) A number N ($N \le M$) of individuals are selected, usually the fittest.
(iii) Thirdly, the n-dimensional probabilistic model that better expresses the interdependencies between the n variables is induced.
(iv) The new population of M new individuals is obtained by simulating the probability distribution learnt in the previous step.

Steps (ii), (iii) and (iv) are repeated until a stopping condition is verified. The most important step is to find the interdependencies between the variables (step (iii)), and this is done using techniques from probabilistic graphical models.

The selection step can be carried out by using any existing strategies in evolutionary computation such as truncation selection. Once the individuals are selected, they are treated as a data-base from which their joint probability distribution are estimated. In the pseudo-code above, the distribution of the l-th generation is represented by $\rho_l(x) = \rho(x \mid D_{l-1}^N)$. Finally, $\rho_l(x)$ is simulated to create the individuals of the next population. These steps are repeated until a previously defined stopping criterion is met.

The main problem of EDAs lies on how the probability distribution $\rho_l(x)$ is estimated. Obviously the computation of all the parameters needed to specify the probability model is impractical. This has led to several approximations where the probability is assumed to factorise according to a probability model. Thus, according to the complexity of the probability model, EDA can be classified as univariate, bivariate and multivariate. Univariate EDAs may assume that the n-dimensional joint probability distribution is decomposed as a product of n univariate independent probability distribution, that is,

$$\rho_l(x) = \prod_{i=1}^{n} \rho_l(x_i), \text{ where } x = (x_1, \ldots, x_n). \tag{1}$$

This type of EDA is known as Univariate Marginal Distribution Algorithm (UMDA).

5. METHODOLOGY

Our algorithm first generates a random population of 50 individuals. The random population is then evaluated using the Frequency Exhaustive strategy (FEA) shown in Figure 1.

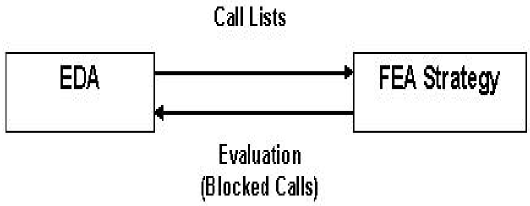

Figure 1: Frequency Exhaustive Algorithm (FEA)

The FEA strategy determines the number of calls without an allocated frequency; in other words one checks how many calls have not been assigned a proper frequency without violating the interference constraints; such calls are known as blocked calls denoted by *b*. A low value of *b* corresponds to a high solution quality. The 50 individuals were then sorted out in terms of number of blocked calls(in ascending order as the best individual are those represented by a lower number of blocked calls). The best 25 individuals were sorted out and these were retained. The probability distribution of an individual being among the selected individuals is estimated and a new population of 50 individuals is sampled. This new population becomes the population to be evaluated by the FEA strategy. The procedure is repeated until we get a solution of the desired quality or terminate the search due to a reached maximum number of iterations. The optimal solution is one which has a zero number of blocked calls within the minimum theoretical lower bounds (Gamst, 1986) and which satisfy the interference constraints as well as the traffic demand.

6. SIMULATION RESULTS

The new approach is tested on the Philadelphia problem, the 21 cells system shown in Figure 1.

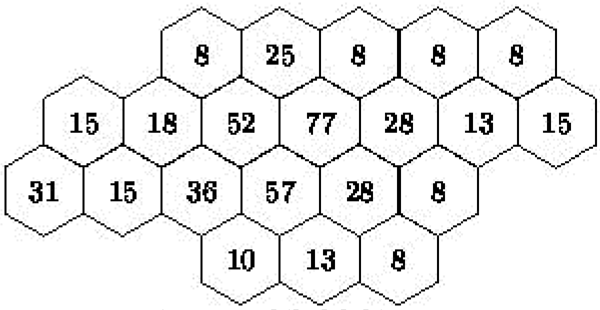

Figure 2: Philadelphia system

This system has been dealt by many authors and hence it is possible to carry out a comparison with reported solutions. A set of 12 problems has been considered by altering the adjacent and co-site constraints described earlier and the demand vector M_i, i =1,2. The demand vectors are shown in Table 1 and the different Interference conditions are shown in Table 2.

Table 1 Frequency Demand Vectors D1 and D2

	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}	m_{11}
M_1	8	25	8	8	8	15	18	52	77	28	13
M_2	5	5	5	8	12	25	30	25	30	40	40
	m_{12}	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{18}	m_{19}	m_{20}	m_{21}	
M_1	15	31	15	36	57	28	8	10	13	8	
M_2	45	20	30	25	15	15	30	20	20	25	

The algorithm has been used using 50 different seed values. The algorithm is stopped when either the solution obtained is of desired quality or the maximum number of iterations chosen as 25 has been reached.
Each problem was run 20 times from different seed values, and the best result was retained. For each problem the admissible frequency is generated which satisfies conditions (v) in section 2 within the theoretical lower bound.

Table 2 Interference Conditions

Problem Case	1	2	3	4	5	6	7	8	9	10	11	12
N_c	7	7	7	12	12	7	7	7	7	12	12	7
ACC	1	1	2	1	1	2	1	1	2	1	1	2
c_{ii}	5	7	7	5	7	5	5	7	7	5	7	5
Demand Vector	D_1	D_1	D_1	D_1	D_1	D_1	D_2	D_2	D_2	D_2	D_2	D_2

Table 3 shows the complete channel assignment for problem 2. A comparison with the earlier work is presented in Table 4.

Table 3: Frequency Assignment

Cells	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	1	1	1	2	2	1	1	1	1	2	2	1	1	1	1	1	1	2	1	1	1
	8	8	8	9	9	8	8	8	8	9	9	8	8	8	8	8	8	9	8	8	8
	15	15	15	16	16	15	15	15	15	16	16	15	15	15	15	15	15	16	15	15	15
	22	22	22	23	23	22	22	22	22	23	23	22	22	22	22	22	22	23	22	22	22
	29	29	29	30	30	29	29	29	29	30	30	29	29	29	29	29	29	30	29	29	29
	36	36	36	37	37	36	36	36	36	37	37	36	36	36	36	36	36	37	36	36	36
	43	43	43	44	44	43	43	43	43	44	44	43	43	43	43	43	43	44	43	43	43
	50	50	50	51	51	50	50	50	50	51	51	50	50	50	50	50	50	51	50	50	50
		57				57	57	57	57	58	58	57	57	57	57	57	57		57	57	
		64				64	64	64	64	65	65	64	64	64	64	64	64		64	64	
		71				71	71	71	71	72	72	71	71	71	71	71	71			71	
		78				78	78	78	78	79	79	78	78	78	78	78	78			78	
		85				85	85	85	85	86	86	85	85	85	85	85	85			85	
		92				92	92	92	92	93		92	92	92	92	92	92				
		99				99	99	99	99	100		99	99	99	99	99	99				
		106					106	106	106	107			106		106	106	106				
		113					113	113	113	114			113		113	113	113				
		120					120	120	120	121			120		120	120	120				
		127						127	127	128			127		127	127	127				
		134						134	134	135			134		134	134	134				
		141						141	141	142			141		141	141	141				
		148						148	148	149			148		148	148	148				
		155						155	155	156			155		155	155	155				
		162						162	162	163			162		162	162	162				
		169						169	169	170			169		169	169	169				
								176	176	177			176		176	176	176				
								183	183	184			183		183	183	183				
								190	190	191			190		190	190	190				
								197	197				197		197	197					
								204	204				204		204	204					
								211	211				211		211	211					
								218	218						218	218					
								225	225						225	225					
								232	232						232	232					
								239	239						239	239					
								246	246						246	246					
								253	253							253					
								260	260							260					
								267	267							267					
								274	274							274					
								281	281							281					
								288	288							288					
								295	295							295					
								302	302							302					
								309	309							309					
								316	316							316					
								323	323							323					
								330	330							330					
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D	8	25	8	8	8	15	18	52	77	28	13	15	31	15	36	57	28	8	10	13	8

Table 4: Performance Comparison

Problem	1	2	3	6	7	8	9	12
Lower Bound	381	533	533	427	221	309	309	253
Our Approach	381	533	533	427	221	309	309	253
2007 Revuelta	381	533	533	427	221	309	309	253
2003 Ghosh	381	533	533	427	221	309	309	253
2001 Chakraborty	381	533	533	463	221	309	309	273
2001 Battiti	381	533	533	427	221	309	309	254
1999 Beckmann	381	533	533	427	221	309	309	253
1997 Kim	381	533	533	-	221	309	309	-

7. CONCLUSIONS

We have presented a new approach to solve the Channel Assignment Problem. The proposed technique is able to achieve the optimal solution for all the well-known benchmark problems where the given bandwidth is equal to the lower bound of the corresponding problem. The main advantage the new algorithm is that UMDA is an easy method to implement compared to other existing procedures that have been used and the average number of iterations to get the optimal solution is also very fast.

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