*UNIVERSITY OF MAURITIUS RESEARCH JOURNAL – Volume 16 – 2010*
*University of Mauritius, Réduit, Mauritius*

*Research Week 2009/2010*

# Comparison of NSGA-II and SPEA2 on the Multiobjective Environmental/Economic Dispatch Problem

**R T F Ah King** *
*Faculty of Engineering,*
*University of Mauritius*
Email: r.ahking@uom.ac.mu

**K Deb**
*Indian Institute of Technology,*
*India*
Email: deb@iitk.ac.in

**H C S Rughooputh**
*Faculty of Engineering,*
*University of Mauritius*
Email: r.rughooputh@uom.ac.mu

*Paper Accepted on 11 October 2010*

**Abstract**

Two of the state of the art multiobjective evolutionary algorithms have been used to solve the environmental/economic dispatch problem. The Fast and Elitist Nondominated Sorting Genetic Algorithm (NSGA-II) and the Strength Pareto Evolutionary Algorithm 2 (SPEA2) have been compared for the IEEE 30-bus system using normalized values of the objectives by generational distance as convergence metric, spread as diversity metric and actual computational times. A further investigation was carried using tools for statistical comparison of multiobjective optimizers. Results are presented for two cases: lossless system and system with transmission losses.

**Keywords**: Power Systems, Environmental/Economic Dispatch, Nondominated Sorting Genetic Algorithm, Strength Pareto Evolutionary Algorithm.

**For correspondences and reprints*

## 1. INTRODUCTION

Most real world problems are multiobjective in nature and have often been considered as single objective problems for their solutions because of limitations in the solution methods. With the advent of new optimization tools due to the development of evolutionary algorithms, these problems can now be handled with the consideration of multiple objectives. Research has also helped in effectively handling the equality and inequality constraints of such problems.

As presented in Momoh (2001), power system optimization has been performed by mathematical programming techniques and heuristic methods. Many applications of modern heuristic methods to the power systems area have been tried in the past decades (Nara 2000, Bansal 2005). Among these heuristic methods, evolutionary algorithms have attracted many researchers to consider them as robust optimization tools for various applications (Bansal 2006).

Some researchers have carried out simultaneous optimization of multiple objectives in the environmental/economic dispatch problem using evolutionary algorithms. The environmental/economic dispatch problem is a multiobjective optimization problem where the fuel cost for power generation and the emissions from the generating plant are simultaneously minimized. Srinivasan and Tettamanzi (1997) used a hybrid genetic algorithm using an indirect representation for solutions and a decoding procedure that always generates a feasible solution. However, the approach did not yield a good distribution on the Pareto-optimal front. Das and Patvardhan (1998) have proposed a Multi-Objective Stochastic Search Technique (MOSST) for the multi-objective economic dispatch problem. The method is based on a hybrid combination of real coded genetic algorithms and simulated annealing. Promising results have been obtained by Abido (2001 and 2003a) by using a Non-dominated Sorting Genetic Algorithm (NSGA) to locate the Pareto-optimal solutions with a good diversity. Subsequently, a niched Pareto genetic algorithm (NPGA) (Abido 2003b), the Strength Pareto Evolutionary Algorithm (Abido 2003c) and a comparative study of NSGA, NPGA and SPEA (Abido 2006) were performed by the same author who concluded that SPEA is better than the other algorithms. Perez Guerrero (2004) presented differential evolution for solving real and reactive power dispatch problems. Penalty function method was used to handle the constraints and this required multiple runs for finding the best solutions. Ah King et al. (2005) applied NSGA-II to the problem under consideration but also considered stochastic generation, loads and objectives. Multiobjective particle swarm optimization techniques to solve the environmental/economic dispatch problem have been proposed by Zhao and Cao (2005), Wang and Singh (2007), Victoire and Suganthan (2007) and Agrawal et al. (2008).

In this paper, the multiobjective environmental/economic dispatch problem has been solved for a typical test system using two state of the art multiobjective evolutionary algorithms: the fast and elitist Nondominated Sorting Genetic Algorithm (NSGA-II) and the strength Pareto evolutionary algorithm 2 (SPEA2). The two algorithms are compared using different performance indices.

## 2. ENVIRONMENTAL/ECONOMIC DISPATCH PROBLEM

Environmental/economic dispatch is a multiobjective problem with conflicting objectives because pollution minimization is conflicting with minimum cost of generation. The environmental/economic dispatch involves the simultaneous optimization of fuel cost and emission objectives which are conflicting ones. The problem is formulated as described below.

### 2.1 Fuel Cost Objective

The classical economic dispatch problem of finding the optimal combination of power generation which minimizes the total fuel cost while satisfying the total required demand can be mathematically stated as follows (Wood and Wollenberg 1996):

$$C = \sum_{i=1}^{n} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \tag{1}$$

Where $C$ is total fuel cost ($/hr); $a_i$, $b_i$, $c_i$ are fuel cost coefficients of generator $i$; $P_{Gi}$: is real power generated by generator $i$ (MW); and $n$ is number of generators.

### 2.2 Emission Objective

The minimum emission dispatch as opposed to the above classical economic dispatch minimizes the $NO_x$ emission objective which can be modeled using second order polynomial functions (Talaq et al. 1994a):

$$E = \sum_{i=1}^{n} (a_{iN} + b_{iN} P_{Gi} + c_{iN} P_{Gi}^2 + d_{iN} \exp(e_{iN} P_{Gi})) \tag{2}$$

where $a_{iN}$, $b_{iN}$, $c_{iN}$, $d_{iN}$ and $e_{iN}$ are $NO_x$ emission coefficients of generator $i$. Unit of $E$ is ton/hr.

The optimization problem is bounded by the following constraints.

### 2.3 Power Balance Constraint

This is represented by an equality constraint:

$$\sum_{i=1}^{n} P_{Gi} - P_D - P_L = 0 \tag{3}$$

where $P_D$ is total load (MW), and $P_L$ is transmission losses (MW).

A loadflow or powerflow solution has to be carried out and this involves an iterative process using Newton-Raphson method. In this case, the equality constraints on real and reactive power at each bus are as follows (Abido 2006):

$$P_{Gi} - P_{Di} - V_i \sum_{j=1}^{NB} V_j [G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)] = 0 \tag{4}$$

$$Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^{NB} V_j [G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)] = 0 \tag{5}$$

where $V_i$ and $V_j$ are voltages at bus $i$ and $j$; $\delta_i$ and $\delta_j$ are voltage angles at bus $i$ and $j$; $Q_{Gi}$ and $Q_{Di}$ are reactive power generated and reactive power demand at bus $i$; $G_{ij}$ and $B_{ij}$ are real and imaginary components of $Y_{BUS}$ and $NB$ is the number of buses in the power system.

### 2.4 Power Generation Limits Constraint

The power generated $P_{Gi}$ by each generator should lie between its minimum and maximum limits, i.e.,

$$P_{Gi\min} \le P_{Gi} \le P_{Gi\max} \tag{6}$$

where $P_{Gi\min}$ is minimum power generated; and $P_{Gi\max}$ is maximum power generated.

### 2.5 Multiobjective Formulation

The multiobjective environmental/economic dispatch optimization problem is therefore formulated as:

Minimize $[C, E]$ (7)

subject to: $\sum_{i=1}^{n} P_{Gi} - P_D - P_L = 0$

$$P_{Gi\,\min} \le P_{Gi} \le P_{Gi\,\max}$$

## 3. ELITIST NON-DOMINATED SORTING GENETIC ALGORITHM (NSGA-II)

Deb et al. (2000) have proposed an elitist Non-dominated Sorting Genetic Algorithm known as NSGA-II which uses both elite-preserving and diversity-preserving mechanisms. The two distinct goals in multiobjective optimization are:

(i) discover solutions as close to the Pareto-optimal solutions as possible

(ii) find solutions as diverse as possible in the obtained non-dominated front

It has been shown that NSGA-II can achieve these two goals well (Deb et al.2000).

A description of the NSGA-II algorithm is given in this section. Initially a random population $P_o$ is created. The population is sorted into different non-domination levels. Each solution is assigned a fitness equal to its non-domination level where level 1 is the best level. Binary tournament selection with a crowded tournament operator, recombination, and mutation operators are used to create an offspring population $Q_o$ of size $N$. The NSGA-II procedure (as in Deb et al. (2000)) is outlined below and shown schematically in Figure 1. Figure 2 illustrates the crowding distance calculation.

**NSGA-II**

Step 1 Combine parent and offspring populations and create $R_t = P_t \cup Q_t$

Perform a non-dominated sorting to $R_t$ and identify different fronts: $F_i$, $i = 1, 2, \ldots$

Step 2 Set new population $P_{t+1}$ = null. Set a counter $i = 1$.

Until $|P_{t+1}| + |F_i| < N$, perform $P_{t+1} = P_{t+1} \cup F_i$ and $i = i + 1$.

Step 3 Perform the Crowding-sort($F_i$,<c) procedure given below and include the most widely spread ($N - |P_{t+1}|$) solutions by using the crowding distance values in the sorted $F_i$ to $P_{t+1}$.

Step 4 Create offspring population $Q_{t+1}$ from $P_{t+1}$ by using the crowded tournament selection, crossover and mutation operators.

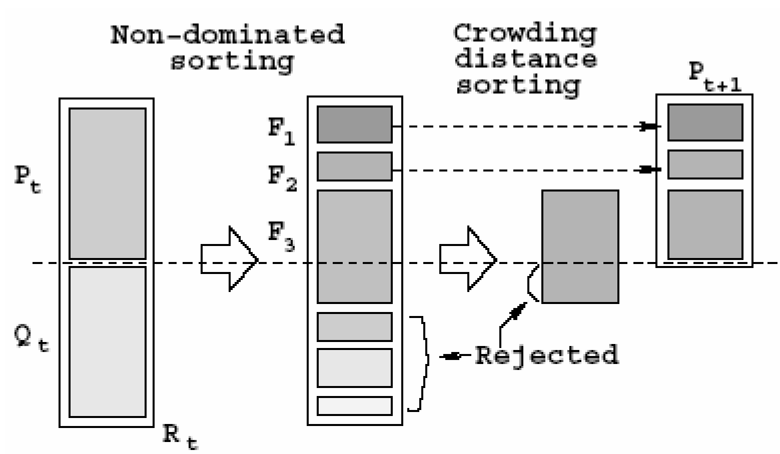



Figure 1: A schematic of the NSGA-II procedure (Deb et al. 2002)

**Crowding-sort($F_i$<c)**

Step 1 Call the number of solutions in $F$ as $l = |F|$. For each $i$ in the set, first assign crowding distance, $d_i = 0$.

Step 2 For each objective function $m = 1,2,...,M$, sort the set in worse order of $f_m$ or, find the sorted indices vector:

$I^m$ =sort($f_m$,>)

Step 3 For $m = 1,2,...,M$, assign a large distance to the boundary solutions, or $d_{I_1^m} = d_{I_l^m} = \infty$, and for all other solutions $j$ = 2 to ($l$ - 1), assign:

$$d_{I_j^m} = d_{I_j^m} + \frac{f_m^{(I_{j+1}^m)} - f_m^{(I_{j-1}^m)}}{f_m^{max} - f_m^{min}}.$$

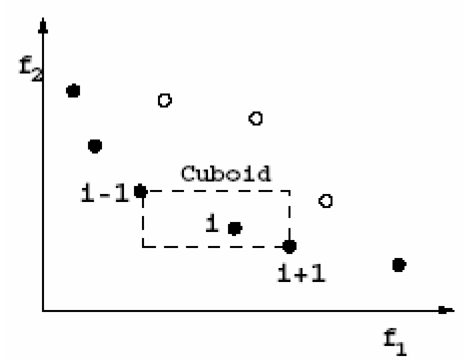



Figure 2: The crowding distance calculation (Deb et al. 2002)

NSGA-II performs a non-dominated sorting of the combined parent and offspring population. Elitism is introduced by maintaining the best non-dominated solutions in fronts until all $P$ population slots are filled. A crowded distance-based niching strategy is used to find solutions from the last front that are to be carried over to the next generation.

### 3.1 Simulated Binary Crossover and Parameter-based Mutation

The use of real-valued genes in GAs offers a number of advantages in numerical function optimization over binary encodings (Wright 1991). The variables are therefore represented as real numbers and the simulated binary crossover (Deb and Agrawal 1995) and the real-parameter mutation operator are used. With simulated binary crossover (SBX), two children solutions ($c_1$ and $c_2$) are created from two parents ($p_1$ and $p_2$) as follows (Deb and Agrawal 1999):

**Simulated Binary Crossover**

Step 1 Choose a random number $u \in [0,1)$.

Step 2 Calculate

$$\beta_q = \begin{cases} (u\alpha)^{\frac{1}{\eta_c+1}}, & if\ u \le \frac{1}{\alpha}; \\ \left(\frac{1}{2-u\alpha}\right)^{\frac{1}{\eta_c+1}}, & otherwise, \end{cases} \tag{9}$$

where

$$\alpha = 2 - \beta^{-(\eta_c+1)},$$

$$\beta = 1 + \frac{2}{y_2 - y_1} min[(y_1 - y_l),(y_u - y_2)].$$

$y_l$ and $y_u$: lower and upper limits of $y$

$\eta_c$ : distribution index for crossover

Step 3 Compute children solutions:

$$c_1 = 0.5[(y_1 + y_2) - \beta_q|y_2 - y_1|], \\ c_2 = 0.5[(y_1 + y_2) + \beta_q|y_2 - y_1|]. \tag{10}$$

The mutation operator (Deb and Agrawal 1999) is applied as follows:

**Parameter-based Mutation**

Step 1 Choose a random number $u \in [0,1)$.

Step 2 Calculate

$$\delta_q = \begin{cases} \left[2u + (1-2u)(1-\delta)^{\eta_m+1}\right]^{\frac{1}{\eta_m+1}} - 1, & \text{if } u \le 0.5, \\ 1 - \left[2(1-u) + 2(u-0.5)(1-\delta)^{\eta_m+1}\right]^{\frac{1}{\eta_m+1}}, & \text{otherwise}, \end{cases} \tag{11}$$

where

$\delta = min[(y - y_l), (y_u - y)] / (y_u - y_l)$

$\eta_m$ : distribution index for mutation

Step 3 Calculate the mutated child:

$$c = y + \delta_q ( y_u - y_l ).$$

### 3.2 Constrained Tournament Method

In this method, two solutions are picked from the population and the better solution is chosen. With constraints, each solution can be either feasible or infeasible. The constrain-domination principle (Deb et al. 2002) is defined as follows:

A solution *i* is said to constrained-dominate a solution *j* if any of the following conditions is true.

1) Solution *i* is feasible and solution *j* is not.
2) Solutions *i* and *j* are both infeasible, but solution *i* has a smaller overall constraint violation.
3) Solutions *i* and *j* are feasible and solution *i* dominates solution *j*.

Thus, feasible solutions are ranked according to their nondomination level based on the objective function values such that feasible solutions have better ranks than infeasible solutions. The infeasible solution with a smaller constraint violation is chosen when the tournament takes place between two infeasible solutions.

## 4. STRENGTH PARETO EVOLUTIONARY ALGORITHM 2 (SPEA2)

An improved version of SPEA, namely SPEA2, was proposed by Zitzler et al. (2001). SPEA2 uses a fine-grained fitness assignment strategy, a density estimation technique, and an enhanced archive truncation method. The SPEA2 algorithm as in Zitzler et al. (2001) is given below.

SPEA2 Algorithm

Input: $N$ (population size)

$\overline{N}$ (archive size)

$T$ (maximum number of generations)

Output: **A** (nondominated set)

Step 1 **Initialization**: Generate an initial population $P_0$ and create the empty archive (external set) $\overline{P}_0 = \phi$. Set $t = 0$.

Step 2 **Fitness assignment**: Calculate fitness values of individuals in $P_t$ and $\overline{P}_t$.

Step 3 **Environmental selection**: Copy all nondominated individuals in $P_t$ and $\overline{P}_t$ to $\overline{P}_{t+1}$. If size of $\overline{P}_{t+1}$ exceeds $\overline{N}$ then reduce $\overline{P}_{t+1}$ by means of the truncation operator, otherwise if size of $\overline{P}_{t+1}$ is less than $\overline{N}$ then fill $\overline{P}_{t+1}$ with dominated individuals in $P_t$ and $\overline{P}_t$.

Step 4 **Termination**: If $t \geq T$ or another stopping criterion is satisfied then set **A** to the set of decision vectors represented by the nondominated individuals in $\overline{P}_{t+1}$. Stop.

Step 5 **Mating selection**: Perform binary tournament selection with replacement on $\overline{P}_{t+1}$ in order to fill the mating pool.

| | |
|---|---|
| Step 6 | **Variation**: Apply recombination and mutation operators to the mating pool and set $\overline{P}_{t+1}$ to the resulting population. Increment generation counter ($t = t + 1$) and go to Step 2. |

### 4.1 Fitness Assignment

Each individual $i$ in the archive $\overline{P}_t$ and the population $P_t$ is assigned a strength value $S(i)$, representing the number of solutions it dominates:

$$S(i) = \left| \{ j \mid j \in P_t + \overline{P}_t \wedge i \phi \; j \} \right| \tag{12}$$

where $|\,.\,|$ denotes the cardinality of a set, + stands for multiset union and the symbol

$\phi$ corresponds to the Pareto dominance relation. On the basis of the $S$ values, the raw fitness $R(i)$ of an individual $i$ is calculated:

$$R(i) = \sum_{j \in P_t + \overline{P}_t,\, j \phi i} S(j) \tag{13}$$

The density $D(i)$ corresponding to $i$ is defined by

$$D(i) = \frac{1}{\sigma_i^k + 2} \tag{14}$$

where $\sigma_i^k$ is the $k$-th element for each individual $i$ the distances (in objective space) to all individuals in archive and population after sorting the list in increasing order.

Finally, adding $D(i)$ to the raw fitness value $R(i)$ of an individual $i$ yields its fitness $F(i)$:

$$F(i) = R(i) + D(i) \tag{15}$$

### 4.2 Environmental Selection

Copy all nondominated individuals, i.e., those which have a fitness lower than one, from archive and population to the archive of the next generation:

$$\overline{P}_{t+1} = \{i \mid i \in P_t + \overline{P}_t \wedge F(i) < 1\}$$

If the nondominated front fits exactly into the archive $\left|\overline{P}_{t+1}\right| = \overline{N}$ the environmental selection step is completed.

If the archive is too small ($\left|\overline{P}_{t+1}\right| < \overline{N}$), the best $\overline{N} - \left|\overline{P}_{t+1}\right|$ dominated individuals in the previous archive and population are copied to the new archive.

If the archive is too large ($\left|\overline{P}_{t+1}\right| > \overline{N}$),an archive truncation procedure is invoked which iteratively removes individuals from $\left|\overline{P}_{t+1}\right|$ until $\left|\overline{P}_{t+1}\right| = \overline{N}$ , that is, the individual which has the minimum distance to another individual is chosen at each stage; if there are several individuals with minimum distance the tie is broken by considering the second smallest distances and so forth.

## 5. PERFORMANCE METRICS

The two distinct goals in multi-objective optimization are (i) to discover solutions as close to the Pareto-optimal solutions as possible and (ii) to find solutions as diverse as possible in the obtained non-dominated front. For comparing two algorithms, at least two performance metrics (one evaluating the progress towards the Pareto-optimal front and the other evaluating the spread of solutions) need to be used (Deb 2001). Two such performance metrics together with two combined weighted metric for the overall performance of an algorithm are described below. A critical review of performance metrics was performed by Okabe (2003) whereas Knowles et al. (2006) summarized the state of the art in performance assessment of stochastic multiobjective optimizers.

### 5.1 Generational Distance for Convergence

Given a nondominated set $Q$ and a Pareto-optimal set $P^*$, the Generational Distance (GD) metric calculates the average distance of the solutions of $Q$ from $P^*$, as follows (Veldhuizen 1999):

$$GD = \frac{(\sum_{i=1}^{|Q|} d_i^p)^{1/p}}{|Q|} \tag{16}$$

For $p = 2$, the parameter $d_i$ is the Euclidean distance (in the objective space) between the solution in $Q$ and the nearest member of $P^*$:

$$d_i = \min_{k=1}^{|P^*|} \sqrt{\sum (f_m^{(i)} - f_m^{*(k)})^2} \tag{17}$$

where $f_m^{*(k)}$ is the $m$-th objective function value of the $k$-th member of $P^*$. An algorithm having a small value of GD is better. If the objective function values are of differing magnitude, they should be normalized before calculating the distance measure. A large number of solutions in $P^*$ is recommended in order to make the distance calculations reliable.

### 5.2 Spread for Diversity

A spread metric for evaluating the diversity among non-dominated solutions was proposed by Deb (Deb et al. 2000):

$$\Delta = \frac{\sum_{m=1}^{M} d_m^e + \sum_{i=1}^{Q} \left| d_i - \bar{d} \right|}{\sum_{m=1}^{M} d_m^e + |Q|\bar{d}} \tag{18}$$

where $d_i$ can be any distance measure between neighboring solutions and $\overline{d}$ is the mean value of these distance measures. In this paper, $d_i$ is calculated using Schott's difference distance measure (Schott 1995), that is, the minimum of the sum of the absolute difference in objective function values between the i-th solution and any other solution in the obtained non-dominated set. Alternatively the Euclidean distance or the crowding distance can be used to calculate $d_i$. $d_m^e$ is the distance between the extreme solutions of $Q$ and $P^*$ for each objective. For a two-objective problem, the term $|Q|$ is replaced by $|Q-1|$. For an ideal distribution of solutions, $\Delta$ = 0 and for bad distributions, $\Delta$ can take a value greater than 1. Thus, an algorithm finding a smaller $\Delta$ value is able to find a better diverse set of non-dominated solutions.

### 5.3 Combined Convergence and Diversity Metrics

Currently, there are only a few metrics combining both convergence and diversity evaluations. Such metrics are the attainment suface based statistical metric, weighted metric and non-dominated evaluation metric (Deb 2001). The last method is simply considering the convergence and diversity evaluations as a two-objective problem and if one algorithm dominates the other in both objectives, then the former is undoubtedly better than the other. Otherwise, no conclusions can be made. The attainment suface based statistical and weighted metrics are described in detail in the following.

#### 5.3.1 Attainment Surface Based Statistical Metric

Once a set of non-dominated solutions are obtained, a curve cannot be drawn to join the solutions since the intermediate points are not guaranteed to be feasible or even Pareto-optimal. A concept to represent the non-dominated solutions was suggested by Fonseca and Fleming (Fonseca 1996) whereby instead of joining the obtained non-dominated solutions by a curve, an envelope which represents the search space which are dominated by the obtained non-dominated solutions is created. The generated envelope is called an attainment surface and is a measure of both convergence and diversity of obtained solutions.

The attainment surface metric is particularly useful to represent the outcome of multiple runs. The attainment surface of each run is created and a number of diagonal imaginary lines, running in the direction of improvement in all objectives, are drawn. The intersection of the diagonal lines and the attainment surfaces are calculated, from which a frequency distribution is obtained based on the statistics. Thus, 0%, 50% and 100% attainment surfaces represent the region of the objective space which are dominated by 0%, 50% and 100% of the simulation runs.

The attainment function can be estimated from a sample of *r* independent runs of an optimizer via the empirical attainment surface (EAF) defined as (Knowles et al. 2006, Fonseca et al. 2005):

$$\alpha_r(z) = \frac{1}{r}\sum_{i=1}^{r} I(A^i \underline{\pi} \{z\}) \tag{19}$$

where $A^i$ is the *i*th approximation set (run) of the optimizer and $I(.)$ is the indicator function, wguch evaluates to one if its arguments is true and zero if its argument is false. Furthermore, the authors mentioned it may be interesting to plot all the goals that have been attained (independently) in 50% of the runs and defined the *k*%-approximation set *A* of an EAF $\alpha_r(z)$ if and only if it weakly dominates exactly those objective vectors that have been attained in at least *k* percent of the r runs:

$$\forall z \in Z : \alpha_r(z) \geq k/100 \Leftrightarrow A\underline{\pi}\{z\} \tag{20}$$

The attainment surface of such an approximation set *A* is the union of all the tightest goals that are known to be attainable as a result of *A*. Thus, the *k*%-attainment surface divides the objective space in two parts: the goals that have been attained and the goals that have not been attained with a frequency of at least *k* percent (Knowles et al. 2006).

### 5.3.2 Weighted Metric

In order to evaluate both goals of convergence and diversity, a weighted metric combining the Generational Distance metric, GD and spread metric, $\Delta$ is suggested (Deb 2001):

$$W = w_1 GD + w_2 \Delta \tag{21}$$

with $w_1 + w_2 = 1$. An algorithm having an overall small value of $W$ means that the algorithm is good in both convergence and diversity-preserving ability. It is recommended to use a normalized pair of metrics.

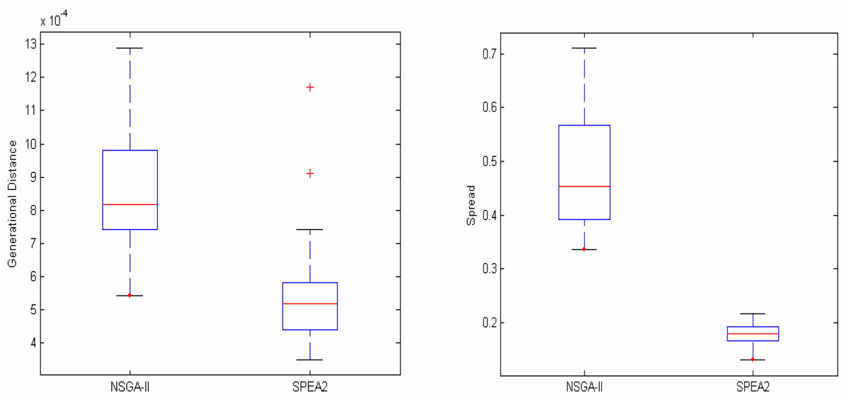
## 6. SIMULATION RESULTS

Two cases: system without transmission losses referred as Case 1 and system with transmission losses referred as Case 2 have been considered. For both Case 1 and Case 2 of the standard system, NSGA-II and SPEA2 algorithms have been compared using normalized values of the objectives by generational distance (*GD*) (Veldhuizen 1999) as convergence metric, spread ($\Delta$) (Deb 2000) as diversity metric, weighted convergence and diversity metrics (*W*) (Deb 2001) with equal weightage for each metric, and actual computational time on a 3.2 GHz Pentium 4 PC with 1 GB RAM running Linux operating system. The best nondominated Pareto front obtained from the combined Pareto fronts of 30 independent runs of each algorithm for 200 generations were used to calculate the metrics. The statistics (Mean, Standard Deviation, Minimum and Maximum values) are given in Tables 1 and 2. Furthermore, Figures 3 and 5 show the box and whisker plots (box plots) for the performance metrics for each case. Figures 4 and 6 also show the 0% attainment surface (best nondominated front) and 100% attainment surface (worst nondominated front) of each algorithm for all 30 runs for each case.

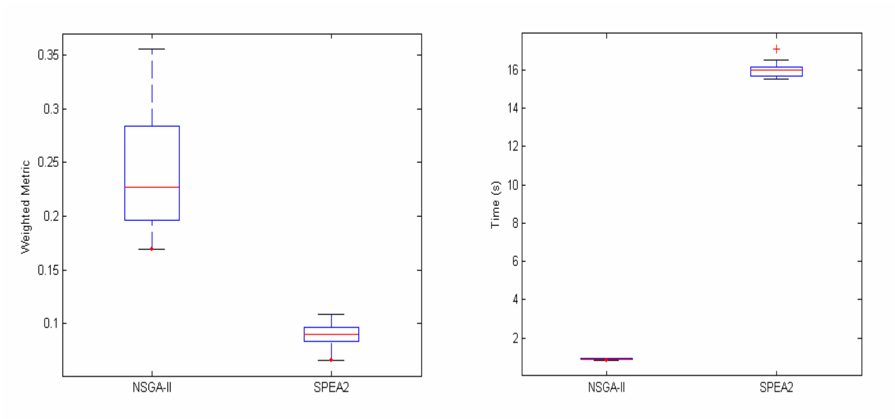
An interesting observation from Figures 5.30 and 5.32 is that the extreme values (minimum fuel cost and minimum NOx emission) are attained by all runs of both algorithms and this is certainly not the case for the other solutions as confirmed by the 0% and 100% attainment surfaces 'gap'.

Table 1: Performance metrics for deterministic Case 1

| | Generational Distance, *GD* | | Spread, $\Delta$ | | Weighted Metric, *W* | | Time (s) | |
|---|---|---|---|---|---|---|---|---|
| | NSGA-II | SPEA2 | NSGA-II | SPEA2 | NSGA-II | SPEA2 | NSGA-II | SPEA2 |
| Mean | 0.000851 | 0.000547 | 0.473265 | 0.177034 | 0.237058 | 0.088791 | 0.891333 | 15.987333 |
| SD | 0.000182 | 0.000171 | 0.093398 | 0.021224 | 0.046697 | 0.010597 | 0.028856 | 0.352302 |
| Min | 0.000543 | 0.000348 | 0.336502 | 0.130890 | 0.168828 | 0.065720 | 0.830000 | 15.520000 |
| Max | 0.001289 | 0.001170 | 0.710375 | 0.216421 | 0.355561 | 0.108427 | 0.950000 | 17.080000 |




(a) (b)




(c) (d)

Figure 3: Box plots of performance metrics for Case 1

(a) Generational Distance (b) Spread (c) Weighted Metric (d) Time

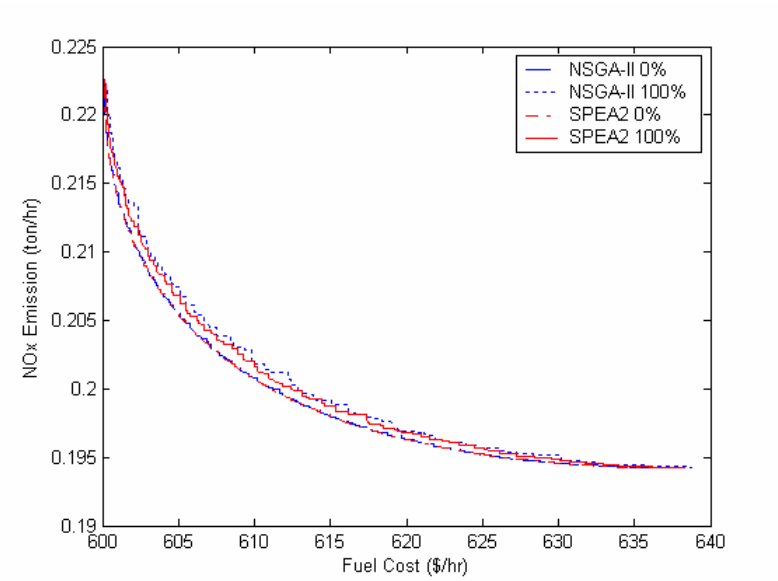
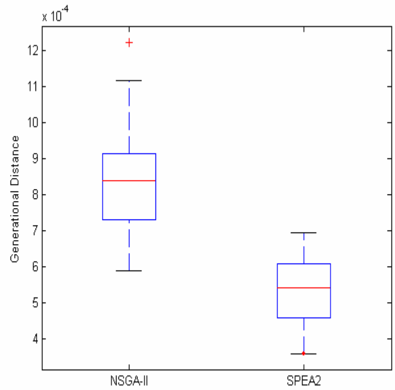


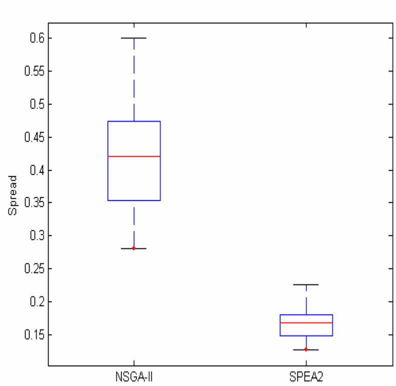

Figure 4: 0% and 100% attainment surface plots for NSGA-II and SPEA2 for Case 1

Table 2: Performance metrics for deterministic Case 2

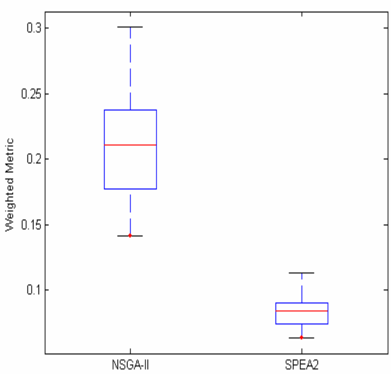
| | Generational Distance, *GD* | | Spread, $\Delta$ | | Weighted Metric, *W* | | Time (s) | |
|---|---|---|---|---|---|---|---|---|
| | NSGA-II | SPEA2 | NSGA-II | SPEA2 | NSGA-II | SPEA2 | NSGA-II | SPEA2 |
| Mean | 0.000834 | 0.000536 | 0.427368 | 0.168292 | 0.214101 | 0.084414 | 68.943667 | 87.024333 |
| SD | 0.000142 | 0.000093 | 0.078230 | 0.024900 | 0.039092 | 0.012442 | 0.096149 | 0.378643 |
| Min | 0.000589 | 0.000358 | 0.280877 | 0.126628 | 0.140997 | 0.063589 | 68.550000 | 86.290000 |
| Max | 0.001221 | 0.000693 | 0.600633 | 0.225991 | 0.300786 | 0.113200 | 69.110000 | 87.760000 |



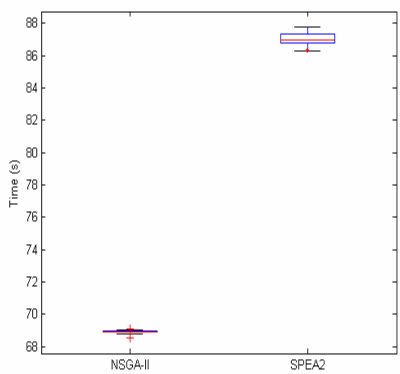

(a)




(b)




(c)




(d)

Figure 5: Box plots of performance metrics for Case 2

(a) Generational Distance (b) Spread (c) Weighted Metric (d) Time

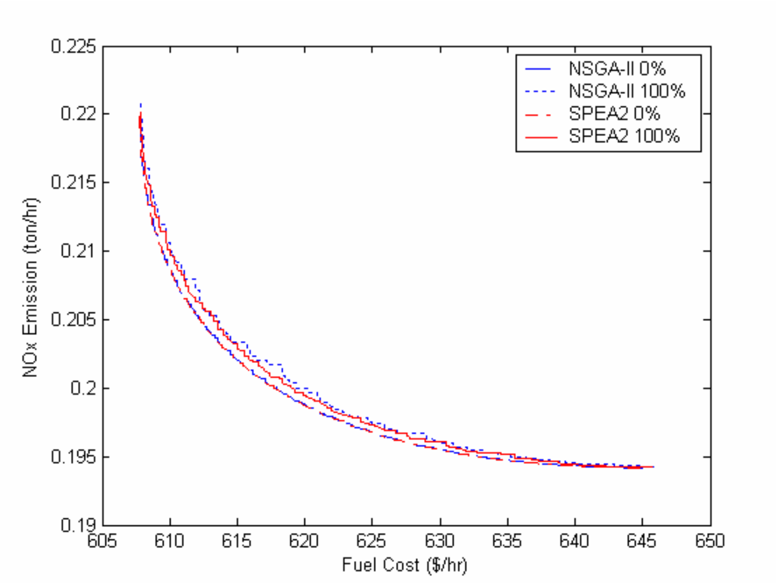



Figure 6: 0% and 100% attainment surface plots of NSGA-II and SPEA2 for Case 2

It can be observed from the tables and box plots that the solutions obtained using SPEA2 are better both in terms of convergence and diversity (and combined weighted metric) than those obtained by NSGA-II in both deterministic cases. In fact, solutions by SPEA2 have significantly better diversity than NSGA-II ones due to the better diversity maintenance strategy of SPEA2. In addition, SPEA2 has a better distributing ability that may help convergence. However, SPEA2 is more computationally expensive to run than NSGA-II and is therefore more time consuming (average of 15 s) in the simulation runs as shown by Case 1. This observation has also been highlighted in Deb et al. (2003). In Case 2, the additional computation time is due to executing the loadflow algorithm for each candidate solution. Nevertheless, there is on average an average additional time of 18 seconds taken by SPEA2 to complete a run compared to NSGA-II. It is also interesting to note that the computational time of NSGA-II has a much smaller standard deviation than SPEA2 in both cases, the larger variation of the latter algorithm being attributed to its archiving technique. The box plots show that SPEA2 is more consistent in terms of both convergence and diversity at the expense of computational time to NSGA-II. It should be pointed out that the 0% attainment surface plots for NSGA-II and SPEA2 in both cases considered are practically similar as observed in Figures 4 and 6.

A further investigation was carried using tools for statistical comparison of multiobjective optimizers (mostats5.c) by Corne and Knowles (1999) to compare the solutions obtained by NSGA-II and SPEA2 on the 30 runs. According to Corne and Knowles (1999), the output of the tools is a 2 by $N$ matrix, where $N$ is the number of objectives (here, $N$=2). Entry $i$ in the first row gives the percentage of the space on which algoirthm $i$'s performance is unbeaten by any of the other algorithms compared, that is, the percentage of the fitness space for which we cannot be 95% confident (based on a non-parametric test - The Mann-Whitney Rank test) that any other algoirthm beat it. Entry $i$ in the second row gives the percentage of the space on which we can be 95% confident that algorithm $i$ beats all of the other algorithms compared.

The outputs of mostats5.c obtained are as follows:

<u>Case 1</u>

| | |
|---|---|
| 32.1 | 100 |
| 0 | 67.9 |

<u>Case 2</u>

| | |
|---|---|
| 44.5 | 100 |
| 0 | 55.5 |

Thus, for Case 1, SPEA2 was not beaten anywhere in the objective space by NSGA-II and SPEA2 beat NSGA-II on 67.9% of the space for sure. NSGA-II performed well being unbeaten on 32.1% of space, however this was not enough to confidently beat SPEA2. For Case 2, again SPEA2 was not beaten anywhere in the objective space by NSGA-II and SPEA2 beat NSGA-II on 55.5% of the space for sure. In this case, however, the performance of NSGA-II was relatively better, being unbeaten by SPEA2 on 44.5% of the space.

## 7. CONCLUSIONS

The multiobjective environmental/economic dispatch for two cases of the IEEE 30-bus system has been solved using two state of the art multiobjective evolutionary algorithms: NSGA-II and SPEA2. The two algorithms have been compared using normalized values of the objectives by generational distance as convergence metric, spread as diversity metric, weighted convergence and diversity metrics with equal weightage for each metric, and actual computational times. In addition, tools for statistical comparison of multiobjective optimizers have been applied to complete the analysis. It is found that the nondominated solutions obtained by SPEA2 are better than NSGA-II both in terms of convergence and diversity but at the expense of computational time. The difference in computational time is one magnitude higher for SPEA2 for this particular problem as demonstrated in the case without transmission losses.

## 8. REFERENCES

ABIDO, M. A., 2001. A new multiobjective evolutionary algorithm for environmental/economic dispatch. *In: IEEE Power Engineering Society Summer Meeting* **2**, 1263-1268.

ABIDO, M. A., 2003a. A novel multiobjective evolutionary algorithm for environmental/economic power dispatch. *Electric Power Systems Research* **65** (1), 71-81.

ABIDO, M. A., 2003b. A niched Pareto genetic algorithm for multiobjective environmental/economic dispatch. *Electrical Power and Energy Systems* **25** (2), 97-105.

ABIDO, M. A., 2003c. Environmental/economic power dispatch using multiobjective evolutionary algorithms. *IEEE Transactions on Power Systems* **18** (4), 1529-1537.

ABIDO, M. A., 2006. Multiobjective evolutionary algorithms for electric power dispatch problem. *IEEE Transactions on Evolutionary Computation* **10** (3), 315-329.

AGRAWAL, S., PANIGRAHI, B. K. AND TIWARI, M. K., 2008. Multiobjective particle swarm algorithm with fuzzy clustering for electrical power dispatch. *IEEE Transactions on Evolutionary Computation* **12** (5), 529-541.

AH KING, R. T. F., RUGHOOPUTH, H. C. S. AND DEB K., 2005. Evolutionary multi-objective environmental/economic dispatch: Stochastic versus deterministic approaches. *In:* C. A. COELLO COELLO, A. HERNANDEZ AGUIRRE and E. ZITZLER, eds. *Third International Conference on Evolutionary Multi-Criterion Optimization (EMO 2005), 9-11 March 2005, Guanajuato, Mexico*. Lecture Notes in Computer Science 3410, Berlin Heidelberg: Springer-Verlag, 677-691.

BANSAL, R. C., 2005. Optimization methods for electric power systems: An overview. *International Journal of Emerging Electric Power Systems* **2** (1), Article 1021, 1-25.

BANSAL, R. C., 2006. A bibliographical survey of evolutionary computation applications in power systems (1994-2003). *International Journal of Power and Energy Systems* **26** (3), 216-225.

CORNE, D. AND KNOWLES, J., 1999. *mostats5.c - Tools for statistical comparison of multiobjective optimizers*. Available from:

http://dbkgroup.org/knowles/multi/mostats5.c [Accessed 15 March 2006]

DAS, D. B. AND PATVARDHAN, C., 1998. New multi-objective stochastic search technique for economic load dispatch. *IEE Proceedings. C, Generation, Transmission, and Distribution* **145** (6), 747–752.

DEB, K. (2001). Multi-Objective Optimization using Evolutionary Algorithms. John Wiley & Sons Ltd., Chichester.

DEB, K. AND AGRAWAL, R. B., 1995. Simulated binary crossover for continuous search space, *Complex Systems* **9**, 115-148.

DEB K. AND AGRAWAL, S., 1999. A niched-penalty approach for constraint handling in genetic algorithms. *Fourth International Conference on Neural Networks and Genetic Algorithms* (*ICANNGA-99), 6-9 April 1999, Portoroz, Slovenia.*

DEB, K., AGRAWAL, S., PRATAP, A. AND MEYARIVAN, T., 2000. A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II, *Parallel Problem Solving from Nature VI Conference, 16-20 September 2000, Paris, France*. 849-858.

DEB, K., MOHAN, M. and MISHRA, S. (2003). Towards a quick computation of well-spread Pareto-optimal solutions. *Proceedings of the Second Evolutionary Multi-Criterion Optimization (EMO-03) Conference*, 8-11 April, Faro, Portugal. 222-236. (Also Lecture Notes in Computer Science (LNCS) 2632).

DEB, K., PRATAP, A. AGRAWAL S. AND MEYARIVAN, T., 2002. A fast and elitist multiobjective genetic algorithm: NSGA-II, *IEEE Transactions on Evolutionary Computation*, **6** (2), 182-197.

FONSECA, C. M., GRUNERT DA FONSECA, V. AND PAQUETE, L., 2005. Exploring the performance of stochastic multiobjective optimisers with the second-order attainment function, *In:* C. A. COELLO COELLO, A. HERNANDEZ AGUIRRE and E.

ZITZLER, eds. *Third International Conference on Evolutionary Multi-Criterion Optimization (EMO 2005), 9-11 March 2005, Guanajuato, Mexico*. Lecture Notes in Computer Science 3410, Berlin Heidelberg: Springer-Verlag, 250-264.

KNOWLES, J. D., THIELE, L. AND ZITZLER, E., 2006. A tutorial on the performance assessment of stochastic multiobjective optimizers. *TIK-Report No. 214, Computer Engineering and Networks Laboratory, ETH Zurich, Switzerland*, 1-34.

NARA, K., 2000. State of the arts of the modern heuristics Application to power systems. *IEEE Power Engineering Society Winter Meeting* **2**, 1279-1283.

OKABE, T., JIN, Y. AND SENDHOFF, B., 2003. A critical survey of performance indices for multi-objective optimization. *In: Congress on Evolutionary Computation (CEC'2003), 8-12 December 2003, Canberra, Australia*, 2, 878-885.

PEREZ GUERRERO, R., 2004. *Differential Evolution based Power Dispatch Algorithms*. Master of Science Thesis, University of Puerto Rico, Mayaguez Campus.

SCHOTT, J. R., 1995. *Fault Tolerant Design Using Single and Multi-Criteria Genetic Algorithms*. Master's Thesis, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Boston, MA.

SRINIVASAN D. AND TETTAMANZI, A. G. B., 1997. An evolutionary algorithm for evaluation of emission compliance options in view of the clean air act amendments. *IEEE Transactions on Power Systems*, 12 (1), 336-341.

TALAQ, J. H., EL-HAWARY, F. AND EL-HAWARY, M. E., 1994a. A summary of environmental/economic dispatch algorithms. *IEEE Transactions on Power Systems*, 9 (3), 1508-1516.

VAN VELDHUIZEN, D., 1999. Multiobjective evolutionary algorithms: Classifications, analyzes, and new innovations, Air Force Inst. Technol., Dayton, OH, Tech. Rep. AFIT/DS/ENG/99-01, 1999.

VICTOIRE, T. A. A. AND SUGANTHAN, P. N., 2007. Improved MOCLPSO algorithm for environmental/economic dispatch. *In: IEEE Congress on Evolutionary Computation, CEC 2007, 25-28 September 2007, Singapore*, 3072-3076.

WANG, L. AND SINGH, C., 2007. Environmental/economic power dispatch using a fuzzified multi-objective particle swarm optimization algorithm. *Electric Power Systems Research*, 77 (12), 1654-1664.

WOOD, A. J. AND WOLLENBERG, B. F., 1996. *Power Generation, Operation, and Control*. John Wiley & Sons, Inc.

WRIGHT, A. H., 1991. Genetic algorithms for real parameter optimization, *In:* J. E. RAWLINS, ed. *Foundations of Genetic Algorithms*. Morgan Kaufmann, 205-218.

ZHAO B. AND CAO Y.-J., 2005. Multiple objective particle swarm optimization technique for economic load dispatch. *Journal of Zhejiang University SCIENCE*, 6A (5), 420-427.

ZITZLER, E., LAUMANNS, M. AND THIELE, L., 2001. *SPEA2: Improving the Strength Pareto Evolutionary Algorithm*. TIK Report 103, Computer Engineering and Networks Laboratory (TIK), ETH Zurich, Zurich, Switzerland.